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Empirical Validation of Field Estimated Dispersion Coefficient for a Tropical River

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Abstract: A dye dispersion study conducted for the study reach during low flows has been resolved to provide information on the dispersion coefficient (D_L) of the Hindon river. Logistic reasons often constrain the field study for this parameter, considered crucial for any water quality modelling study. It, therefore, becomes imperative to identify, amongst several empirical formulas, the most appropriate formulation for the site specific area. In the present study, the classical methods have been used to identify the McQuivey & Keefer (1974) and Liu (1977) as the most suitable empirical relationship for estimation of D_L for Hindon river reaches, for which field study was not practically possible.

Key words: Tracer, Rhodomine B**,** time of travel**,** dispersion coefficient.

Introduction

Fate of contaminants discharged into any riverine system apart from the flow, is largely controlled by the travel time and dispersion characteristics of the system. Water quality modelling draws the inputs of these variables to simulate the impact of the contaminants on the general health of the water system, thus necessitating their estimation.

Tracers, usually dyes or salts, are consistently being used to predict the behaviour of soluble contaminants entering the system. The field procedure involves injection of dye, instantaneous or continuous as the case may be, and then tracing the movement of the dye cloud considered analogous to the contaminant cloud as it spreads and smears, giving a concentration profile along the stream.

For an instantaneous release of dye, the dye cloud movement at the centroid with the mean flow of the stream defines the time of travel while changing dye cloud distribution defines dispersion (Martin and McCutcheon, 1999).

Where time of travel studies provide an insight to the carrying capacity of the stream/river, dispersion or mixing characteristics of the stream/river affects the chemical gradients of the constituents and has been a subject of extensive research over the past 50 years. Longitudinal dispersion coefficient or simply dispersion coefficient is an important parameter in the advection-diffusion equation of the solute transport in the rivers and is given by:

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D_L \frac{\partial^2 C}{\partial x^2} \quad (1)$$

where C is the mean concentration [ML^{-3}], U is the mean velocity [LT^{-1}], x the longitudinal length [L], t is time [T], and D_L is a constant longitudinal dispersion coefficient [$L^2 T^{-1}$].

Estimation of dispersion coefficient (D_L), as a necessary input to modelling study, includes usage of simple empirical relations relating dispersion coefficient to the shear stress (Fischer, 1967, 1968; Elder, 1959; Liu, 1977) or by conducting dye tracer experiments in the field (Fischer, 1967, 1968). Method of moments and

Fischer routing method utilize the time concentration curves obtained in the field tracer study to estimate the dispersion coefficient.

This paper documents a tracer study on a river reach conducted with the following objectives: (1) estimating the dispersion coefficient using classical methods viz., Method of moments and Fischer routing method, and (2) identifying the most appropriate empirical relation for the estimation of dispersion coefficient by way of comparing the results of both experimental and empirical methods of estimation for the study reach.

Study Location

The river Hindon, located within the bounds of 29° and 30°15' Latitude and 77°15' and 78° longitudes, is a rain-fed river that originates in the foothills of Shivaliks (lower Himalayas) in India. It traverses a distance of some 110 km beyond the study reach before it joins river Yamuna which eventually is a major tributary of the mighty Ganges. During its course of travel, the river Hindon along with its two main tributaries river Kali and river Krishni, drains four agriculturally and industrially developed districts of Uttar Pradesh, India namely: Saharanpur, Muzaffarangar, Baghpat and Meerut (Figure 1).

