Asian Journal of Water, Environment and Pollution, Vol. 15, No. 2 (2018), pp. 127–140.
DOI 10.3233/AJW-180025

An Approach to Select the Optimum Rock Failure Criterion for Determining a Safe Mud Window through Wellbore Stability Analysis

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Received August 6, 2017; revised and accepted January 20, 2018

Abstract: Instability of a well means for whatever reason, the well shape deformed from its circular condition, which may be accompanied by collapse or fracture initiation. The problem of the instability of the well in the production and exploration wells imposed extra costs in the drilling projects; so the proper predictions to prevent instability are imperative. Several analytical and numerical methods have been proposed to analyze the stability of wellbores so far, but in this paper, an analytical study was presented to determine the appropriate rock failure criterion through wellbore stability analysis. Two important factors affecting the analyses are failure criteria and rock behaviour. In this paper, with regard to attain the most suitable failure criterion, 15 rock failure criteria, including Hoek-Brown and Mohr-Coulomb, were compared with the achieved data from 14 different rock samples and finally, the modified Lade criterion with standard deviation of 2% was considered as the most accurate equation. By integrating this failure criterion through the stresses analysis around the well that is in accordance with the Kirsch analytical equations, a new equation was developed to predict the minimum weight of the mud to prevent the wellbore collapse.

In order to verify the model, the equation is compared with the actual data of two wells: one in Norouz field in Iran and another one in the field in Indonesia. In both cases, only the modified Lade equation predicted the proper mud weight to keep stability of the well, correctly. In well No. 34 of the Norouz field, the outcast caliper log and the diameter of the well were recorded, as well as those parts of the wellbore which were unstable. The results showed that the Modified Lade criterion can be viewed as most accurate equation and Hoek-Brown, Mohr-Coulomb and Mogi-Coulomb criteria regarded as the most useful equations, predicting the unstable points. The findings demonstrated that the only modified Lade criterion defines the unstable points adequately. In the other case in shale formation, drilling operations in 4800 to 6200 feet depth with the mud weight of 5.10 pound per gallon in Indonesia, the instability was not reported. According to Mohr-Coulomb criterion, the wellbore was meant to be instable while these criteria show the suitable mud weight is found 1.2 lb/gal more than current weight in which there is the possibility of fluid lost. Using Mogi-Coulomb criterion through the analyses, the wellbore was found stable at the deviation of more than 35 degrees. But according to modified Lade criteria, the well is found stable in any dip in azimuth, which coincided with the case in real terms.

Key words: Failure criterion, mud window, wellbore, stability analysis.

Introduction

Wellbore stability analysis plays a major role in successful drilling operations so that a well with no stability problem has a great influence on the productivity of drilling operations. In the process of drilling oil and gas wells, one of the main goals is to drill at the lowest possible cost, which contradicts the instability of the

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wellbore wall. Instability of wellbores can impose costs to projects around 100 million dollars in all over the world which could rise to one billion dollars (Aadnoy and Ong, 2003). Improper mud window design could lead to collapse or fracture in the well. Millions of dollars are spent annually to tackle the wellbores instability in different formations. Various methods have been presented for analyzing the stability of wells by different petroleum companies, which, together with various software, try to reduce the time and cost of the wellbore instability problem. In mud window design, since all rock failure criteria cannot be suitable for all conditions and rocks solely, engineers should be able to choose a suitable failure criterion based on a given rock formation characteristics to predict the drilling direction and optimal mud pressure. More than 20 different failure criteria such as Mohr-Coulomb, Hoek-Brown, Mogi-Coulomb, modified lade, Drucker-Prager etc. have been developed to predict rock failure, but so far there is few comprehensive comparison between the accuracy of these failure criteria, the task which is investigated in this study.

In order to compare the precision of different criteria, it is necessary to establish a database from laboratory tests data published in the literature. In order to accurately recognize and predict the factors of the well wall instability during drilling and to prevent them, a universal model which stems from whole gathered data is needed. The model should consist of geological data, in-situ stresses, formation geomechanical properties and pore pressure, which is used as a tool to rapid interpretation of the field condition. In some cases, the Mohr-Coulomb criterion with a high error rate, or the Mogi-Coulomb criterion with relatively less error is used for such analysis, which may not provide acceptable results. Therefore, the well stability analysis needs to be carried out based on the most accurate criteria in accordance with the site condition.

Wang (2007) used a finite element method to analyze the stability of a horizontal well that was drilled in poor sandstones, considering the effect of the role of mud cake. Salehi (2010) also studied the stability of wellbore in under-balanced drilling and used finite element and difference finite methods to verify the results. Lee (2011) also applied a numerical finite element method for assessing the stability of a wellbore. There are many wellbore stability analyses using numerical methods so far, but the number of articles that have been merely analytic to this issue are limited. Due to diversity of results achieved from various failure criteria, choosing the appropriate failure criterion to examine wellbore stability is a complex and difficult task (McLean and Addis, 1990). Among rock failure criteria, because of generality and liability of Mohr-Coulomb and Hoek-Brown criteria, these two criteria are first investigated.

In addition to these the most commonly-used criteria, 13 other criteria are also studied in order to achieve the optimal rock failure criterion, which is described in the following sections. Therefore, in this research, it has been tried to choose an approach to select the optimal failure criterion in the wellbore stability analysis in such a way that it is the best match with the actual rock failure. In the next step, according to the assumptions considered for rock behaviour, the equations for calculating the radial, axial and tangential stresses around the wellbore are presented. Afterward, the failure criteria for the results of laboratory tests of 14 rock type samples have been investigated and consequently the best fitted rock failure criterion has been proposed. In order to verify the suggested criterion, the results of application of this criterion for stability analysis of two wells at Nowruz field and a field in Indonesia are presented.

Laboratory Tests Database

In this paper, 14 different type of rocks have been used for comparing and fitting the results of different criteria. Due to the fact that sedimentary rocks are normally encountered in the drilling path, the largest number of rock samples are selected from this category. These samples are as six sedimentary rocks including sandstone, limestone, dolomite, shale, siltstone and salt rock, four igneous rocks, granite, granodiorite, trachyte and andesite, and four metamorphic rocks such as amphibole, marble, hornfels and metapelitesn (Table 1).

Triaxial compressive test information for all rocks is extracted from Table 1. For some rocks, the test results are presented numerically in the articles, but in a number of cases (salt rock, granodiorite, siltstone, hornfels and metapelites), the graphs are numerically coded by using the Plot digitizer software.

