Asian Journal of Water, Environment and Pollution, Vol. 4, No. 2, pp. 49-55.

A Coupled Mathematical Water and Salt Balance Model of Flat Bay

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Received November 7, 2005; revised and accepted June 22, 2006

Abstract: In coastal environment, the availability of good quality surface water draws significance because of salinity problems that mainly arise because of salt-water intrusion and tidal flow. The temporal extent of salinity and dynamics of surface water through a lake are basic information needed for water resources management. In this study, a coupled mathematical model for water and salt balance has been presented to simulate the bank storage, outflow and outflow salt concentration. The model has been applied to the lake Flat bay, situated on the northern side of Port Blair in Andaman and Nicobar Islands, India. Water and salt budgets are used to predict the behaviour of this natural lake in terms of both quality and quantity. Convolution relation is used to simplify the mathematics of bay-bank interaction for estimating bank storage flow. Flow in and out of the aquifer at the bay bank is determined for the semi-infinite aquifer for different aquifer parameters to investigate the sensitivity of the bank storage flow. The model developed herein will be an efficient tool for appropriate development of coastal bays and reservoir regulation strategies to meet the fresh water demand in coastal areas and also to ensure appropriate management of water resources in terms of both quality and quantity.

Key words: Bank storage, water budget, salt budget, Flat bay, lake modelling.

Introduction

Increased demand for water associated with population growth has heightened the importance of proper management of limited water resources. Most of groundwater aquifers and surface water reservoirs are connected to one another; therefore, the use of one can affect the quantity and quality of the other. It is perhaps because of this reason that water resource managers have taken considerable interest in quantification of the interaction of surface water and ground water (Moench and Barlow, 2000). Incorporating the role of bank storage flow is important for gaining insight into the surface water quantity and quality dynamics through a water body system. Bank storage flow may considerably smoothen the water dynamics and contribute in the salt balance of a lake system (Chen and Chen, 2003). Flow into and out of a bay bank and volumes of bay water are dependent on a number of geologic and hydrologic parameters of a particular bay-bank system.

Venetish (1970) considered certain aspects of the convolution relation as applied to finite and semi-infinite aquifers. He showed that the convolution relation giving the response of an aquifer at a specified distance from a stream could be written in two ways: (i) when the system instantaneous unit impulse response function is used, the system input is the fluctuation of head at the source, and (ii) when the unit step response function is used, the system input is the time derivative of the head at the source. Hall and Moench (1972) briefly reviewed the use of convolution relation and represented expressions in discrete form for solving the diffusion equation. Moench and Barlow (2000) presented Laplace transform step-response functions for various homogeneous confined

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and leaky aquifers and for anisotropic, homogeneous unconfined aquifers interacting with perennial streams. In their stream-aquifer model, the stream was assumed to penetrate the full thickness of the aquifer, and the infiltrated stream water during the flood was accumulated and stored on both sides of the stream. Barlow et al. (2000) applied the analytical step-response functions of Moench and Barlow (2000) for the analysis of stream-aquifer interactions along the Blackstone River in central Massachusetts and the Cedar river in the eastern Iowa.

A dynamic hydrologic model was presented by Crowe (1993) to assess lake-groundwater systems to provide insight into the effects of climatic variability on the Wabamun lake and its watershed in Alberta, Canada. Sensitivity analysis indicated that small changes in temperature or precipitation throughout the watershed, might significantly impact upon the quality of the lake water and the availability of groundwater resources. Specifically, a long-term temperature rise that increases evaporation, or a decrease in precipitation, will reduce the amount of water available to recharge the groundwater-flow regime. Groundwater storage in the watershed will further decline due to continued discharge to the lake. The effect of climatic variability on the quantity of water in the Wabamun lake was relatively small. However, the salinity of the lake will increase dramatically.

Benduhn and Renard (2004) investigated the dynamic evolution of the sea using a coupled mathematical model of water and salt balance of the Aral Sea. The water balance considered river inflow, groundwater inflow, atmospheric precipitation and evaporation. The salt balance considered the dominant ions dynamics. The evaporation rates were calculated with a modified Penman equation accounting for the salinity of the lake and using statistical climatic data. Model developed by them was inspired from the work of Asmar and Ergenzinger (2002) for the Dead Sea. These analyses, based on inland drainage flow system, only partially revealed the interactions between the bank and sea. They assumed one directional flow between bank and sea; while it is true that water also flows from sea to bank under rising stage condition. The objective of this study is to develop a mathematical model to investigate the general behaviour of a natural coastal bay in terms of both quantity and quality by taking two-way bay-bank interaction for predicting the outflow responses.

