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Estimation of Hazard Rate Function for Building Second Order Mixed Model Using Fuzzy Techniques

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Abstract: This research deals with constructing second order mixed model, from Exponential (θ), and Gamma (3,θ), where the mixing proportions are $\left(\frac{\alpha}{\alpha+1}\right), \left(\frac{1}{\alpha+1}\right)$.

The p.d.f. is derived, and also CDF and reliability function. Then the parameters are estimated by method of moments and maximum likelihood as well as some proposed method. Where from Table 1 we find the first best fuzzy hazard rate is moment estimator with percentage and the second one proposed while the third best one is maximum likelihood estimation. We observe that the first best fuzzy hazard rate estimator is proposed one, and the second best one is maximum likelihood estimation and finally the moment estimator is best, according to the results of fuzzy hazard rate function, we find that the first best is the proposed one, while the second best one is moments estimator, and finally the third one is maximum likelihood estimation. All the derivation required are explained, and results of comparison are explained in tables.

Key words: Maximum likelihood estimation, moment estimation, proposed method estimation, second order mixed model.

Introduction

The reliability of system can be considered as the probability that the system, must be strong enough to overcome the stress imposed on it. Bamber (1975) discussed the ordinal dominance graph and the area below the receiver operating graph, and published his paper in the *Journal of Mathematical Psychology* . The problems of estimating reliability as well as hazard rate function have attracted much attention from researchers, for example, Mom and Cheng (1991) introduced fuzzy system reliability analysis for components, using different types of membership function. and explained fuzzy system and sets. Cai et al. (1991) introduced fuzzy reliability depending on the fuzzy variable as a basis for a theory of fuzzy reliability and its applications of fuzzy sets and system, while Cheng and Mom (1993) introduced fuzzy system reliability analysis using fuzzy number. While Chen (1994) introduced an analysis of fuzzy system reliability, applying fuzzy number arithmetic operation and working on estimating fuzzy reliability of different types of system.

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Jiang and Chen (2003) introduced a numerical algorithm of fuzzy reliability in Reliability Engineering and system safety.

Wang and Shih (2008) introduced fuzzy reliability for systems with triangular fuzzy numbers based on statistical data, as well as fuzzy probability distribution for reliability of concrete structures. The Rayleigh distribution has a number of applications in settings where magnitudes of normal variables are important (Chansoo and Keunhee, 2009).

Mahmoudi and Zakerzadeh (2010) worked on the generalized Poisson-Lindley distribution and presented various methods for estimating and comparing the results through reliability optimization of a parallel series system. Ristić and Balakrishnan (2011) introduced a new distribution generated by gamma random variables and showed that the distribution contains as a special case of lower record value from random variables from a population with exponential distribution.

Also Shanker and Mishra (2013) introduced a research about quasi lindley distribution. El-Damcese and Ramadan (2015) investigated some statistics properties of the mixture generalised linear failure rate distribution (or MGLFRD) and MGLFRD with fuzzy parameters methods. Formulas of a fuzzy reliability function, fuzzy hazard function and their α-cut set are presented.

Hussian and Essam (2016) estimated models for strength stress, models under fuzzy environment of exponential random variables with different parameters. An application for the Rayleigh distribution is the analysis of wind velocity (Pessanha et al., 2016). Zeghdoudi and Nedjar (2016) worked on deriving gamma Lindley Distribution and studied its properties by simulation.

Shafiq and Veirtl (2017) studied generalised estimators for the parameters and hazard rates proposed for bathtub failure rate distributions to model fuzzy life time data effectively.

The mixture distributions have vital role in practical applications for researches that deal with economics, medicine, agriculture, life testing, and reliability for classical reliability theory. There are several methods and models in which the parameters are assumed to be precise. Nevertheless, there are difficulties in handling reliability and hazard functions in the real world due to the vague, random distribution of fuzzy parameters in lifetime distribution. Many researchers have worked on fuzzy reliability and developed this field. In their study, Hussian and Essam (2017) discussed about fuzzy exponential distribution and how to compute reliability in case of stress-strength model and ranked set sampling.

Jasaim and Saleemah (2018) compared different fuzzy estimators for hazard rate function under mixed quasi – Lindely distribution. These estimators are maximum likelihood, moments, and frequency ratio method.

Theoretical Part

This part of research deals with defining the random variable (×1), which is distribution as exponential with parameter (θ), where its probability density function pdf and cumulative distribution function CDF, are explained in equations (1) and (2).