Determining the Mud Window by Using the Fracture Criterion to Analyze the Stability of the Well

Determination of the Mud Window for Vertical Wells

In drilling, there are two basic problems for the well stability, including tensile fracture and collapse in the well. It can often be prevented by determining the

Table 1: Sources of laboratory tests

Main source	*Number of tests*	*Rock type*
Takahashi and Koide, 1989	42	Sandstone
Mogi, 1971	52	Limestone
Mogi, 1971	45	Dolomite
Takahashi and Koide, 1989	26	Shale
Chang and Haimson, 2000	30	Amphibolite
Mogi, 1971	28	Trachyte
Haimson and Chang, 2000	18	Granite
Mogi, 1971	40	Andesite
Michelis, 1987	35	Marble
Sriapai, Walsri et al. 2013	36	Salt
Haimson, 2009	19	Granodiorite
Haimson, 2009	31	Siltstone
Chang and Haimson, 2005	14	Hornfels
Chang and Haimson, 2005	14	Metapelite

proper weight of the mud. According to the models previously described, an isotropic linear elastic model is used to do this. The reason for using this model is that it requires less input information than other complex models, and can be utilized easily in practical work. According to this model, the greatest concentration of stress occurs on the well wall. As suggested by Kirsch, stresses around the vertical wells are obtained by the following equations (Aadnoy and Looyeh, 2010; Terzaghi, 1943):

$$\sigma_r = P_w \tag{1}$$

$$\sigma_\theta = \sigma_H + \sigma_h - 2(\sigma_H - \sigma_h)\cos 2\theta - P_w \tag{2}$$

$$\sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\theta \tag{3}$$

The θ angle is the angle rather to σ_H (clockwise) and vary from 0 to 360 degrees. According to this equation and Figure 1, the axial and tangential stresses are functions of θ, so it can be said these stresses vary sinus around the well and the maximum magnitude of them is seen at $\theta = \pm\frac{\pi}{2}$ and the minimum magnitude is at θ = 0 and π. This critical location at all magnitude of in-situ stresses are constant, so it can be stated that any fracture or collapse happened at this points.

Also with respect to aforementioned equations and Figures 2 and 3, it is obvious P_w actually is pressure of drilling fluid and only effects radius and tangential stresses and does not change the axial stress. Increasing of mud weight causes increase of radius stress, decreases the tangential stress and leads the well to the fracture; so thanks to modification in mud weight, the magnitude of these stresses can be changed. Whatever the difference of these two stresses be more or in other words the stress concentration be intensive, the probability of well wall instability increases.

If the magnitude of tangential stress be more than radial stress, the maximum probability of collapse occurs at $\theta = \pm\frac{\pi}{2}$. On the contrary, if the radial stress be more than tangential stress, the maximum probability occurs at θ = 0 and π.

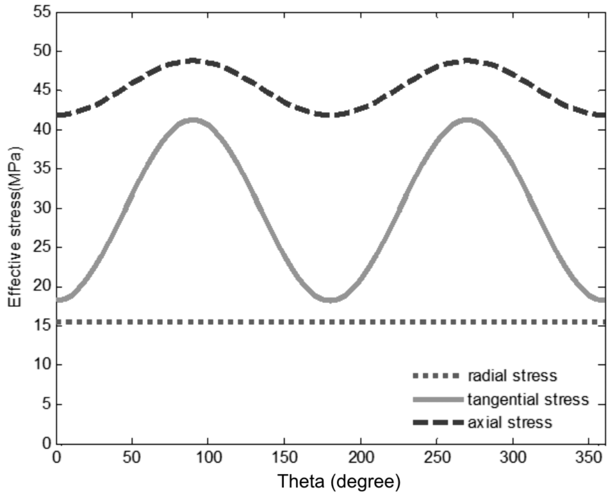


Figure 1: The magnitude of radial, tangential and axial stresses with θ (P_w = 50 MPa).

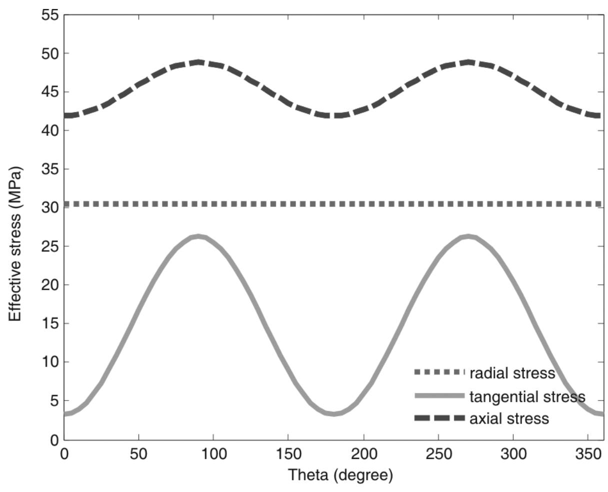


Figure 2: The magnitude of radius, tangential and axial stresses with θ (P_w = 60 MPa).

Figure 4 shows the distribution of radial, tangential and axial stresses with distance from well. This graph is drawn based on Krisch equation and for deviation angle of 30 degrees, azimuth 67 degrees and $\theta = \frac{\pi}{2}$. The radial and tangential stresses vary with the distance far from well collar and at far distance (almost five times of well radius) converge to constant stresses. Axial stress almost is constant and at the distance of 1.5 times of well radius be equal to maximum principal stress. Because at all states the tangential stress is more than radial stress then at $\theta = \frac{\pi}{2}$ stresses be concentrated and the probability of collapse in the well be more than other magnitude of θ.

Figure 5 shows the same well. At this state at the distance of 3.5 times of well radius, the tangential stress be more than the radius stress which at more distance becomes tensile and the probability of wall tensile fracture is more than other magnitude of θ.

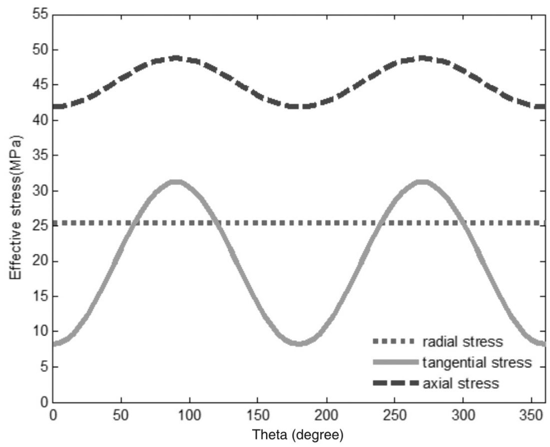


Figure 3: The magnitude of radial, tangential and axial stresses with θ (P_w = 65 MPa).

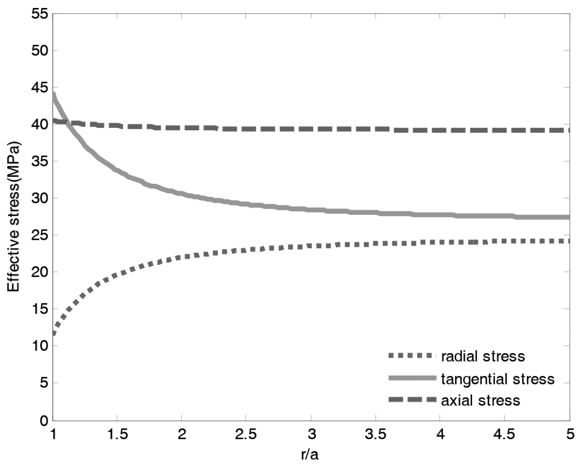


Figure 4: Distribution of stresses with distance from well wall ($\theta = \frac{\pi}{2}$).