Model Description

The developed mathematical model consists of two mass balance equations; one for water and another for salt. A monthly-basis simulation model is developed to obtain the bank storage flow, velocity of outflow, salt stored in the lake and outflow salt concentration. The input parameters for this deterministic model are monthly inflows into the lake, evaporation, precipitation, lake characteristics and aquifer parameters.

Water Budget

A prerequisite to analyzing the system's water quantitative behaviour is accounting of all the water movement through it. The most straightforward approach is a simple accounting of inflows, outflows and storage change. The water budget considers inflow from different streams and tributaries, precipitation, bank-storage, evaporation, outflow and surface storage. All these factors are combined into one equation by taking a mass balance of water in the system. This yields:

$$S_{i+1}^{w} = S_{i}^{w} + \left(\sum_{j=1}^{n} I_{i,j} - \sum_{j=1}^{m} O_{i,j} + P_i - B_i - E_i + \varepsilon_i\right) * \Delta t \tag{1}$$

where S^w is the total surface storage [L^3], i is the index for time step, j is the index for number of streams, n is the number of inflow streams and m is the number of outflow streams, I is inflow rate [L^3T^{-1}], O is the outflow rate [L^3T^{-1}], Δt is the time interval [T^{-1}], P is the precipitation rate [L^3T^{-1}], E is the evaporation rate [L^3T^{-1}], B is the bank storage flow [L^3T^{-1}], and ε is the error term.

Stream water infiltrates into a hydraulically connected aquifer during a rising flood stage and its reverse motion during stream flow recession recharges the stream; the volume of water so stored and released after the flood is referred to as bank storage (Singh, 1968). Formulation for bank storage flow begins with one-dimensional diffusion equation (Hall and Moench, 1972).

$$\frac{\partial^2 h}{\partial x^2} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{2}$$

where h is the piezometric head [L], S is the aquifer storage coefficient, T is the aquifer transmissivity [L^2T^{-1}], x is the distance from bay bank and t is the time.

For a linear boundary value problem discussed by Moench and Barlow (2000), the total response of a stream-aquifer system to a time series of individual input can be determined by superposition or convolution of the system's response to that individual input. The convolution integral can be written in general form as

$$h(x,t) = \int_0^t F(\tau)U(x,t-\tau)d\tau \tag{3}$$

or as

$$h(x,t) = \int_0^t F'(\tau)P(x,t-\tau)d\tau \tag{4}$$

where $F(\tau)$ is the system input [L], $U(x, t)$ is the instantaneous unit impulse response function of the system [T^{-1}], $h(x, t)$ is the water table elevation [L], $p(x, t)$ is the unit step response of the system (dimensionless) and $F'(\tau)$ is the time derivative system input.

The flow in or out of the bay bank is obtained by applying Darcy's law to the above equation, and thus the flow can be expressed as:

$$Q(t) = T\int_0^t F'(\tau)\frac{\partial P(0,t-\tau)}{\partial x}d\tau \tag{5}$$

where $Q(t)$ is the discharge and T is the aquifer transmissivity.

For a semi-infinite aquifer, the unit step response can be expressed as (Hall and Moench, 1972):

$$U(x,t) = \frac{x}{(4\pi\alpha t)^{1/2} t^{3/2}} \exp\left[-(\frac{x^2}{4\alpha t})\right] \tag{6}$$

$$P(x,t) = \operatorname{erfc}\frac{x}{(4\alpha t)^{1/2}} \tag{7}$$

$$\frac{\partial P(0,t)}{\partial x} = \frac{1}{-(\pi\alpha t)^{1/2}} \tag{8}$$

where α is the aquifer diffusivity and erfc is the complimentary error function.

Equation (5) in discretized form is applied to the bay-bank system of Flat Bay considered in the present application. The response function is terminated when the ordinates are relatively insignificant. After estimating all terms in the water budget equation to account for all inflows and outflows, applying the budget consecutively in finite step fashion from an initial condition can make a good approximation of the water in storage at any time.