While the second random variable (×2) is distribution as Gamma (3, θ), and its pdf and CDF, are explained in equations (3)and (4).

Let x_1 be r.v. ~Exponential (θ) with p.d.f and CDF, given as:

$$f(x_1) = \theta e^{-\theta x} \tag{1}$$

$$F(x_1) = 1 - e^{-\theta x} \tag{2}$$

While ×2 is r.v~Gamma(3,θ)

$$f(x_2) = \frac{\theta^3}{2} x^2 e^{-\theta x} \tag{3}$$

And its CDF is

$$F(x_2) = 1 - e^{-\theta x} - \theta x e^{-\theta x} - \frac{x^2}{2}\theta^2 e^{-\theta x} \tag{4}$$

Now the mixed p.d.f is

$$f_x(x) = \frac{\alpha}{\alpha+1}\theta e^{-\theta x} + \frac{1}{(\alpha+1)}\frac{\theta^3}{2}x^2 e^{-\theta x}$$

$$f_x(x) = \frac{\theta}{\alpha+1}e^{-\theta x}\left[\alpha + \frac{\theta^2}{2}x^2\right] \tag{5}$$

While the cumulative distribution function of this second order mixed model is:

$$F_x(x) = \frac{\alpha}{\alpha+1}\left(1-e^{-\theta x}\right) + \frac{1}{(\alpha+1)}\left[1 - e^{-\theta x}\left(1+\theta x+\frac{\theta^2 x^2}{2}\right)\right] \tag{6}$$

And corresponding Reliability function is

$$R_x(x) = 1 - F_x(x)$$

$$= \frac{\alpha}{(\alpha+1)}e^{-\theta x} + \frac{1}{(\alpha+1)e^{-\theta x}}\left(1+\theta x+\frac{\theta^2 x^2}{2}\right) \tag{7}$$

And the hazard rate function

$$h(x,\ \alpha,\ \theta) = \frac{f_x(x,\alpha,\theta)}{R_x(x,\alpha,\theta)} \tag{8}$$

Therefore, the function to be estimated is fuzzy hazard rate function $(\tilde{k}_i h(x_i))$ were

$$h_x(x_i) = \frac{f(x_i)}{R_x(x_i)}$$

$$= \frac{\frac{\theta e^{-\theta x}}{(\alpha+1)}\left(\alpha + \frac{\theta^2 x^2}{2}\right)}{\frac{e^{-\theta x}}{\alpha+1}\left[\alpha + 1 + \theta x + \frac{\theta^2 x^2}{2}\right]}$$

$$= \frac{\hat{\theta}\left(\alpha + \frac{\hat{\theta}^2 x_i^2}{2}\right)}{\left[\hat{\alpha} + 1 + \hat{\theta} x_i + \frac{\hat{\theta}^2 x_i^2}{2}\right]}$$

and fuzzy hazard of mixed Exponential-Gamma is

$$\tilde{h}_x(\tilde{k}_i x_i) = \frac{\hat{\theta}\left(\hat{\alpha} + \hat{\theta}^2 \frac{\tilde{k}_i x_i^2}{2}\right)}{\hat{\alpha} + 1 + \hat{\theta}\tilde{k}_i x_i + \frac{\hat{\theta}^2 (\tilde{k}_i x_i)^2}{2}} \tag{9}$$

where θ, α are estimated by moments, and maximum likelihood, and proposed one.

After the second order failure to time model is derived from exponential and Gamma and its mean and variance and Reliability function, and hazard rate function are derived, we introduce, estimation methods to estimate (α, θ) and then compare different fuzzy estimators of hazard rate function.

Estimation

This section deals with estimating the two parameters (θ, α) of second order failure to time model, by methods of maximum likelihood, and moments as well as proposed one.