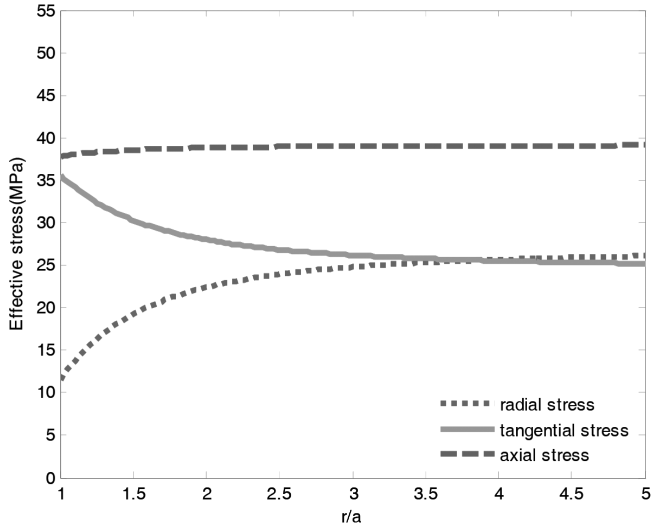


Figure 5: Distribution of stresses with distance from well wall (θ = 0).

Determination of the Mud Window for Horizontal Wells

Thanks to tremendous advances in drilling technology, wells are being drilled at difficult and complicated surrounding situations. At existence of technical challenges, directional drilling is being conducted at many oil fields, the most important reason of that is, increasing of production field radius for more recovery and decreasing the number of required rigs to covering the entire field. Directional drilling without considering the instability of well can result in at least 10% increase in cost of drilling.

For one rock type with linear elastic behaviour, the maximum stress concentration occurred at wellbore wall. The stress around horizontal well can be obtained via Krisch equations:

$$\sigma_r = P_w \tag{4}$$

$$\sigma_\theta = (\sigma_v + \sigma_H \sin^2\alpha + \sigma_h \cos^2 \alpha) - 2\,(\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2 \alpha) \cos 2\theta - P_w \tag{5}$$

$$\sigma_z = \sigma_H \cos^2\alpha + \sigma_h \sin^2 \alpha - 2v\,(\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2 \alpha) \cos 2\theta \tag{6}$$

$$\sigma_{\theta z} = (\sigma_h - \sigma_H) \sin^2\alpha \cos \theta \tag{7}$$

$$\sigma_{r\theta} = 0 \tag{8}$$

$$\sigma_{rz} = 0 \tag{9}$$

The tangential and axial stresses are functions of θ and vary sinus. The wall collapse happens when the tangential stress is maximum. With respect to upside equations, tangential and axial stresses can happen at $\theta = \frac{\pi}{2}$ and $\theta = 0$ and π; so at this magnitude of θ, two states can be considered for tangential stress:

$$\sigma_{\theta-0} = 3\sigma_H \sin^2\alpha + 3\sigma_h \cos^2\alpha - \sigma_v - P_w \tag{10}$$

$$\sigma_{\theta-\pi} = 3\sigma_v - \sigma_H \sin^2\alpha - \sigma_h \cos^2\alpha - P_w \tag{11}$$

In order to see the maximum tangential stress at $\theta = 0$, it is required to satisfy below condition:

$$\sigma_{\theta-0} - \sigma_{\theta-\pi} \geq 0 \tag{12}$$

Substituting the equations, we reach to:

$$\sigma_v \leq \sigma_H \sin^2\alpha + \sigma_h \cos^2\alpha \tag{13}$$

With respect to the range of σ_H/σ_h is 1 to 2 and α varies from 0 to 90 degree, the vertical stress must be less than horizontal stress so at normal stress regime. This state is impossible; in other words, the maximum tangential stress at normal stress regime happens at $\theta = \frac{\pi}{2}$.

At the other hand, in order to see the maximum tangential stress at $\theta = \frac{\pi}{2}$, the below condition is necessary to satisfy:

$$\sigma_{\theta-\pi} - \sigma_{\theta-0} \geq 0 \tag{14}$$

With substituting equation, this result gained:

$$\sigma_v \geq \sigma_H + (\sigma_h - \sigma_H)\cos^2\alpha \tag{15}$$

Therefore, according to cited terms, the vertical stress must be more than horizontal stress. That is why in the reversed stress regime, this state is impossible. It means the maximum tangential stress in reversed state happens at $\theta = 0$.

Determination of the Mud Window for Deviated Wells

The stresses on the wall of a deviated well can be calculated by using Kirsch equations in terms of radial, tangential and axial stresses. The minimum limits of mud window for vertical and horizontal wells are obtained from comparing relevant equations with failure criteria. Although the axial and tangential stresses cannot be substituted with principal stresses because the magnitude of $\sigma_{\theta z}$ is not zero.

Before replacing at failure criteria this stress must be zero. If the normal and shear stresses on a surface appears with angle of θ and considered on x-y plane, as the same at Figure 6 shows that the triangle is fix and the total of applied stresses on it is zero. In this case σ and τ can be obtained from:

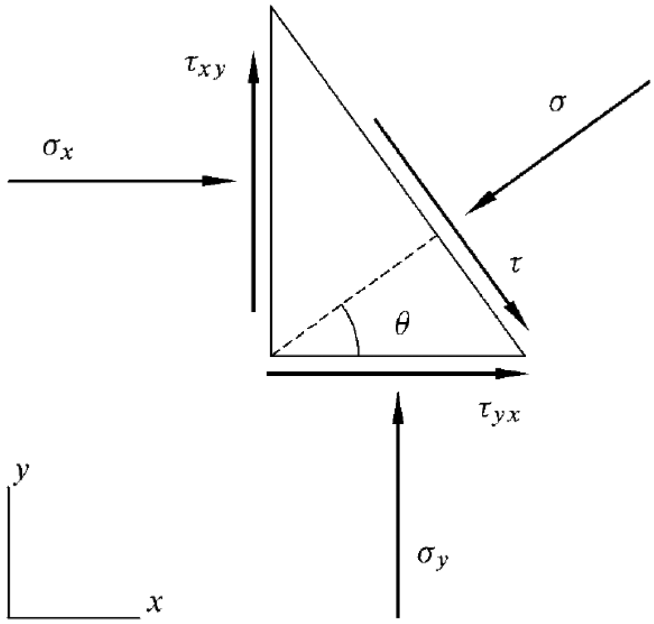


Figure 6: Normal and shear stresses acting on a plane (Fjaer et el., 2008).

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta \tag{16}$$

$$\tau = \sigma_y \sin\theta \cos\theta - \sigma_x \sin\theta \cos\theta + \tau_{xy} \cos^2\theta - \tau_{yx} \sin^2\theta \tag{17}$$

These two equations can be simplified as:

$$\sigma = \frac{1}{2}(\sigma_x + \sigma_y) + ½(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy} \sin 2\theta \tag{18}$$

$$\tau = \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta \tag{19}$$

By choosing proper θ, the magnitude of τ can be zero. As it is obvious in above equation, this happens when:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \tag{20}$$

This equation has two solutions called principal stresses as below:

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\tau_{xy} + \frac{(\sigma_x - \sigma_y)^2}{4}} \tag{21}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy} + \frac{(\sigma_x - \sigma_y)^2}{4}} \tag{22}$$