Salt Budget

Salt budget is parallel to the one described for water except dissolved salt is accounted for instead of water. This budget is also calculated on monthly basis. The major system features are inflow salt from different streams and tributaries, bank-storage contribution, outflow salt and surface salt storage. The outflow response to a variety of inflow time series is given by the following equation obtained by mass balance of dissolved salt.

$$S_{i+1}^s = S_i^s + [\sum_{j=1}^n (I_{i,j} \times C_{i,j}^{IS}) - (\sum_{j=1}^m O_{i,j})C_i - B_i \times C_i + k \times C_i \times S^w]\Delta t \tag{9}$$

where S^s is the total salt storage in the lake [M], C^{IS} is the inflow salt concentration [ML^{-3}] and C is the outflow salt concentration [ML^{-3}] and k is the decay rate coefficient [T^{-1}], which is zero for salt. It is assumed that the salt concentration in water entering and leaving the bank storage is equal to the mean lake concentration.

The prediction of salt outflow concentration is based upon the assumption that the inflow is instantaneously and thoroughly mixed with water in the bay. It means that the concentration is the same throughout the reservoir and in the outflow at a given time. Dingman and Johnson (1971) have applied this assumption to mixing models of water quality from several New Hampshire lakes and developed equations expressing the outflow concentrations as function of lake volume, lake concentration and several other parameters. The analytical results were applied to 23 specific lakes to estimate their residence time and equilibrium concentration for a constant inflow concentration. Thus, such assumption appears to be valid within the desired accuracy for real life problems.

The system output at any time t will be the integral of the responses from past pulse inputs, which are expressed via the Markov model as:

$$C_i = \sum_{j=1}^i (Z \times j)K(i-j+1) \tag{10}$$

where $Z(j)$ is the system input at time j and $K(t)$ is the response function given as below:

$$K(t) = \frac{1}{Q}[e^{-(t-\Delta t)/T} - e^{-t/T}] \tag{11}$$

where Q is the total outflow, T is the ratio of the volume to the flow which represents the hydraulic detention time, that is, the time it would take to empty out the lake if all inputs of water to the lake ceased.

The response function is terminated when the ordinates are relatively insignificant. After estimating all terms in the salt budget equation to account for all incoming and outgoing salt, the salt budget is applied consecutively in finite step fashion from an initial salt concentration to make a good approximation of the salt storage in the bay at any time.

Application To Flat Bay

The mathematical model presented in this paper is applied to a coastal lake, Flat bay, to evaluate bank storage flow and outflow salt concentration. The Flat bay (Figure 1) is situated on the northern side of Port Blair, capital city of Union Territory of Andaman and Nicobar Islands, India. The Government of India is considering the bay for the development of a fresh water lake (WAPCOS, 1999; Keshari, 2001). Andaman and Nicobar Islands consist of approximately 500 islands and are located at latitudes 6° N to 14° N and longitudes 92° E to 94° E in the Bay of Bengal. They have prominent topographical features with reserve forests, numerous creeks and bays. In recent times, the islands have acquired immense importance due to their strategic location, natural resources and tourism. The Flat bay lies between latitudes 11°36′37″ N to 11°39′19″ N and longitudes 92°39′47″ E to 92°42′26″ E with total drainage area of about 50 km^2. Highlands and hillocks sloping towards the bay surround the lake.

Data Availability and Processing

The entire drainage area of the considered water body falls in the tropical region with considerable annual rainfall, approximately 3000 mm. The depth of unconfined aquifer of the study area varies from 30 m to 50 m. The subsoil is silty clay underlain by rock formations. The map of Andaman and Nicobar Islands of 1:50000 is used as a base map in this study. The geomorphology of lakes including creeks, command area and high and low water areas are obtained from this base map. Contour lines are used for the determination of the cross sectional area of the outflow of the Flat bay. The approximate high and low water areas of the Flat bay are 12.5 Mm^2 and 4.25 Mm^2, respectively (WAPCOS, 1999; Keshari, 2001). Table 1 shows the catchments area, yield and salt concentration of four creeks joining the Flat bay, namely, Bumlitan, Dhanikhari, Chouldhari and