Maximum Likelihood Estimation (MLE)

Let $x_1, x_2, \ldots x_n$ be a r.s from $f_x(x_i, \theta, \alpha)$

$$L = \prod_{i=1}^{n} f_x(x_i, \theta, \alpha)$$

$$\theta^n e^{-\theta \Sigma x_i} (\alpha+1)^{-n} \prod_{i=1}^{n} \left[\alpha + \frac{\theta^2}{2} x_i^2\right] \tag{10}$$

Then

$$\log L = n \log\theta - \theta \Sigma\, x_i - n \log(\alpha+1) + \sum_{i=1}^{n} \log\left(\alpha + \frac{\theta^2}{2} x_i^2\right) \tag{11}$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n}{\theta} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{\theta x_i^2}{\left(\alpha + \frac{\theta^2}{2} x_i^2\right)} \tag{12}$$

From $\frac{\partial \log L}{\partial \theta} \to 0$

$$\frac{n}{\theta} = \sum_{\iota=1}^{\nu} \xi_\iota - \theta \frac{\sum_{i=1}^{n} x_i^2}{\left(\alpha + \frac{\theta^2}{2} x_i^2\right)} \tag{13}$$

$$\hat{\theta}_{MLE} = \frac{n}{\sum_{i=1}^{n} x_i - \hat{\theta} \frac{\sum_{i=1}^{n} x_i^2}{\left(\alpha + \frac{\theta^2}{2} x_i^2\right)}} \tag{14}$$

While from

$$\frac{\partial \log L}{\partial \alpha} = \frac{-n}{(\alpha+1)} + \sum_{i=1}^{n} \frac{1}{\left(\alpha + \frac{\theta^2}{2} x_i^2\right)}$$

$$\frac{n}{(\hat{\alpha}+1)} = \sum_{i=1}^{n} \left(\hat{\alpha} + \frac{\hat{\theta}^2}{2} x_i^2\right)^{-1}$$

$$(\hat{\alpha}_{MLE} + 1) = \frac{n}{\sum_{i=1}^{n} \left(\hat{\alpha} + \frac{\hat{\theta}^2}{2} x_i^2\right)^{-1}} \tag{15}$$

Solving equation 12 numerically gives $(\hat{\theta}_{MLE})$ but for α, we have

$$\frac{\partial \log L}{\partial \alpha} = \frac{-n}{(\alpha+1)} + \sum_{i=1}^{n} \frac{1}{\left(\alpha + \frac{\theta^2}{2} x_i^2\right)} = 0$$

Which is an implicit function solved numerically to find $(\hat{\alpha}_{MLE})$ from simplified equation below

$$\sum_{i=1}^{n} \left(\hat{\alpha} + \frac{\hat{\theta}^2}{2} x_i^2\right)^{-1} = \frac{n}{(\hat{\alpha}_{MLE} + 1)} \tag{16}$$

Moments Method Estimation (MOM):
The estimation by moment's method depend on

$$\mu_r' = E(x^r) = \frac{\sum_{i=1}^{n} x_i^r}{n}$$

For r=1 and r=2, since we have two parameters:

Since $E(x) = \dfrac{\alpha+3}{\theta(\alpha+1)}$ and

$$E(x) = \left(\frac{\alpha}{\alpha+1}\right)\frac{1}{\theta} + \frac{1}{(\alpha+1)}\left(\frac{3}{\theta}\right)$$

$$= \frac{\alpha+3}{\theta(\alpha+1)} = E(x)$$

$$V(x) = \left(\frac{\alpha}{\alpha+1}\right)^2\left(\frac{1}{\theta^2}\right) + \frac{1}{(\alpha+1)^2}\left(\frac{3}{\theta^2}\right)$$

$$V(x) = \frac{\alpha^2+3}{(\alpha+1)^2\theta^2}$$

$$E(x^2) = \frac{\alpha^2+3}{(\alpha+1)^2} + \frac{(\alpha+3)^2}{\theta^2(\alpha+1)^2}$$

$$= \frac{(\alpha^2+3+\alpha^2+6\alpha+9)}{(\alpha+1)^2\theta^2}$$

$$= \frac{2\alpha^1+6\alpha+12}{(\alpha+1)^2\theta^2}$$

$$E(x^2) = \frac{2(\alpha^2+3\alpha+6)}{(\alpha+1)^2\theta^2}$$

Solving

$$\frac{\sum_{i=1}^{n} x_i}{n} = E(x) \text{ and } \frac{\sum_{i=1}^{n} x_i^2}{n} = E(x^2)$$

to find $\hat{\alpha}_{mom}, \hat{\theta}_{mom}$ as

$$\frac{\hat{\alpha}+3}{\hat{\theta}(\hat{\alpha}+1)} = \frac{\Sigma x_i}{n_i}$$

$$\frac{2\hat{\alpha}^2+6\hat{\alpha}+12}{(\hat{\alpha}+1)_2\hat{\theta}^2} = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