The principal stresses can be written in terms of radial, tangential and axial stresses as follow:

$$\sigma_r = P_w \tag{23}$$

$$\sigma_2 = \frac{\sigma_{\theta d} - P_w + \sigma_{zz}}{2} - \sqrt{\sigma_{\theta z}^{2} + \frac{(\sigma_{\theta d} - P_w + \sigma_{zz})^2}{4}} \tag{24}$$

$$\sigma_1 = \frac{\sigma_{\theta d} - P_w + \sigma_{zz}}{2} + \sqrt{\sigma_{\theta z}^{2} + \frac{(\sigma_{\theta d} - P_w + \sigma_{zz})^2}{4}} \tag{25}$$

where σ_{zz}, $\sigma_{\theta z}$ and $\sigma_{\theta d}$ are:

$$\sigma_{\theta d} = \sigma_x + \sigma_y - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\sigma_{xy}\sin 2\theta \tag{26}$$

$$\sigma_{zz} = \sigma_z - \nu\left[2(\sigma_x - \sigma_y)\cos 2\theta + 4\sigma_{xy}\sin 2\theta\right] \tag{27}$$

$$\sigma_{\theta z} = 2(\sigma_{yz}\cos\theta - \sigma_{xz}\sin\theta) \tag{28}$$

As it is obvious, the maximum and mean stresses are functions of θ. In vertical and horizontal wells, the angle of θ defined is based on the angle related to horizontal stress; in the same way for deviated wells, it is better the θ angle defined is related to maximum stress concentration. Location of maximum stress concentration obtained from Kaarstad equation (Kaarstad and Aadnoy, 2005):

$$\theta_1 = \frac{1}{2}\arctan\left(\frac{2\sigma_{xy}}{\sigma_x - \sigma_y}\right) \tag{29}$$

$$\theta_2 = \theta_1 + \frac{\pi}{2} \tag{30}$$

Substituting these two θ at equations (26-28) for assessment of stress, the one which represents bigger magnitude of stress, is acceptable and known as the θ_{Max}. With replacing equations (23-25) to failure criteria and substituting of $\theta = \theta_{Max}$, the minimum magnitude of P_w can be obtained.

In order to better study of this analytical method, it needs numerical code to analyse the results well. Results are represented in next section. Matlab software is used which is common, powerful and practical code. Using this code, effect of each parameter is seen easily and graphically. Also in order to verify the results, two different well data are used.

Analytical Analysis of Possible Terms

As mentioned at previous section for determination of minimum proper mud weight to prevent well wall collapse, there are three different states. In order to choose between this three magnitudes, each of these should be used to gain the magnitude of radial, tangential and axial stresses to check whether they satisfy their first condition or not. If they satisfy more than one first condition, minimum amount of P_w is acceptable. Although with using code we can analyse this condition simply, in following, the possible condition in different stress regimes studied.

A. *Normal stress regime:* As mentioned in this stress regime, the maximum tangential stress is seen at $\theta = \pi/2$, so stresses can be obtained as:

$$\sigma_r = P_w \tag{31}$$

$$\sigma_\theta = 3\sigma_v - \sigma_h - P_w \tag{32}$$

$$\sigma_z = \sigma_h + 2\nu(\sigma_v - \sigma_h) \tag{33}$$

In order to this term happens $\sigma_z \geq \sigma_\theta \geq \sigma_r$, the below condition should be satisfied:

$$\sigma_z = \sigma_\theta \geq 2\nu \tag{34}$$

With substituting upside equation:

$$(3 - 2\nu)\sigma_v \leq (2 - 2\nu)\sigma_h + P_w \tag{35}$$

According to Possion ratio is 0 to 0.5 and also P_w could not be more than minimum principal stress; in this state the vertical stress should be less than the horizontal stress which is in contrast with normal regime. So for normal stress regime, two stress states as $\sigma_\theta \geq \sigma_z \geq \sigma_r$ and $\sigma_\theta \geq \sigma_r \geq \sigma_z$ are available.

B. *Reverse stress regime:* As mentioned, the maximum tangential can be seen at $\theta = 0$. So stresses in this θ is equal:

$$\sigma_r = P_w \tag{36}$$

$$\sigma_\theta = 3\sigma_h - \sigma_v - P_w \tag{37}$$

$$\sigma_z = \sigma_h + 2\nu(\sigma_v - \sigma_h) \tag{38}$$

In order to this term happens $\sigma_\theta \geq \sigma_r \geq \sigma_z$, the below condition should be satisfied:

$$\sigma_r = \sigma_z \geq 0 \tag{39}$$

With substitution of above equations:

$$(2\nu)\sigma_v \geq (1 + 2\nu)\sigma_h - P_w \tag{40}$$

Also in order to this term happens $\sigma_z \geq \sigma_\theta \geq \sigma_r$, the below condition should be satisfied:

$$(1 - 2\nu)\sigma_v \geq (2 - 2\nu)\sigma_h - P_w \tag{41}$$

With replacing all parameters, for satisfying equations (40) and (41), the vertical stress should be more than the horizontal stress which is contrast with condition of reverse stress regime. Thus, only this term $\sigma_\theta \geq \sigma_z \geq \sigma_r$ is possible in this term.

Mud Window Design Using Hoek-Brown Criterion

Researchers have proved that the Hoek-Brown criterion represents results with more accuracy rather than Mohr-Coulomb criterion (Jiang and Xie, 2011). The criteria which do not consider the mean principal stress effect are very simple and they are very ideal if can be used to study well stability and therefore represent good results. Similar method, cited before, can be used for Hock-Brown criterion. In order to appointment of lower limit of mud weight, $\sigma_z \geq \sigma_\theta \geq \sigma_r$ is the first state that studied. In this state, it can be said:

$$\sigma_3 = \sigma_r = P_w - P_o \tag{42}$$

$$\sigma_2 = \sigma_\theta = A - P_w - P_o \tag{43}$$

$$\sigma_1 = \sigma_z = B - P_o \tag{44}$$

where $A = 3\sigma_H - \sigma_h$ and $B = \sigma_v + 2\theta\ (\sigma_h - \sigma_H)$. Substituting these equations in Hock-Brown criterion and simplifying them, below quadratic equation can be reached:

$$P_w^{\ 2} + \left(m\sigma_c + 2B\right)P_w - \left(m\sigma_c P_o - B^2 + \sigma_c^{\ 2}\right) = 0 \tag{45}$$

Upper equation has two roots. According to the minimum mud pressure is the aim, the minor root is acceptable and the minor root of upper equation is:

$$P_w = \frac{-\left(X + 2B\right) - \sqrt{(X + 2B)^2 - 4(XP_o - B^2 + Y)}}{2} \tag{46}$$

X and Y are defined as:

$$X = m\sigma_c \tag{47}$$

$$Y = \sigma_c^2 \tag{48}$$

In this way, for other five scenarios maximum and minimum of proper mud weight can be determined. Results of these six scenarios are shown in Tables 2 and 3, briefly.