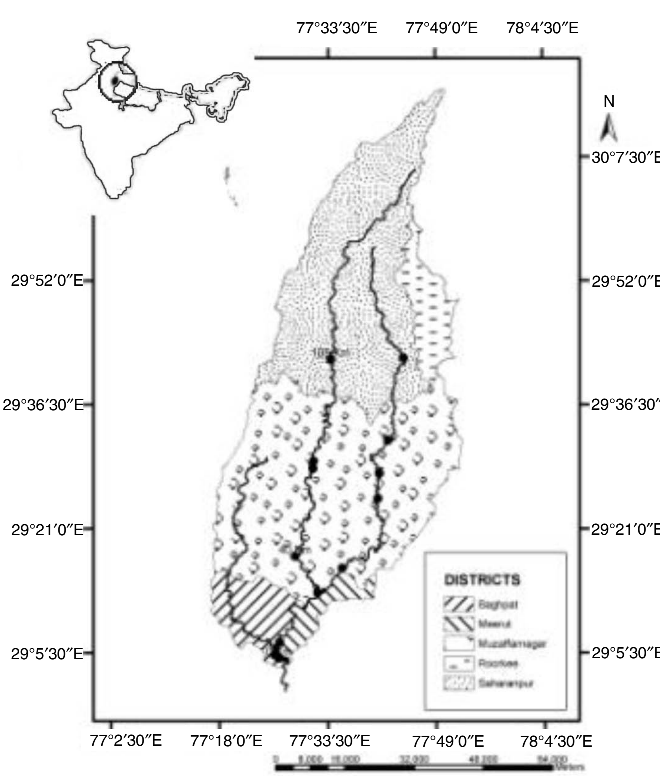

Figure 1: Study location.

Water quality of the river along with its tributaries reflects the impact of number of small and medium scale industries viz. sugar mill, paper and paper board along with the municipal effluents emptying into the river.

In the absence of any literature available on the dispersion characteristics of the river, dye dispersion study was conducted as a part of steady state water quality modelling exercise of the river stretch. However, lack of gauging station anywhere on the study reach except at the terminal point together with the logistics reasons restricted the study to 7.5 km of the river reach under study.

Planning Dye Dispersion Study

Preliminary Survey

Tracer study was coordinated with the routine water quality sampling programme, for Hindon river, during low flow season. With fairly good estimates of the geometry of the study reach in hand determined during several sampling events, an injection site and two sampling stations were fixed prior to the day of the field experiment. Figure 2 shows the geometry of the reach along with the schematic representation of the sampling stations and the injection point.

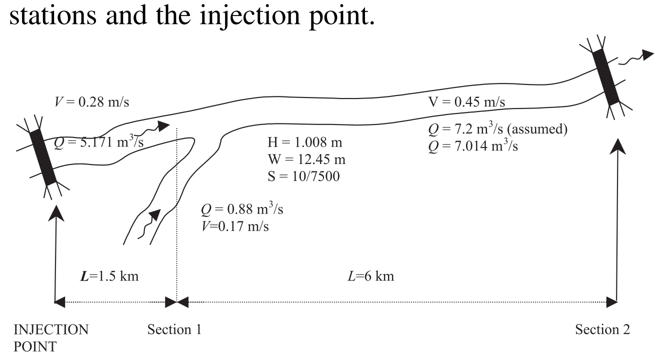

Figure 2: Schematic diagram of study reach.

Dye and Dye Dosage Requirement

Rhodamine B was used as a dye because of its ease of availability along with it fulfilling the requirements of an effective tracer such as: water solubility, harmless in low concentration and detectability at low concentrations. However, its tendency to adsorb to solids was duly taken into account.

The dye dosage requirement was determined from the following relation (USGS, 1989):

$$V = 2.0 \times 10^{-3} \left(\frac{Qm \times L}{v} \right)^{0.93} \times C_p \qquad (2)$$

where V is the volume of Rhodamine B in litres; Q_m is the maximum stream discharge at the downstream site, in cubic metre per second; L is the distance to the downstream site, in km; v is the mean stream velocity, in metre per second, and C_p is the peak concentration at the downstream sampling site, in micrograms per litre.

Given the properties of the dye viz. solubility (= 20 gm/L) and specific weight (=1.12) and with first estimate of reach characteristics available viz. discharge and velocity (Figure 2), the dye dosage requirement for expected peak concentration of one microgram per litre was determined as 192 gm. Thus total dye dosage concentration actually used was 200 grams per 10 litre of distilled water.

Estimation of Travel Times

The sampling schedule for each section was planned on the basis of the initial estimate of travel times. The following relations were used to estimate at each section the time to begin sampling and duration of sampling (USGS, 1989):

$$T_p = 0.28 \times \frac{L}{V_p} \tag{3}$$

$$T_{D10} = 0.7 \times T_p \times \exp(0.86) \tag{4}$$

$$T_L = T_p - \frac{1}{3} \times T_{D10} \tag{5}$$

$$T_{t10} = Tp + \frac{2}{3} T_{d10} \tag{6}$$

where L is in km; and v_p is velocity of peak in m/sec; T_{D10}, in min is the duration corresponding the time when the receding concentration reaches 10 percent of peak; T_L, in min is the elapsed time to the leading edge of the dye cloud, T_{t10}, in min is the time elapsed to the trailing edge to the 10 percent level.

