

# Investigation of Temperature Effects on the Salt Penetration in Groundwater

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**Abstract:** Groundwater is the water located in the soil porous matrix or in the fractures of rock formations. Many different factors affected the quality of the groundwater. One of the effective factors on groundwater quality is the vicinity of pollution source to the aquifer.

In this paper, transport of salt in groundwater sources is numerically investigated. A horizontal porous matrix saturated with water is selected to simulate the double diffusive natural convection effects in mass transfer rate. Darcy model is used for prediction of velocity field in groundwater source. It is shown that temperature difference between salt and water source accelerate the transport of pollutants.

**Key words:** Salt penetration, underground water, temperature.

## Introduction

Groundwater contamination or groundwater pollution occurs when contaminants, pollutants, unwanted constituent, or impurity make their way in soil and release to the underground and into groundwater. Many natural materials such as soil, rocks, zeolites, tissues, and man-made substances such as cements and ceramics can be categorized as porous media. Therefore the concept of porous media is used in pollution transport in groundwater.

Transport phenomena in porous media are widely investigated by researchers. In 1977, Chang examined mass flux effects on heat transfer. He demonstrated that thermal boundary layer thickness increases as a function of injection rate.

In 1984, Cheng and Pop studied unsteady laminar flow equation over a vertical flat plate embedded in anisotropic porous media. They showed that increasing the permeability in part of porous medium increases temperature field in such domain.

In 1986, Ingham and Brown studied natural convection boundary layer over a vertical flat plate submerged in a porous media under a heat flux applied to the surface as a function of direction ( $x^\lambda$ ,  $\lambda = \text{constant}$ ). They showed that the velocity and temperature at the large distance of the wall depend on the value of  $\lambda$ .

In 2005, Vadasz et al. investigated a transition form of Darcy regime for prediction of flow behaviour under natural convection in porous media.

In 2007, Saeed et al. numerically studied combination of natural convection and conduction in a finite two-dimensional porous media. They found that average Nusselt number increases by increasing the Rayleigh number and thermal conductivity or decreasing the wall thickness.

In 2008, Hooman and Gurgenci studied fluid flow and convective heat transfer by considering variable viscosity in porous medium. They used gas and liquid as the fluid to evaluate the effect of viscosity on flow regime and heat transfer pattern.

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In 2009, Basak et al., studied free convection on the trapezoidal porous media isolated from top, heated uniformly from below and linearly from the sides in order to obtain isothermal lines, flow function and Nusselt number.

In 2009, Baytas et al., evaluated double diffusive natural convection passing over a flat plate embedded in saturated porous media by assuming non-Darcy flow. They demonstrated that Rayleigh number and Darcy number have significant effects on heat and mass transfer rate.

In 2014, Bera et al., numerically investigated the effect of thermal non-equilibrium state on double-diffusive natural convection in a saturated square porous medium. They employed Darcy-Brinkman-Forchheimer model for prediction of velocity profile in the cavity. They illustrated that the influence of the presence of Brinkman term in the momentum equation is negligible for low permeable media.

In 2015, Mondal and Sibanda investigated the effects of buoyancy ratio on double-diffusive natural convection in a saturated porous medium under uniform and non-uniform boundary conditions. They studied a rectangular enclosure that the two crossover walls were concentrated and heated, while the top wall is insulated and the right vertical wall is under cold temperature. They numerically evaluated average heat and mass transfer.

In the recent years, Urmia salt lake located in North West of Iran is going to dry. So the vast amount of salt remains on the surface of ground. The remained salt gradually penetrates to the underground water, so they become salty. In this paper, effect of warming of remained salt due to shining the sun on mass transfer rate is investigated.