Proposed Method Estimation (PROP):
Since the studied (compound distribution) have one shape parameter (α) and one scale parameter (θ), we can use some non-parametric proposed estimator like (Median estimator) where

$$\hat{\theta} = Min(x_i) = x_{(i)}$$

and $$\hat{\alpha} = \ln 2 / \ln\left(\frac{Me}{\hat{\theta}}\right)$$

also we can use probability weighted moment (PWM) estimator of α where

$$PWM(i, j, k) = E[X^i\{F_x(x)\}^j\{1 - F_x(x)\}^k]$$

$$= \int x^i\{F_x(x)\}^j\{1-F_x(x)\}^k dF_x(x)$$

This can alternatively be written as:

$$PWM(i, j, k) = \int_0^1 \{x(F_x)^i F_x^j (1-F_x)^k dF_x\}$$

where xF_x is the quantile function. PWM (1,0,0) are conventional moments
we use (PWM) as:

$$\hat{\alpha} = \frac{\widehat{M}_{1,0,1} - \widehat{M}_{1,0,0}}{2\widehat{M}_{1,0,1} - \widehat{M}_{1,0,0}}$$

where $\widehat{M}_{1,0,0} = \overline{y}$

$$\widehat{M}_{1,0,1} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(i-1)y_{(i)}$$

where $y_{(1)}<y_{(2)}<\ldots y_{(n)}$ order observation of sample and the scale parameter

$$\hat{\theta} = \frac{\overline{y}(+1)}{\hat{\alpha}}$$

Simulation Procedure

The comparison fuzzy hazard rate function of second order model is done through simulation, where (n = 25, 50, 75) and take the values of $0 < ti < \infty$

1. Generate the random variable according to (Quasi Lindley) with two parameters (α, θ) using method of rejection and acceptation.
2. Generate random variable ui ~uniform (0, 1)
3. Generate two random variables v_i~ exp (θ), and wi ~ Gamma (3, θ).
4. If $\mu_i \le p = \dfrac{\alpha}{\alpha+1}$ then $x_i = v_i$ otherwise $x_i = w_i$.

Third step
Estimate second order fuzzy hazard rate function using $\hat{\alpha}, \hat{\theta}$ and given values of (k_i) (fuzzy parameter) as well as values of random variables (x_i).

Summary of initial values are α>-1, θ>0

α	θ	$\tilde{k}_i$	n = 25, 50, 75
0.4	0.7	0.3	
0.25	0.6	0.6	
0.25	0.8	0.3	

Conclusions

So finally we can conclude that first best fuzzy hazard rate function estimator is proposed one and second best is moment and Maximum Likelihood is third best one. According to the values of fuzzy hazard rate function

Table 1: Estimators of fuzzy hazard rate function when α = 0, 4, θ = 0.7, $\tilde{k}$ = 0.3

N	t_i	$R_{real}\ H_i$	$\widehat{H}_{mle}$	$\widehat{H}_{Mom}$	$\widehat{H}_{prop}$	*Best*
	1.5	0.5500	0.5832	0.5443	0.5262	Prop
	2.5	0.6000	0.6336	0.6051	0.6203	MOM
	3.5	0.6323	0.6686	0.6124	0.6634	MOM
25	4.5	0.6562	0.6927	0.6443	0.6721	MOM
	5.5	0.6742	0.7083	0.6728	0.6822	Prop
	6.5	0.6889	0.7234	0.7186	0.7203	MOM
	7.5	0.7003	0.7335	0.7321	0.7384	MOM
	8.5	0.7082	0.7439	0.7421	0.7455	MOM
	9.5	0.7176	0.7525	0.7504	0.7555	MOM
	10.5	0.7242	0.7578	0.7644	0.7621	MLE
	1.5	0.5500	0.5635	0.5428	0.5642	MOM
	2.5	0.6000	0.6122	0.6019	0.6142	MLE
	3.5	0.6323	0.6468	0.6399	0.6468	MOM
	4.5	0.6562	0.6703	0.6636	0.6710	MOM
50	5.5	0.6742	0.6884	0.6867	0.6890	MOM
	6.5	0.6889	0.7028	0.7025	0.7024	Prop
	7.5	0.7003	0.7141	0.7134	0.7121	Prop
	8.5	0.7082	0.7233	0.7219	0.7233	MOM
	9.5	0.7176	0.7307	0.7288	0.7309	MOM
	10.5	0.7242	0.7371	0.7365	0.7374	MOM
	1.5	0.5500	0.5557	0.6064	0.5737	MLE
	2.5	0.6000	0.6061	0.6426	0.6016	Prop
	3.5	0.6323	0.6382	0.6736	0.6542	MLE
	4.5	0.6562	0.6814	0.7042	0.6772	Prop
75	5.5	0.6742	0.6894	0.7192	0.6946	MLE
	6.5	0.6889	0.6993	0.7319	0.6936	Prop
	7.5	0.7003	0.7130	0.7431	0.7081	Prop
	8.5	0.7082	0.7243	0.7504	0.7089	Prop
	9.5	0.7176	0.7333	0.7574	0.7188	Prop
	10.5	0.7242	0.7462	0.7623	0.7277	Prop