Mud Window with Using Mohr-Coulomb

To determine the lower limit of mud weight and to prevent wall collapse, $\sigma_z \geq \sigma_\theta \geq \sigma_r$ is the first state studied. With replacing equations (42–44) in Mohr-Coulomb criterion, the below equation to determine the minimum of mud pressure is gained:

Table 2: Determination of lower limit of mud window using Hoek-Brown failure criterion

State	*Lower limit of mud window*
$\sigma_z \geq \sigma_\theta \geq \sigma_r$	$P_{wb1} = \dfrac{\left(X + 2B\right) - \sqrt{(X + 2B)^2 - 4(XP_0 + B^2 - Y)}}{2}$
$\sigma_\theta \geq \sigma_z \geq \sigma_r$	$P_{wb2} = \dfrac{\left(4A + X\right) - \sqrt{(4A + X)^2 - 16(A^2 + XP_0 - Y)}}{8}$
$\sigma_\theta \geq \sigma_r \geq \sigma_z$	$P_{wb3} = A - B - \sqrt{X\left(B - P_0\right) - Y}$

Table 3: Determination of upper limit of mud window using Hoek-Brown failure criterion

State	*Upper limit of mud window*
$\sigma_r \geq \sigma_\theta \geq \sigma_z$	$P_{wf1} = B + \sqrt{X\left(B - P_0\right) + Y}$
$\sigma_r \geq \sigma_z \geq \sigma_\theta$	$P_{wf2} = \dfrac{\left(4A - X\right) + \sqrt{(4A - X)^2 - 16(A^2 - X\left(A - P_0\right) + Y)}}{8}$
$\sigma_z \geq \sigma_r \geq \sigma_\theta$	$P_{wf3} = \dfrac{(2A - 2B - X)\sqrt{(2A - 2B - X)^2 - 4\left(A - B\right)^2 - X\left(A - P_0\right) + Y}}{2}$

$$P_w = \frac{B - P_o + qP_o - C_o}{q} \quad (49)$$

Similarly, for other five scenarios the minimum and maximum of proper mud weight can be determined. The results of these six scenarios for Mohr-Coulomb criteria are shown in Tables 4 and 5.

Table 4: Determination of lower limit of mud window using Mohr-Coulomb failure criteria

State	*Lower limit of mud window*
$\sigma_z \geq \sigma_\theta \geq \sigma_r$	$P_{wb1} = \frac{B - P_o + qP_o - C_o}{q}$
$\sigma_\theta \geq \sigma_z \geq \sigma_r$	$P_{wb2} = \frac{A - C_o + (q+1)P_o}{q+1}$
$\sigma_\theta \geq \sigma_r \geq \sigma_z$	$P_{wb3} = A - C_o + (q-1)P_o - Bq$

Table 5: Determination of upper limit of mud window using Mohr-Coulomb failure criteria

State	*Upper limit of mud window*
$\sigma_r \geq \sigma_\theta \geq \sigma_z$	$P_{wf1} = C_o + Bq + (1-q)P_o$
$\sigma_r \geq \sigma_z \geq \sigma_\theta$	$P_{wf2} = \frac{C_o + Aq + (1-q)P_o}{q+1}$
$\sigma_z \geq \sigma_r \geq \sigma_\theta$	$P_{wf3} = \frac{C_o + Aq + (1-q)P_o - B}{q}$

Selecting Proper Failure Criterion (Study of 13 other Failure Criteria)

The failure criteria should not be such complicated that make the study of well stability very difficult; in other words, it should have a tangible physical concept to be verified using conventional laboratory tests. In this section, 13 failure criteria are compared. These criteria are selected from the most applicable and reliable criteria including: Bieniawski-Yudhbir (BY), linear forms, quadratic and power law Mogi 1961 (LM 1961, SM 1961 and PM 1961), quadratic and power law Mogi 1971 (SM 1971 and PM 1971), modified Lade (ML), Drucker-Prager (DP), modified Wiebols-Cook (WC), Hoek-Brown (HB), triaxial Hoek-Brown (3D HB), Hoek-Brown Matsuoka Nakai (HBMN), Mohr-Coulomb (MC) and modified Mohr-Coulomb (MMC).

The first step to verification of these 13 criteria with experimental data, is to determine the equation coefficients. With this respect, two methods of axial coordinate transformation and grid search have been applied. Usually to solve such problems, the grid search method is utilized which is time-consuming method. That is why for some criteria the axial coordinate transformation is used to get results of verification faster. This process was accomplished for all criteria and experimental data. For instance, Table 6 shows the fit of multi axial tests data with quadratic Mogi 1971 criterion.

In order to reduce the number of diagrams and to achieve more comprehensive results, the rocks are classified in pairs in binary groups. Dolomite and limestone in the group of carbonate rocks, sandstone, shale, siltstone and salt rock in sedimentary rocks group, trachyte and andesite in the group of extrusive igneous rocks, granite and granodiorite in the group of intrusive igneous rocks, amphibolite and marble in the group of metamorphic rocks with carbonate sources and hornfels and metapelites in the group of proximity metamorphic rocks were classified.

After extensive analysis and modelling, the modified Lade model is finally chosen as the best criterion that has the closest results and the best match with experimental tests data for rock samples. Figure 7 shows the results of these analyses.

Mud Window Design Using Modified Lade Criterion

According to analysis cited before and selected modified Lade failure criterion, in the following, some equations are presented based on this criterion to determine proper mud weight.

The modified Lade criteria defined as:

$$(I_1')^3 = (27 + \eta) I_3' \quad (50)$$

where the stresses invariants are calculated as:

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \quad (51)$$

$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \quad (52)$$

where S and η denote rock cohesion and internal friction angle, respectively. With reference to the above equations, it could be concluded that the quantity of stress invariants (I'_3 and I'_1) in all conditions are constant and do not change; in other words, there is no difference which one of principal stresses replace in vertical stress position. In this criterion with one scenario the minimum mud weight can be determined to prevent well wall collapse.

Table 6: The best fit of multi axial test data with quadratic Mogi 1971 criterion

Rock type	*Failure criterion (Mogi, 1971)*
Sandstone	$\tau_{oct} = -0.0012\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.77\left(\frac{\sigma_1+\sigma_3}{2}\right) + 3.55$
Dolomite	$\tau_{oct} = -0.0002\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.6\left(\frac{\sigma_1+\sigma_3}{2}\right) + 62.86$
Granite	$\tau_{oct} = -0.0002\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.86\left(\frac{\sigma_1+\sigma_3}{2}\right) + 8.15$
Shale	$\tau_{oct} = -0.0002\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.49\left(\frac{\sigma_1+\sigma_3}{2}\right) + 19.94$
Limestone	$\tau_{oct} = 0.0005\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.16\left(\frac{\sigma_1+\sigma_3}{2}\right) + 127.7$
Trachyte	$\tau_{oct} = 0.0003\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.009\left(\frac{\sigma_1+\sigma_3}{2}\right) + 11.1$
Andesite	$\tau_{oct} = -0.0006\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.99\left(\frac{\sigma_1+\sigma_3}{2}\right) - 7.2$
Amphibolite	$\tau_{oct} = -0.0009\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.72\left(\frac{\sigma_1+\sigma_3}{2}\right) + 24$
Marble	$\tau_{oct} = -0.0005\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.74\left(\frac{\sigma_1+\sigma_3}{2}\right) + 4.52$
Salt rock	$\tau_{oct} = -0.005\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.99\left(\frac{\sigma_1+\sigma_3}{2}\right) - 0.63$
Granodiorite	$\tau_{oct} = 0.0008\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.49\left(\frac{\sigma_1+\sigma_3}{2}\right) + 49.7$
Siltstone	$\tau_{oct} = -0.0006\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.67\left(\frac{\sigma_1+\sigma_3}{2}\right) + 14.5$
Hornfels	$\tau_{oct} = -0.0027\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 1.62\left(\frac{\sigma_1+\sigma_3}{2}\right) - 78.5$
Metapilite	$\tau_{oct} = -0.0003\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 + 0.65\left(\frac{\sigma_1+\sigma_3}{2}\right) + 18.67$