### Governing Equations

Incompressible, 2-D, double diffusive natural convection in a cavity filled with saturated, isotropic porous medium is studied numerically. It is assumed that the solid matrix and the fluid are in local thermal equilibrium. The schematic of simulated media is shown in Figure 1. The system is a square of height and width  $L$ , filled with a saturated porous medium with binary fluid. Left and right vertical boundaries are exposed to the uniform concentration and temperature  $T_1, C_1$  and  $T_2, C_2$ , respectively ( $T_1 > T_2$  and  $C_1 > C_2$ ). The top and bottom walls are insulated, adiabatic and impermeable. It is assumed that the physical properties are constant

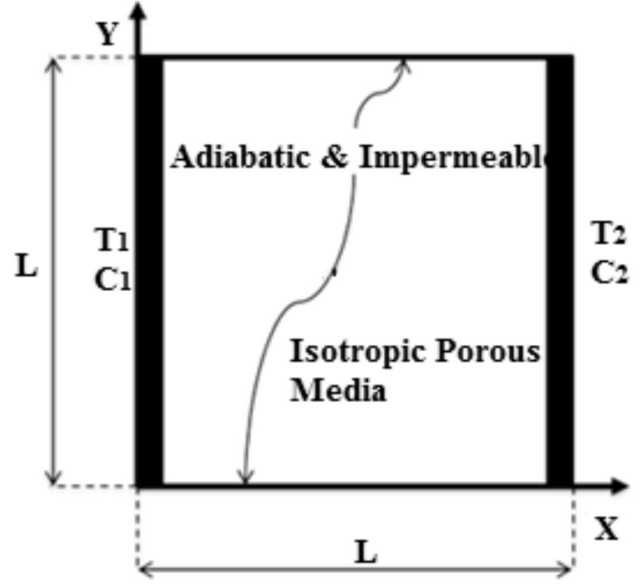


Figure 1: Schematic of the simulated porous medium and its boundary conditions.

in the system except density in buoyancy force term (Oberbeck-Boussinesq approximation):

$$\rho = \rho_0 [1 - \beta_T(T - T_0) - \beta_C(C - C_0)] \quad (1)$$

where  $\rho$  is the density and  $\rho_0$  is the reference density at reference temperature and concentration ( $T_0$  and  $C_0$ ),  $\beta_T$  and  $\beta_C$  are the thermal and concentration expansion coefficients, respectively:

$$\beta_T = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)_{P,C} \quad \beta_C = -\frac{1}{\rho} \left( \frac{\partial \rho}{\partial C} \right)_{P,T} \quad (2)$$

As suggested by Nield and Bejan (2006), the equations that govern the conservation of mass, momentum and energy can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u = \frac{K}{\mu} \frac{\partial p}{\partial x} \quad (4)$$

$$v = \frac{K}{\mu} \frac{\partial p}{\partial y} + \rho g \quad (5)$$

$$\alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \quad (6)$$

$$D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) = u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \quad (7)$$

where  $g$  is the gravity acceleration,  $K$  the permeability of the porous media,  $D$  the mass diffusivity coefficient,

$\nu$  the kinematic viscosity of the fluid,  $\Delta T$  and  $\Delta C$  are the differences of temperature and concentration.

The initial and boundary conditions are presented as follow:

$$\left\{ \begin{array}{ll} t = C = 0; u = v = 0 & \text{for } t = 0, \\ \frac{\partial T}{\partial x} = 0; \frac{\partial C}{\partial x} = 0; u = v = 0 & \text{for } y = 0, \\ \frac{\partial T}{\partial x} = 0; \frac{\partial C}{\partial x} = 0; u = v = 0 & \text{for } y = L, \\ T = T_1; C = C_1; u = v = 0 & \text{for } x = 0, \\ T = T_2; C = C_2; u = v = 0 & \text{for } X = L, \end{array} \right. \quad (8)$$

Nusselt and Sherwood dimensionless numbers are defined as follows:

$$Nu = -\int_0^1 \left( \frac{\partial T}{\partial x} \right)_{x=0} dy \quad Sh = -\int_0^1 \left( \frac{\partial C}{\partial x} \right)_{x=0} dy \quad (9)$$