Table 1 shows the time estimates at each section; also shown in the table are the time values actually observed with the passage of tracer cloud at each section.

Table 1: Expected time estimates

Section	*Distance from the injection point (km)*	*Expected time characteristics (min)*			*Observed time characteristics (min)*		
		T_L	T_p	T_{t10}	T_L	T_p	T_{t10}
1	1.5	58.74	87	144	45	85	140
2	7.5	256	304	398	240	305	410

Sampling and Analysis

Injection of Dye

An instantaneous slug type of injection was made from the bridge at the site of injection station. A line injection was thus made possible by moving across the bridge, thereby allowing for rapid initial mixing.

Sample Collection

Grab samples from the middle section of the river were collected at least half an hour before the expected time of the leading edge at each sampling station. The frequency of the sampling was fixed as 5 min at each station. Background samples were collected before the expected arrival of the cloud. The total number of samples collected were 38 and 47 respectively at each station.

Analysis of Samples

The samples were collected and brought to the laboratory for the purposes of analysis. Spectroflourometer with an appropriate filter and suitably calibrated with the standard solutions was used to analyze the samples. The concentration values so obtained were adjusted for (1) the background concentration at each section, and (2) the inflow of the tributary at section 2. The adjusted concentration values at each station vs. elapsed time were then plotted as a smooth line on a simple graphical scale (Figure 3).

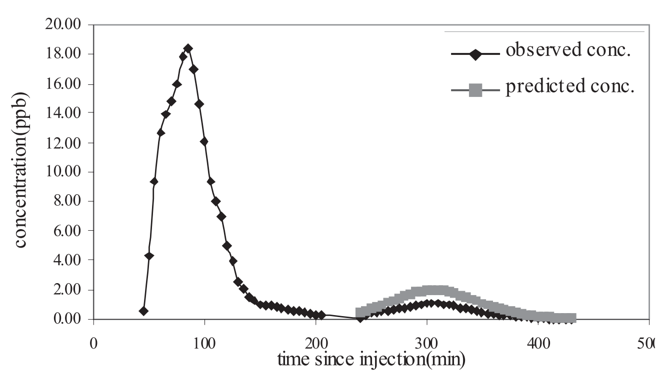

Figure 3: Observed time-concentration curves.

The obtained time concentration curves are characterized by an abrupt leading edge and a long tail, depicting the essential feature of a skewed distribution.

Estimation of Dispersion Coefficient

Dye Tracer Experimental Approach

The spread of the cloud in the response curve forms the basis of the estimation of dispersion coefficient by this approach. Method of moments and the Fischer routing method was used to evaluate the D_L of the study reach.

Method of moments is based on the basic definition of the variance of a material that behaves according to the diffusion equation (Eq. 1) (Martin and McCutcheon, 1999).

The magnitude of the longitudinal dispersion coefficient is estimated from time concentration data obtained from two stations as:

$$D_L = \frac{U^2}{2} \times \frac{\sigma^2_{td} - \sigma^2_{tu}}{t_{cd} - t_{cu}} \tag{7}$$

where U is the mean stream velocity; σ^2_t is the variance of the concentration-time curve (extrapolated to 1% of peak dye concentration); t_c is the time of travel to the centroid of the curve; and the subscripts d and u refer to downstream and upstream respectively. The variances are computed from:

$$\sigma^2 = \frac{\int Ct^2 dt}{\int C dt} - t^2_c \tag{7a}$$

and the centroid time of travel is computed from:

$$t_c = \frac{\int Ctdt}{\int Cdt} \tag{7b}$$

The magnitude of D_L as estimated from Eq. 7 with t_{cu}=89.21 min; t_{cd}= 310.23 min and σ^2_{cu} =683.33 min^2; σ^2_{cd}=1309.98 min^2 was therefore found to be equal to 17.4 m^2/sec.

Fischer routing method is mathematically a convolution of the input distribution with a linearized one dimensional response function as follows (McQuivey and Keefer, 1974):

$$C(x_2, t) = \int_{-\infty}^{\infty} \frac{C(x_1,\tau)U}{\sqrt{4\pi D_L(t_{cd} - t_{cu})}} \exp\left\{-\frac{[U(t_{cd} - t_{cu} - t + \tau)]^2}{4D_L(t_{cd} - t_{cu})}\right\} d\tau \tag{8}$$

where $C(x_2, t)$ is the predicted value of concentration at station x_2 at time t, $C(x_1, \tau)$ is the observed value of concentration at station x_1 at time τ, and t_{cd} and t_{cu} are time to centroid of travel at downstream section x_2 and upstream section x_1 respectively.