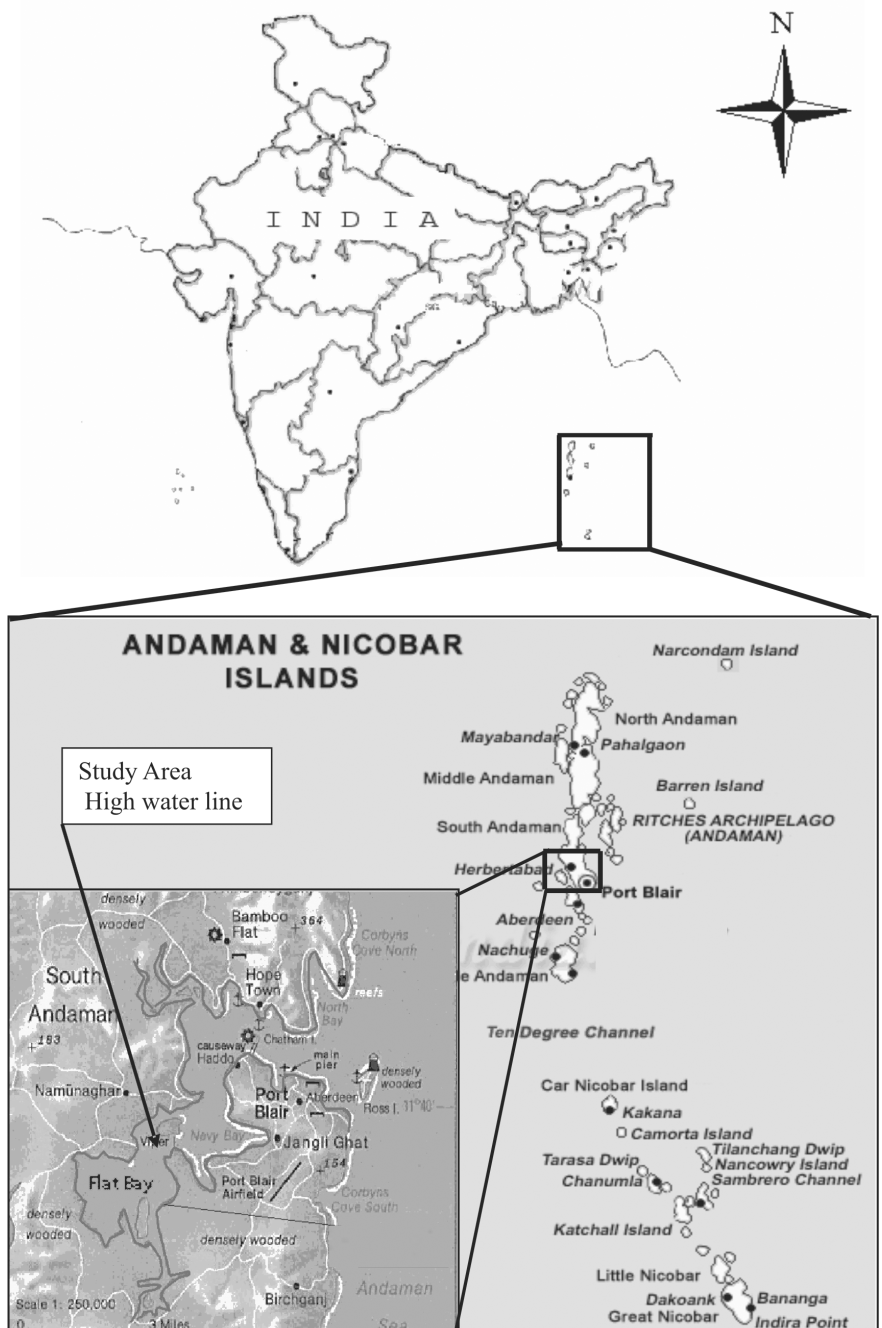

Figure 1: Location of Flat bay.

Table 1: Inflows and quality characteristics of creeks of Flat bay

S. No.	*Catchment*	*Area (Mm^2)*	*Yield (Mm^3/Yr)*	*Salt Concentration (mg/l)*	*Project activity*
1	Bumliton	23	29	60.73	Existing dam takes 6.2 Mm^3/Yr
2	Dhanikhari	13	16	63.82	Dam is planned to draw 1 Mm^3/Yr
3	Chouldhari	7	16	63.82	*
4	Homfreys	7	16	70.91	*

*No dam is existing or planned.

Homfreys. The initial salt concentration of the lake is 25 ppt (parts per thousand). These data are based on measurements taken for these creeks.

Historical evaporation and rainfall data of the Flat bay in Andaman and Nicobar Islands is analyzed and processed for the application of the developed model. The mean monthly evaporation and rainfall data over the study area for a period of five years from 1981 to 1985 are used for the present study. The lake stage and corresponding storage capability are given in Table 2. Accurate computation of runoff amount is difficult, as measurement has not been carried out in the past. No attempt has been made in the past for predicting runoff using conceptual models based on the watershed characteristics and other influencing climatic and soil parameters. For planning and management of water resources in the region, runoff is estimated by field engineers using rational method (Chow, 1988; Wurbs and James, 2002) assuming runoff coefficient as 0.7 (WAPCOS, 1999; Keshari, 2001). This value is based on the past experience and limited observations for the study areas.