In order to solve the above equations, the following dimensionless variables are employed to convert the governing equations to the dimensionless form:

$$x = \frac{x}{L}, Y = \frac{y}{L}, \tau = \frac{t}{\left( \frac{\sigma L^2}{\alpha} \right)}, V = \frac{v}{\left( \frac{\alpha}{L} \right)}, U = \frac{u}{\left( \frac{\alpha}{L} \right)},$$

$$P = \frac{(P - P_0)}{\left[ \rho f \left( \frac{\alpha^2}{L^2} \right) \right]}, \theta = \frac{T - T_1}{T_2 - T_1}, \phi = \frac{C - C_1}{C_2 - C_1}, \alpha = \frac{K_m}{(\rho c)_f} \quad (10)$$

So the equations and boundary conditions will become:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (11)$$

$$U = - \frac{Da}{pr} \frac{\partial P}{\partial X} \quad (12)$$

$$V = \frac{Da}{pr} \frac{\partial P}{\partial Y} \text{ Ra. Pr. } \theta + N. \text{ Ra. Sc. } \phi \quad (13)$$

$$\left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} \quad (14)$$

$$\frac{1}{Le} \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) = U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} \quad (15)$$

And corresponding dimensionless initial and boundary conditions reduce to:

$$\left\{ \begin{array}{ll} \theta = \phi = 0; U = V = 0 & \text{for } t = 0, \\ \frac{\partial \theta}{\partial x} = 0; \frac{\partial \phi}{\partial x} = 0; U = V = 0 & \text{for } Y = 0, \\ \frac{\partial \theta}{\partial x} = 0; \frac{\partial \phi}{\partial x} = 0; U = V = 0 & \text{for } Y = 1, \\ \theta = 0; \phi = 0; U = V = 0 & \text{for } X = 0, \\ \theta = 1; \phi = 1; U = V = 0 & \text{for } X = 1, \end{array} \right. \quad (16)$$

where:

$$N = \frac{(\beta_s \Delta C)}{(\beta_t \Delta T)}, Da = \frac{K}{H^2}, Le = \frac{\alpha}{D},$$

$$Pr = \frac{\nu}{\alpha}, Ra = \frac{g \beta_T \Delta T H^3}{\alpha \nu} \quad (17)$$

where  $N$  is the buoyancy ratio,  $Da$  is Darcy number,  $Le$  is Lewis number,  $\nu$  the kinematic viscosity of the fluid,  $\Delta T$  and  $\Delta C$  are the differences of temperature and concentration.

## Solution Method

In order to solve the governing equations, the above equations are discretized according to central finite difference scheme, and then simultaneously solved in MATLAB. In order to verify the validity of results, obtained Nusselt numbers in present modelling are compared with those in literature. Table 1 shows the comparison of results. It is seen that the results have good agreement with the results of literature.

**Table 1: Comparison of Nusselt numbers obtained in the study with the results presented in the literature**

Ra	Nu number	
	(present study)	(Trevisan and Bejan, 1986)
20	1.51	1.29
40	1.79	1.61

## Results and Discussion

Figures 2 and 3 show the temperature and concentration profile in the rectangular porous medium. It is clear that heat and mass transfer have similar trends; however the purpose of present study is investigating the effect of temperature difference value on mass transfer rate. Therefore, average  $Nu$  and  $Sh$  numbers are plotted according to  $Ra$  number. Figure 4 shows the variation of Nusselt number and Sherwood number verses  $Ra$