The usage of the above equation, within a range of expected value of dispersion coefficient, allows routing of observed concentration distributions at the upstream section to predict distribution at section where a second concentration curve has been observed. The value of dispersion coefficient that produces a best fit of the predicted and observed is selected.

The estimation of D_L by this method, though considered to give best possible match in a statistical sense between predicted and observed concentrations (McQuivey and Keefer, 1974), may however produce erroneous results due to numerical integration of the convolution integral given by Eq. 8 and frozen cloud approximation (Singh and Beck, 2003).

However for the present study, the downstream concentration as predicted using Eq. 8, within a broad range of D_L, identified from the method of moments, yielded an acceptable error corresponding to a D_L value equal to 29.17 m^2/sec. The predicted values were plotted along with the observed profile and shown in Figure 3. The RMSE for the two profiles, viz. predicted and observed, has been evaluated as equal to 0.5.

Several empirical and semi-empirical formulations are available in the literature that, generally, relate the dispersion to the bottom shear stress (τ_0) and thus to the shear velocity (U*) of the channel. These formulations have been developed in various regional settings and are, therefore, more or less confined in their applications.

Table 2 provides a selective list of formulas as developed by various researchers which may provide, given the geometry of the stream, an estimate of the dispersion characteristics of the stream typically for model applications in the absence of measured tracer concentrations in the field. The utility of the similar formulas in evaluating the dispersion coefficient for the study reach revealed wide range of values with each value inheriting the theory of development and limitations posed by the empirical relationships.

The estimations shown in Table 2 indicate the closeness of McQuivey & Keefer (1974) and Liu (1977) formulae with that determined from the observed time-concentration curves described earlier.

Conclusions

The concentration profiles obtained from the dye tracer study, conducted during low flows, have been resolved to provide information on the dispersion characteristics of the study reach.

The dispersion coefficient was determined from the method of moments and subsequently refined using Fischer routing method. The closeness of the predicted concentration and the observed concentration at the downstream section precludes the belief of over-

Table 2: Empirical relationships for the estimation of dispersion coefficient

Reference	*Formulation*	*Formulation developed for:*	*Estimation of D_L for the study reach (m^2/sec)*
Elder (1959)	$D_L = 5.9 \times H \times u_*$	2-D channels	0.72
Glover (1964)	$D_L = 500 \times R \times u_*$	Natural streams	60.48
Parker (1961)	$D_L = 14.3 \times R^{1.5}\sqrt{2 \times g \times S}$	Open channel flow	2.34
Thackston and Krenkel (1966)	$D_L = 7.25 \times H \times u_* \times \frac{U}{u_*}$	2-D channels	1.75
McQuivey & Keefer (1974)	$D_L = 0.058 \times \frac{Q_0}{S_0 \times W_0}$	Natural streams	24.5
Liu (1977)	$D_L = 0.18 \times \frac{\beta Q^2}{u_* R^3}$, $\beta = 0.18 \times (\frac{\sqrt{grs}}{U})^{1.5}$	Wide shallow channels	25.62
Fischer (1975)	$D_L = 0.011 \times \frac{U^2 W^2}{H u_*}$	Open channel flow	0.82

R = hydraulic radius; u_* = shear velocity; H = depth; g = acceleration due to gravity; S = bed slope of the stream; W = width of stream; Q = flow discharge in the stream; and U = mean velocity of stream.

estimation of concentration values using Fischer method for the study reach specifically.

The empirical estimations revealed a wide range of variation of the dispersion coefficient and therefore question the selection of a particular relation for the estimation of dispersion coefficient in the absence of actual field data, that is often constrained by the practicality of employing the similar study in the field weighed down by insufficient gauging stations and monitoring sites across most of the tropical rivers.

The two most widely used empirical formulas, Liu as a special case of Fischer predictive formula and McQuivey and Keefer as a simplified formula verified for a variety of stream types, produce an acceptable estimate of dispersion coefficient for the study reach in line with that obtained from the Fischer Routing method. The validity of dispersion coefficient estimated by these methods thus prove useful for estimating the magnitude of dispersion coefficient for entire river reaches under modelling study within a permissible range of acceptance.

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