Table 2: Variation in lake capacity with stage of Flat bay

Elevation (m)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Capacity (Mm^3)	2.4	3.8	5.5	7.0	8.8	10.8	13.2	15.7

Results and Discussion

Results are obtained for the Flat bay using developed mathematical model to investigate the behaviour of lake in terms of both quality and quantity over the period 1981 to 1985. Figure 2 shows the monthly variation of bank storage for the Flat bay. It is evident from this figure that most of the time flow takes place from the lake to the aquifer, and only for few months prior to monsoon, the flow takes place from the aquifer to the lake. The temporal variation of bank storage is almost same year to year. A comparison of results shown in Figure 3 for different storage characteristics of aquifer indicates that the bank flow is sensitive to the aquifer storage coefficient during the monsoon period. Results are shown for two values of specific yield, $S = 0.005$ and $S = 0.003$. The higher value of storage coefficient allows the aquifer to gain more water from the lake during the high stage period and vice versa. For the same lake stage, it is not necessary that bank storage will be same. It will depend upon the history of water level declining or rising.

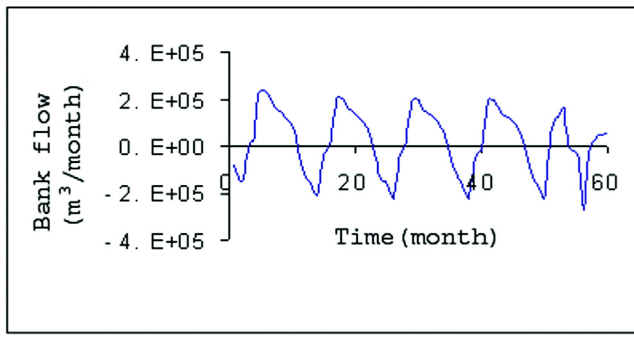

Figure 2: Monthly variation of bank storage.

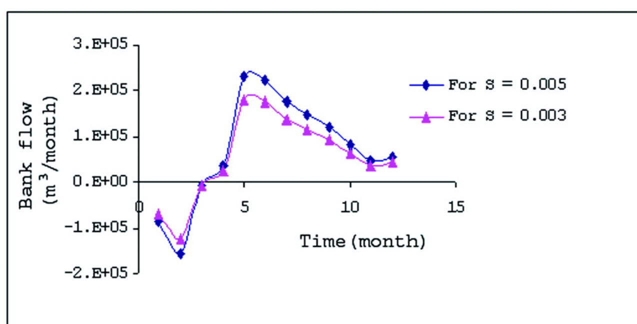

Figure 3: Variation of bank storage flow for different aquifer parameters.

Figure 4 shows monthly variation of outflow from the Flat bay. It indicates that outflow of the lake is always positive i.e. water is flowing from lake to sea over the period 1981 to 1985. Results are validated based on the measured and simulated mean velocity for the year 1985. The simulated velocity is shown in Table 3. The average

Table 3: Simulated mean monthly velocity for the year 1985

Month	*Jan*	*Feb*	*Mar*	*Apr*	*May*	*Jun*	*Jul*	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
Velocity (cm/sec)	0.1	0.15	0.07	0.13	0.37	0.67	0.45	0.53	0.56	0.55	0.35	0.32

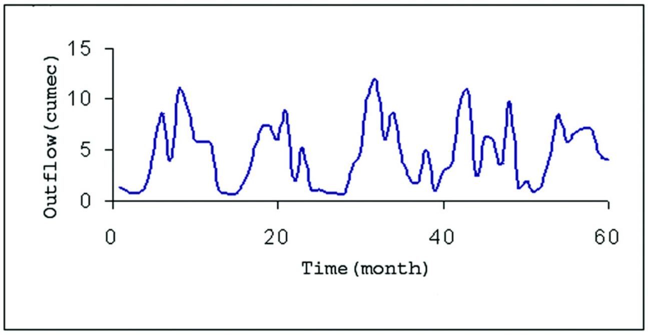

Figure 4: Monthly variation of outflow from the bay.