From Table 1 we find the best first estimator for fuzzy hazard rate function is $\hat{h}_{mom}(t_i)$ with percentage $\left(\frac{14}{30}\right) = 47\%$ while $\hat{h}_{prob}(t_i)$ is best with $\left(\frac{11}{30}\right) = 37\%$ while MLE best with $\left(\frac{}{30}\right)$ 16%. So we prefer the estimation by moments first and then by proposed and finally by MLE.

Table 2: Estimators of fuzzy hazard rate function when a = 0.25, q = 0.6, $\tilde{k}$ = 0.6

N	t_i	$R_{real}\ H_i$	$\widehat{H}_{mle}$	$\widehat{H}_{Mom}$	$\widehat{H}_{prop}$	*Best*
25	1.5	0.551	0.5732	0.6064	0.5737	MOM
	2.5	0.602	0.6226	0.6506	0.6216	Prop
	3.5	0.633	0.6646	0.6826	0.6542	Prop
	4.5	0.6572	0.6827	0.7032	0.6773	Prop
	5.5	0.6750	0.7088	0.7192	0.6946	Prop
	6.5	0.6888	0.7235	0.7329	0.7082	Prop
	7.5	0.7012	0.7348	0.7421	0.7189	Prop
	8.5	0.7019	0.7439	0.7502	0.7267	Prop
	9.5	0.7167	0.7515	0.7564	0.7325	Prop
	10.5	0.7231	0.7634	0.7622	0.7405	Prop
50	1.5	0.551	0.5635	0.5442	0.5536	MOM
	2.5	0.602	0.6133	0.6202	0.6025	Prop
	3.5	0.633	0.6466	0.6262	0.6342	MOM
	4.5	0.6572	0.6704	0.6612	0.6586	Prop
	5.5	0.6750	0.6918	0.6788	0.6720	Prop
	6.5	0.6888	0.7045	0.6705	0.6947	MOM
	7.5	0.7012	0.7126	0.7152	0.7066	Prop
	8.5	0.7019	0.7251	0.7228	0.7156	Prop
	9.5	0.7167	0.7334	0.7283	0.7221	Prop
	10.5	0.7231	0.7387	0.7294	0.7284	Prop
75	1.5	0.551	0.5557	0.5688	0.5688	MLE
	2.5	0.602	0.6062	0.6176	0.6082	MLE
	3.5	0.633	0.6394	0.6512	0.6412	MLE
	4.5	0.6572	0.6645	0.6745	0.6633	Prop
	5.5	0.6750	0.6714	0.6743	0.6622	Prop
	6.5	0.6888	0.6824	0.6915	0.6957	MLE
	7.5	0.7012	0.6934	0.7031	0.7044	MLE
	8.5	0.7019	0.7054	0.7124	0.7156	MLE
	9.5	0.7167	0.7141	0.7223	0.7241	MLE
	10.5	0.7231	0.7221	0.7463	0.7284	MLE

It is clear from Table (2) that the first best estimator of fuzzy hazard rate function is $\hat{h}_{prop}(k_i t_i) = \frac{16}{30} * 100$=0.53%=53% while $\hat{h}_{MLE}(k_i t_i) = 0.2666 = 27\%$ while $\hat{h}_{Mom}(k_i t_i) = 20\%$.