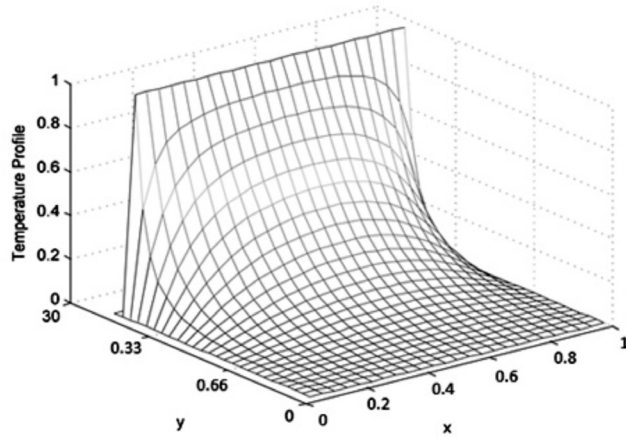


Figure 2: Temperature profile in the porous media.

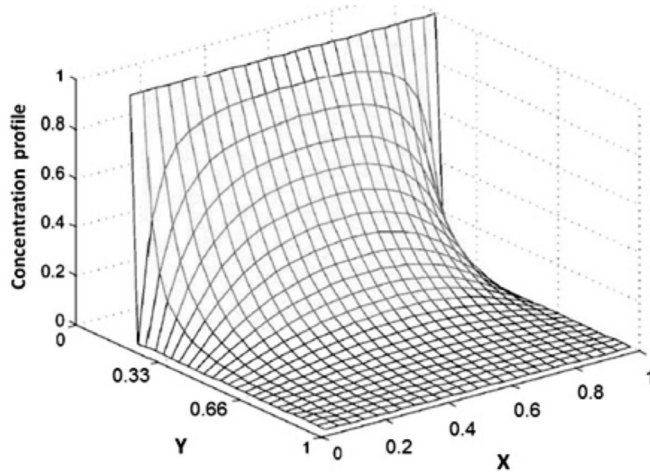


Figure 3: Concentration profile in the porous media.

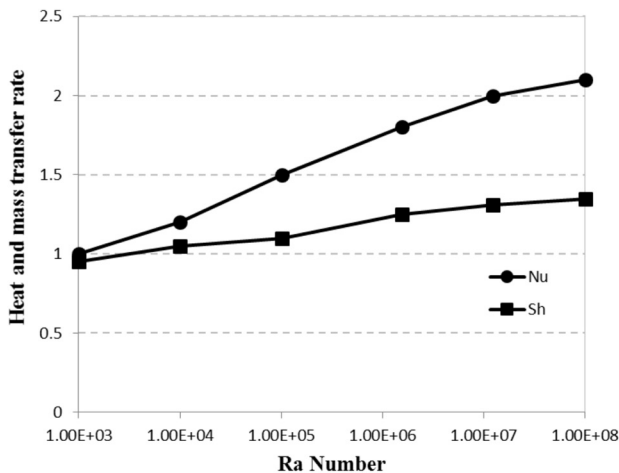


Figure 4: The heat and mass transfer rate versus. thermal Ra number.

number. As it is expected, increasing the thermal Ra number increases the Nusselt number. In addition, increasing the Ra number increases the mass transfer rate. In other words, existence of heat transfer intensify mass transfer rate. For the current study, the results show that increasing the temperature of salt (due to sunshine) on the surface cause the penetration rate of salt into underground water rise.

## Conclusion

A two-dimensional solution of natural convection in a rectangular porous medium is numerically analyzed. The concentration and temperature are applied to the vertical walls, while the top and bottom walls are insulated. Darcy model is employed to predict the velocity field. The governing equations are solved using the finite difference scheme. It is assumed that the porous matrix and the saturated fluid are in thermal equilibrium. It is observed that increasing the temperature difference between salt source and underground water increases the salt penetration into underground water sources.