To simplify, the quantity of stresses assumed as:

$$\sigma_r = P_w \tag{53}$$

$$\sigma_\theta = A - P_w \tag{54}$$

$$\sigma_z = B \tag{55}$$

where A and B are calculated as:

$$A = 3\sigma_H - \sigma_h \tag{56}$$

$$B = \sigma_v + 2\nu(\sigma_h - \sigma_H) \tag{57}$$

The model assumed as poro-elastic; thus, the stresses should first change to effective stresses (Terzaghi, 1943):

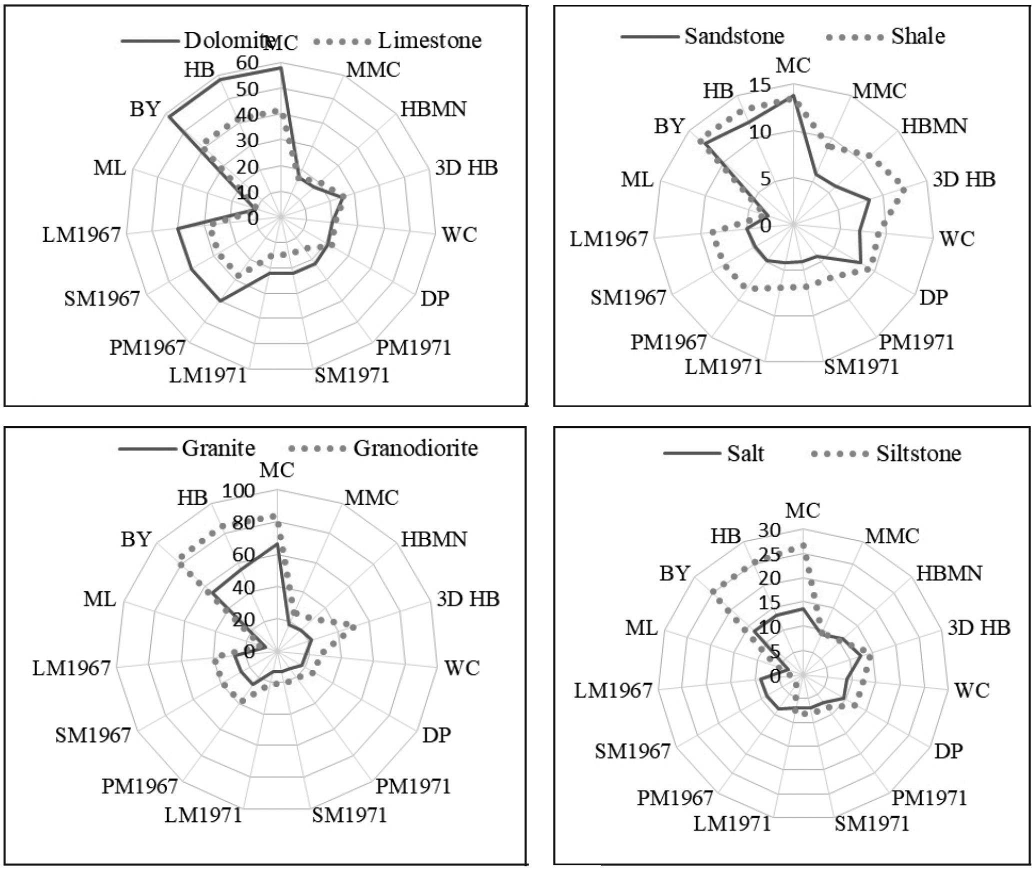


Figure 7: Dual comparison of rock samples and analyzing percent error in presented criteria.

$$\sigma_r = P_w - P_o \tag{58}$$

$$\sigma_\theta = A - P_w - P_o \tag{59}$$

$$\sigma_z = B - P_o \tag{60}$$

By replacing these equations with stress coefficients, they change into:

$$I_1 = A + B + 3S - 3P_o \tag{61}$$

$$I_3 = \left(P_w - P_o + S\right)\left(A - P_w - P_o + S\right)(B - P_o + S) \tag{62}$$

If these coefficients are replaced in modified Lade criterion, after simplifying them, below quadratic equation is created:

$$P_w{}^2 - AP_w + D = \frac{(A + B + 3S - 3P_o)^3}{\left(27 + \eta\right)C} \tag{63}$$

where the quantity of C and D are defined as:

$$C = (P_o - B - S) \tag{64}$$

$$D = \left(P_o - S\right)^2 + A(P_o - S) \tag{65}$$

Because the aim is to determine the minimum mud weight preventing collapse, small root of quadratic equation represents the lower limit of mud window:

$$P_{wb} = \frac{A - \sqrt{A^2 - 4\left(D - \dfrac{E^3}{\left(27 + \eta\right)C}\right)}}{2} \tag{66}$$

where E is:

$$E = A + B + 3S - 3P_o \tag{67}$$

On the other hand, in this scenario, the maximum mud weight to prevent well wall failure can be determined. For simplicity, the quantity of stresses is assumed as:

$$\sigma_r = P_w \tag{68}$$

$$\sigma_\theta = A' - P_w \tag{69}$$

$$\sigma_z = B' \tag{70}$$

With changing upper equations to effective stress and replace them in stress coefficients, they are obtained as below:

$$I_1 = A' + B' + 3S - 3P_o \quad (71)$$

$$I_3 = \left(P_w - P_o + S\right)\left(A' - P_w - P_o + S\right)(B' - P_o + S) \quad (72)$$

If these invariants replace in modified Lad criterion, after broad simplifications, the quadratic equation (the major root of it), is achieved:

$$P_{wf} = \frac{A' - \sqrt{A'^2 - 4\left(D' - \frac{E'^3}{(27+\eta)C'}\right)}}{2} \quad (73)$$

where C', D' and E' are:

$$C' = (P_o - B' - S) \quad (74)$$

$$D' = (P_o - S)^2 + A'(P_o - S) \quad (75)$$

$$E' = A' + B' + 3S - 3P_o \quad (76)$$

Application of the Model to Real Cases and Results Analysis

At first sensitivity analysis was performed based on changes in failure criteria, in-situ stresses and rock properties, then determination of mud window and the proper route for directional drilling were studied and finally by using the data from two case studies (one well in Pagerungen oilfield in Indonesia; another case, well No. 35 of Norouz oilfield in Iran), the accuracy of calculations was investigated.

The Wellbore in Pagerungen Oilfield (Indonesia)

Ramos (1998) reported the drilling operation in Pagerungen oilfield of Indonesia which has been carried out without any instability. A well has been drilled in shale formation in depth of 4800 to 6200 ft with mud weight of 10.5 ppg. Using laboratory tests data, the Poisson's ratio of 0.3, internal friction angle of 35 degree and formation uniaxial compressive strength of 1800 psi, have been reported. Moreover, the amounts of σ_v, σ_h and σ_H were measured using leak-off test and the values found to be 1, 0.87 and 1.22 psi/ft, respectively. This well was drilled with deviation of 25 degree and an azimuth of 47 degree. It should be stated that the azimuth was meant to be considered from 0 to 50.