using three methods, which are Maximum Likelihood estimator and Moment estimator and Proposed estimator one, we note that the best first estimator for fuzzy hazard rate function is $\hat{h}_{mom}(t_i)$ with percentage $\left(\frac{14}{30}\right) = 47\%$ while $\hat{h}_{prob}(t_i)$ is best with $\left(\frac{11}{30}\right) = 37\%$ while MLE best with $\left(\frac{5}{30}\right) = 16\%$ (shown in Table 1). So we prefer the estimation by moments first and then by proposed and finally by MLE. Table 2 clearly shows that the first best estimator of fuzzy hazard rate function is $\hat{h}_{prop}(k_i t_i) = \frac{16}{30} * 100 = 0.53\% = 53\%$ while $\hat{h}_{MLE}(k_i t_i) = 0.2666 = 27\%$ while $\hat{h}_{MOM}(k_i t_i) = 20\%$. Table 3 shows the result of fuzzy estimator of hazard rate function $\hat{h}(k_i t_i)$. We find that the first

Table 3: Estimators of fuzzy hazard rate function when $\alpha = 0.25$, $\theta = 0.8$, $\tilde{k} = 0.3$

N	t_i	$R_{real}\, H_i$	$\widehat{H}_{mle}$	$\widehat{H}_{Mom}$	$\widehat{H}_{prop}$	*Best*
	1.5	0.5511	0.5821	0.6064	0.5737	Prop
	2.5	0.600	0.6334	0.6526	0.6226	Prop
	3.5	0.6674	0.6676	0.6826	0.6452	Prop
25	4.5	0.6927	0.7086	0.6931	0.6532	Prop
	5.5	0.6938	0.7246	0.6994	0.6946	MLE
	6.5	0.7002	0.7358	0.7094	0.7082	MOM
	7.5	0.7082	0.7505	0.7238	0.7188	Prop
	8.5	0.7446	0.7443	0.7345	0.7268	Prop
	9.5	0.7525	0.7575	0.7433	0.7278	Prop
	10.5	0.7578	0.7634	0.7576	0.7325	Prop
	1.5	0.5511	0.5699	0.5624	0.5559	Prop
	2.5	0.600	0.6188	0.6132	0.6059	Prop
	3.5	0.6674	0.6543	0.6468	0.6394	Prop
	4.5	0.6927	0.6625	0.6720	0.6633	MLE
50	5.5	0.6938	0.6684	0.6884	0.6824	MLE
	6.5	0.7002	0.7028	0.7039	0.7962	MLE
	7.5	0.7082	0.7142	0.7042	0.7054	MOM
	8.5	0.7446	0.7243	0.7233	0.7256	MLE
	9.5	0.7525	0.7251	0.7374	0.7242	Prop
	10.5	0.7578	0.7325	0.7394	0.7297	Prop
	1.5	0.5511	0.5582	0.5538	0.5523	Prop
	2.5	0.600	0.6087	0.6245	0.6032	Prop
	3.5	0.6674	0.6427	0.6352	0.6365	MLE
	4.5	0.6927	0.6653	0.6586	0.6549	Prop
75	5.5	0.6938	0.6828	0.6762	0.6913	MOM
	6.5	0.7002	0.6964	0.6889	0.7024	MOM
	7.5	0.7082	0.7075	0.7008	0.7115	MLE
	8.5	0.7446	0.7166	0.7057	0.7192	MOM
	9.5	0.7525	0.7246	0.7152	0.7182	Prop
	10.5	0.7578	0.7403	0.7226	0.7265	MOM

Table (3) shows the result of fuzzy estimator of hazard rate function $\hat{h}(k_i t_i)$. We find that the first best one is $\hat{h}_{prop}(k_i t_i) = \frac{17}{30} * 100 = 57\%$ while $\hat{h}_{Mom}(k_i t_i) = \frac{6}{30} * 100 = 20\%$ and $\hat{h}_{MLE}(k_i t_i) = \frac{7}{30} * 100 = 23\%$ so the first best estimator of fuzzy hazard rate function of second order failure to time model is proposed one and then MLE is second best, while moments is best with third degree.

best one is $\hat{h}_{prob}(k_i t_i) = \frac{17}{30} * 100 = 57\%$ while $\hat{h}_{MOM}(k_i t_i) = \frac{6}{30} * 100 = 20\%$ and $\hat{h}_{MOM}(k_i t_i) = \frac{7}{30} * 100 = 23\%$ so the first best estimator of fuzzy hazard rate function of second order failure to time model is proposed one and then MLE is second best, while moments is best with third degree.

$\hat{h}_{(x_i)}$	*Table (1)*	*Table (2)*	*Table (3)*
MLE	16%	27%	23%
Moments	47%	20%	20%
Proposed	37%	53%	57%

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