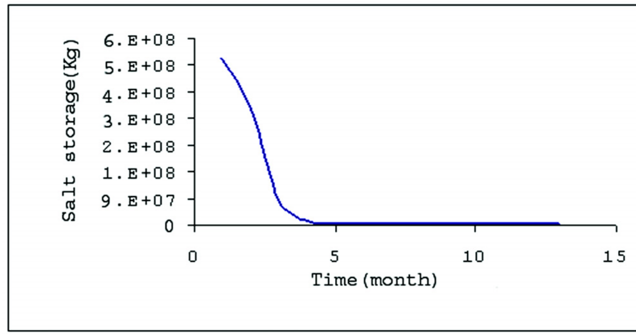

Figure 7: Monthly variation of salt stored in the bay.

value of the simulated mean velocity is 0.35 cm/sec while the observed mean velocity shows a value of 0.25 cm/ sec. Figure 5 shows the variation of bank storage with bay stage. With the increase in bay stage, water flows from lake to aquifer increase appreciably with non-linear behaviour. If measures are taken to control the entry of seawater into the bay, quality of bay water can be improved with time. Figures 6 and 7 show the response of the bay system under such conditions. These figures depict monthly variations of lake salt concentration and salt stored in the bay, respectively. It is evident from these figures that after ten months the bay water quality is guided by the quality of inflows coming to the bay from the catchments. It is observed that the mathematical model predicts less outflow concentration than what actually occurred in the lake. This difference between measured and computed concentration can be attributed to the fact that the model does not consider the effect of tidal flow.

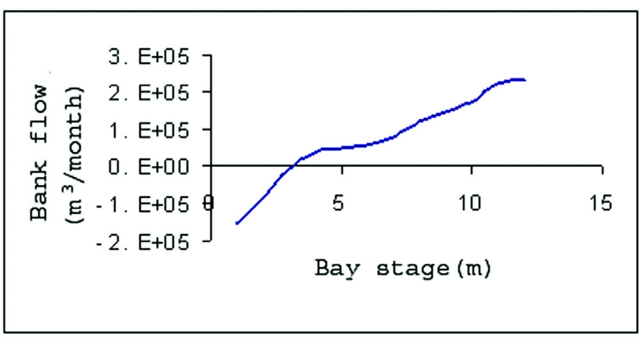

Figure 5: Monthly variation of bank storage flow with bay stage.

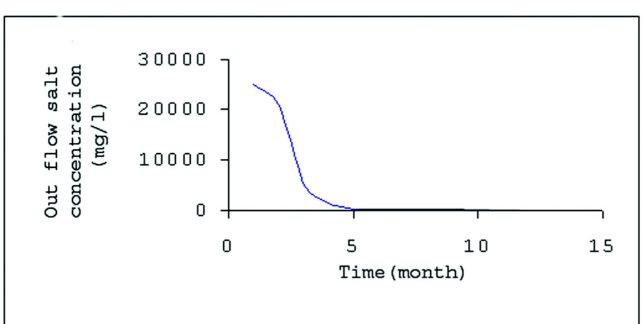

Figure 6: Monthly variation of bay salt concentration.

Conclusions

A coupled water and salt balance deterministic model has been presented to predict the behaviour of a natural lake in terms of both quality and quantity. The model utilizes convolution relation to simplify the bay-bank interaction for simulating bank storage flow. The model has been applied to a natural coastal lake, Flat bay, situated on the northern side of Port Blair in Andaman and Nicobar Islands, India for simulating bank storage flow, bay outflow, salt stored in the bay and outflow salt concentration on monthly basis. It is observed that bank flow is sensitive to the aquifer storage coefficient suggesting for more precise lake water quality modelling; more emphasis should be placed on bay-bank interaction interface. With the increase in lake stage, bank flow from lake to aquifer increases appreciably with non-linear behaviour.

The model's evolution of the bay shows that the quality of the coastal bay can be improved within a year if the seawater ingress is ceased and inflow conditions of 1980s of four creeks are maintained. Thereafter, lake salt concentration depends mainly on the quality of inflow water. The model developed herein is an efficient tool for planning, management and regulation strategies of natural bays, lakes and reservoirs.

Acknowledgements

Authors are thankful to Water and Power Consulting Services (I) (WAPCOS) Ltd., New Delhi and Andaman and Lakshdweep Harbour Works (ALHW), Port Blair for making the data available for carrying out the research work.

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Asian Journal of Water, Environment and Pollution

Volume 3 Number 2 **July – December, 2006**

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