## References

- Basak, T., Roy, S., Singh, A. and B.D. Pandey (2009). Natural convection flow simulation for various angles in a trapezoidal enclosure with linearly heated side wall(s). *International Journal of Heat and Mass Transfer*, **52**: 4413–4425.
- Baytas, A.C., Baytas, A.F., Ingham, D.B. and I. Pop (2009). Double diffusive natural convection in an enclosure filled with a step-type porous layer: Non-Darcy flow. *International Journal of Thermal Sciences*, **48**: 665–673.
- Beg, O.A., Takhar, H.S., Soundalgekar, V.M. and V. Prasad (1998). Thermoconvective flow in a saturated, isotropic, homogeneous porous medium using Brinkman's model—Numerical study. *Int. J. Numer. Methods Heat Fluid Flow*, **8**: 559–589.
- Bera, P., Pippal, S. and A.K. Sharma (2014). Thermal non-equilibrium approach on double-diffusive natural convection in a square porous-medium cavity. *International Journal of Heat and Mass Transfer*, **78**: 1080–1094.
- Cheng, P. (1977). The influence of lateral mass flux on free convection boundary layers in a saturated porous medium. *Int. J. Heat Mass Transfer*, **20**: 201–206.
- Cheng, P. and W.J. Minkowycz (1977). Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. *J. Geophys. Res.*, **82**: 2040–2044.

- Cheng, P. and I. Pop (1984). Transient free convection about a vertical flat plate imbedded in a porous medium. *Int. J. Engng. Sci.*, **22**: 253–264.
- Ghasemi, B. and S.M. Aminossadati (2009). Periodic natural convection in a nanofluid-filled enclosure with oscillating heat flux. *International Journal of Thermal Sciences*, 1–9.
- Gorl, R.S.R. and M. Kumari (1999). Nonsimilar solutions for mixed convection in non-Newtonian fluids along a wedge with variable surface temperature in a porous medium. *Int. J. Numer. Meth. Heat Fluid Flow*, **9**: 601–611.
- Hooman, K. and H. Gurgenci (2008). Effects of temperature-dependent viscosity on Be'nard convection in a porous medium using a non-Darcy model. *International Journal of Heat and Mass Transfer*, **51**: 1139–1149.
- Ingham, D.B. and S.N. Brown (1986). Flow past a suddenly heated vertical plate in a porous medium. *Proc. Roy. Soc. London Ser. A*, **403**: 51–80.
- Krishna, D.J., Basak, T. and S.K. Das (2009). Natural convection in a non-Darcy anisotropic porous cavity with a finite heat source at the bottom wall. *International Journal of Thermal Sciences*, **48**: 1279–1293.
- Nield, D.A. and A. Bejan (2006). *Convection in porous media*. Springer Inc.
- Mondal, S. and P. Sibanda (2015). Effects of buoyancy ratio on unsteady double-diffusive natural convection in a cavity filled with porous medium with non-uniform boundary conditions. *International Journal of Heat and Mass Transfer*, **85**: 401–413.
- Saeid, N.H. (2007). Conjugate natural convection in a porous enclosure: Effect of conduction in one of the vertical walls. *International Journal of Thermal Sciences*, **46**: 531–539.
- Straughan, B. (2002). Effect of property variation and modelling on convection in a fluid overlying a porous layer. *Int. J. Numer. Anal. Meth. Geomech.*, **26**: 75–97.
- Sultana, Z., Hyder, Md. N. (2007). Non-darcy free convection inside a wavy enclosure. *International Communications in Heat and Mass Transfer*, **34**: 136–146.
- Vadasz, J.J., Roy-Aikins, J.E.A. and P. Vadasz (2005). Sudden or smooth transitions in porous media natural convection. *International Journal of Heat and Mass Transfer*, **48**: 1096–1106.
- Wong, K.C. and N.H. Saeid (2009). Numerical study of mixed convection on jet impingement cooling in a horizontal porous layer under local thermal non-equilibrium conditions. *International Journal of Thermal Sciences*, **48**: 860–870.
- Trevisan O.V. and A. Bejan (1986). Mass and heat transfer by natural convection in a vertical slot filled with porous medium. *Int. J. Heat Mass Trans.*, **29(3)**: 403–415.