To study this well, the formation pore pressure was assumed to be normal condition (0.45 psi/ft) and the study performed in 6000 ft depth. Figure 8 illustrates the results based on well dip and azimuth. According to Mohr-Coulomb criterion, well would be in instability condition in which the proper mud weight is overestimated as 1.2 lb/gal more than current weight. If we choose this criterion for mud weight, there is the possibility of loss of drilling fluid. According to Mugi-Coulomb criterion, with this mud weight, well will be stable in deviation of more than 35 degrees. But according to modified Lade criterion, well is stable in whole conditions of dip and azimuth. According to objective observations that indicate stability of well, only modified Lade criterion predicts this condition appropriately.

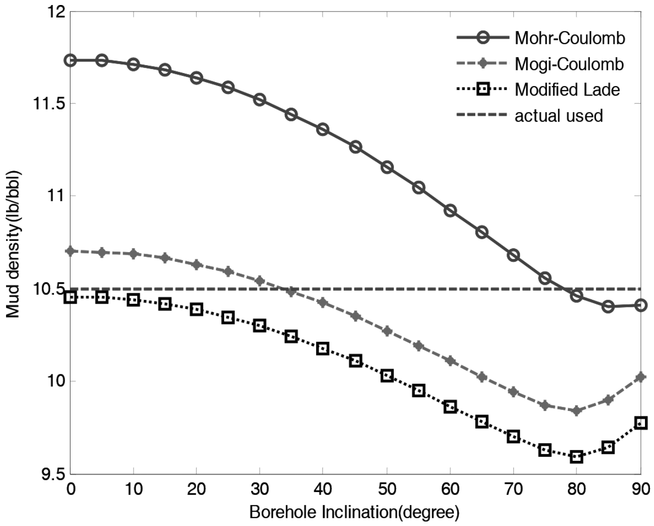


Figure 8: Minimum proper mud weight for Pagerungen oilfield (drilling in depth of 6000 ft).

The Wellbore (No. 35) in Norouz Oilfield (Iran)

The well is located in Norouz oilfield in southern part of Iran. With respect to this well, in-situ stresses can be determined via leak-off test and drilling data but in addition to the costly and time consuming of these tests, the number of tests that can be conducted in an oil well are limited because in most cases, these tests lead to damages on well wall. Due to these problems, in the majority of cases, well logs are used to determine the values of in-situ stresses. Figure 9 shows the acoustic and density logs for well No. 35 of Norouz field from 2347 to 2659 m depth. Regarding this well, the caliper, density and acoustic logs (both pressure and shear waves) were simultaneously applied to investigate the well's stability.

According to rock elastic and dynamic relations, E (Elasticity modulus) and ∂ (Poisson's ratio) of the formation can be calculated as follows (Zoback, 2010):

$$E = \rho V_s^2 \frac{3V_P^2 - 4V_s^2}{V_P^2 - V_s^2} \quad (77)$$

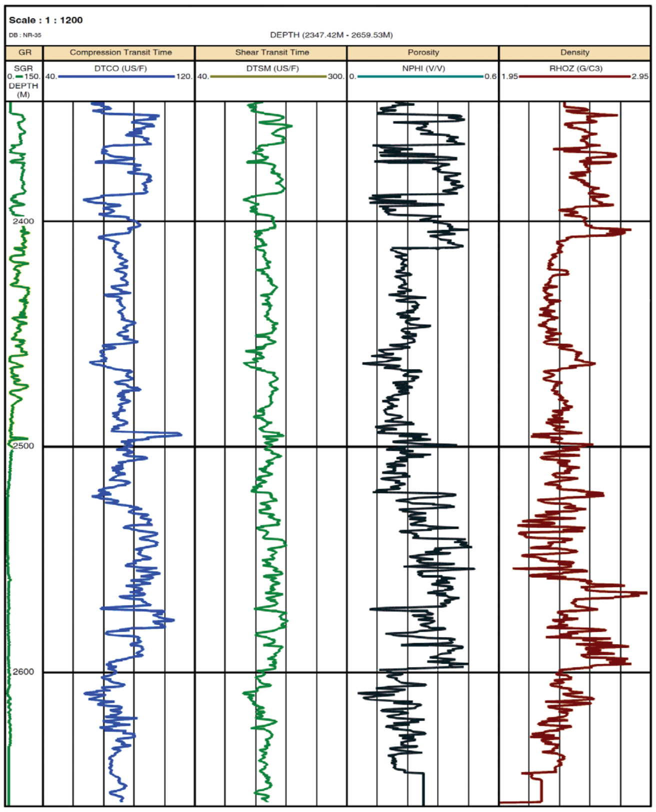
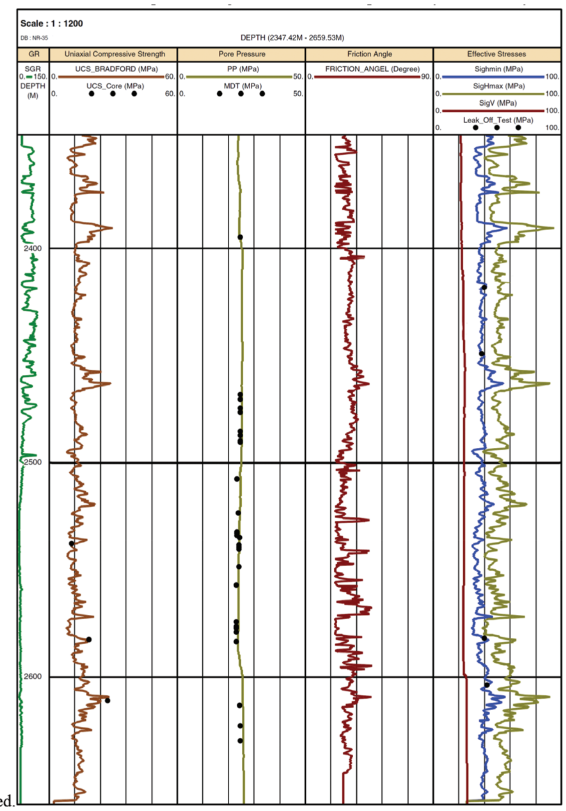


Figure 9: Acoustic and density logs for well No. 35 in Norouz oilfield.



occurred.

Figure 11: Determination of in-situ stresses in Norouz oilfield.

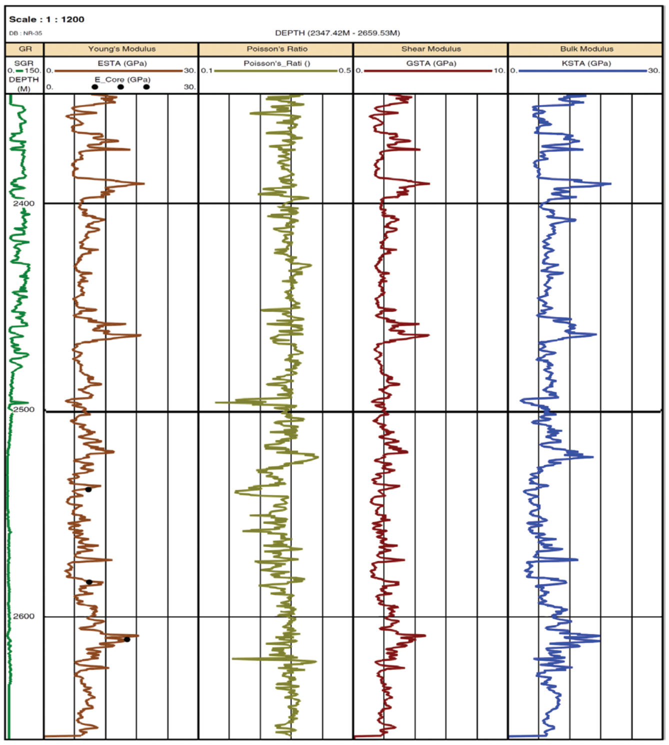


Figure 10: Determination of mechanical properties of encountered formation for well No. 35 in Norouz oilfield.

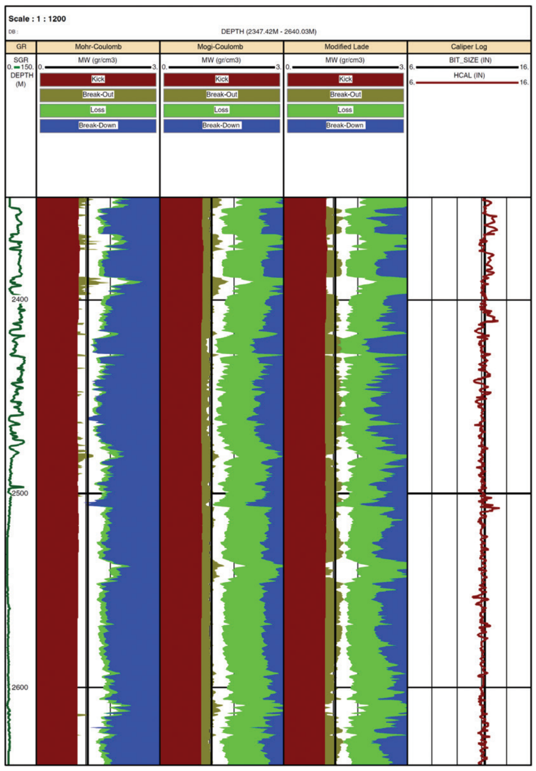


Figure 12: Determination of mud window for well No. 35 of Norouz oilfield using Mohr-Coulomb, Mogi-Coulomb and modified Lade criteria.

$$\vartheta = \frac{V_P^{\,2} - 2V_s^{\,2}}{2(V_P^{\,2} - V_s^{\,2})} \tag{78}$$

where V_P and V_s are the velocity of pressure and shear waves, respectively. As mentioned before, E (Elasticity modulus) and V can be determined using pressure and shear waves logs (Figure 10). With this regard, Geolog software was utilized. Figure 11 represents the physical properties of formation gained from the software. Through the analysis, the formation's uniaxial compressive strength (C) is determined from Bradford equation (Bradford et al., 1998):

$$C = 2.28 + 4.1089E \tag{79}$$

For determination of overburden pressure, density of rock formation is needed. The quantity of vertical stress σ_v gained from integration of formation density from surface to a given depth (Zoback et al., 2003):

$$\sigma_v = \int_0^z \rho(z)\, g dz \tag{80}$$

To analyse the stability of the well, exact amount of horizontal stresses should be determined. This task can be carried out in both direct and indirect ways. In direct method, horizontal stresses are calculated by using rock mechanics laboratory tests data and indirectly by means of drilling logs. Empirical equations can be used to determine these stresses. The most commonly used equations are the poro-elastic equations, which require some data such as vertical pressure σ_v, formation pressure P_f, Poisson's ratio ϑ, and tectonic strains:

$$\sigma_H = \frac{\vartheta}{1-\vartheta}(\sigma_v - \alpha P_f) + \alpha P_f + \frac{E\varepsilon_x}{1-\vartheta^2} + \frac{\vartheta E\varepsilon_y}{1-\vartheta^2} \tag{81}$$

$$\delta_h = \frac{\vartheta}{1-\vartheta}(\sigma_v - \alpha P_f) + \alpha P_f + \frac{E\varepsilon_y}{1-\vartheta^2} + \frac{\vartheta E\varepsilon_x}{1-\vartheta^2} \tag{82}$$

Figure 11 shows the calculated in-situ stresses using the geology software. By calculating the in-situ stresses and the angle of the internal friction, as shown in Figure 11, the information required to verify the stability of the well is completed. This part of well is drilled using a mud with a specific weight of 1.25. According to the results achieved from the Mohr-Coulomb, Mogi-Coulomb and modified Lade criteria, the modified Lade criterion has the best fit with the caliper log data (Figure 12). It is not possible to design a mud without any instability in the well, but almost with a mud with a specific weight of 1.35, the least probability of instability is occurred.

Conclusions

Appropriate selection of rock failure criterion in wellbore stability analysis plays a major role to yield a proper mud window in a specific formation. In this paper, to predict the stability of the well wall and to determine the optimal failure criteria for designing the mud window, 15 failure criteria consist of Bieniawski-Yudhbir (BY), linear forms, quadratic and power law Mogi 1961 (LM 1961, SM 1961 and PM 1961), quadratic and power law Mogi 1971 (SM 1971 and PM 1971), modified Lade (ML), Drucker-Prager (DP), modified Viebols-Cook (WC), Hoek-Brown (HB), triaxial Hoek-Brown (3D HB), Hoek-Brown Matsuoka Nakai (HBMN), Mohr-Coulomb (MC) and modified Mohr-Coulomb (MMC) were studied. With the adaption of different failure criteria with laboratory data, it was proved that the modified Lade criterion with standard deviation of 2% is the most proper criteria for predicting the well wall collapse while none of the other failure criteria provide realistic results for wall failure. Also, in the field scale, it was found that at different stress regimes, modified Lade criterion provides proper results. By integrating the modified Lade failure criterion with the surrounding stresses of the well, calculated from the Kirsch equations, a new equation for predicting minimum weight of the mud to prevent the well wall collapse was obtained.

For verification, the equation was compared with actual data of one well (well No. 35) in the Norouz oilfield in southern Iran and one well in Indonesia; in both cases, only modified Lade equation correctly predicts the proper mud weight to keep the stability of well wall. In the well No. 35 of the Norouz field, the caliper log versus the diameter of the well have been recorded, so the points that are unstable are known. The modified Lade criterion as an accurate equation and Mohr-Coulomb and Mogi-Coulomb criteria as the most useful criteria were chosen to verify the predicted unstable points. It was found that only modified Lade criterion predicts these points properly for well No. 35. Regarding other case in Indonesia, in drilling operation in shale formation (in depth of 4800 to 6200 m and mud weight of 10.5 ppg) no instability was observed but according to Mohr-Coulomb criterion the well is at instability state as well as the criterion provides the suitable mud weight more than current weight leading to possibility of drilling fluid lost. With respect to Mogi-Coulomb criterion using this mud weight, the well was predicted stable at the deviation of more than 35 degrees. But, due to modified Lade criterion, the well is steady in terms of slope and azimuth in any situation. The results, in general, showed that only modified Lade criterion predicted the realistic conditions appropriately.

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