

Ontology of Mathematical Entities: Substantialisation

Matematik Nesnelerin Ontolojisi: Cevherleştirme

MURAT KELİKLİ 

Afyon Kocatepe University

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Abstract: Mathematics embodies a complex universe of abstract concepts in human thought. However, the ontology of mathematical objects requires a deep analysis in order to understand not only the physical world, but also beyond the abstract world. This paper considers the fundamental qualities of mathematical objects and how we can use Aristotle's theory of substance to understand these entities. Aristotle's account of mathematical objects may be incomplete, but by using his understanding of substance, I aim to present my own assessment in the most favourable light to Aristotle. Rather than being a direct assessment of Aristotle, this paper aims to explain my view as the most favourable possible.

Keywords: Aristotle, philosophy of mathematics, substance, substantiation.



Introduction

Mathematical objects are abstract entities that exist independently of the physical world. Therefore, their ontological position and reality have been questioned. This article will focus on the substantial nature of mathematical objects and offer a perspective on how they exist and how they should be understood.

Aristotle recognises substances as the basic building blocks of reality. Considering mathematical objects as substances brings a deeper perspective to their abstractness and universal qualities. Mathematical objects are grasped through reason, not through sensory observation, and thus their level of reality is of a different dimension.

According to Aristotle, whether mathematical objects are substances or not is a mystery. Some of the commentators accept mathematical objects as substance, while others reject it (Katz, 2018, p. 135). This is due to the fact that Aristotle did not engage in deep discussions about mathematics or that we have not been able to access the works in which he did. As far as we can see, Aristotle's views on mathematics are based on his criticism of Plato's view that mathematical objects are unchanging and eternal substances and Pythagoras' view that they are sensible substances in the *Metaphysics*. Nevertheless, taking Aristotle's conception of substance into account, I aim to give my own view, which is not Aristotle's definitive view, but may be. It may be difficult to interpret Aristotle's views and come to a clear conclusion on this issue, because Aristotle's views on mathematics are ambiguous at some points. However, it can be said that his conception of substance is an important perspective to consider when dealing with mathematical objects. Aristotle's approach can help us think about the abstractness of mathematical objects and their level of reality.

Discussions about the reality of abstract objects are also encountered in other fields. For example, social constructionists argue that social institutions do not exist in reality and that they are real. However, no definite explanation has been given about the process of taking mathematical objects as substance and the realisation process of the social institutions we produce. In order to explain this process, we need to look at the structure Aristotle established as non-absolute predication.



Absolute and Non-Absolute Predication

Aristotle determines predicates as absolute(natural) and non-absolute(unnatural). Aristotle considers the use of the accident to show the substance instead of the substance as a type of predication. In the expression "white is wood", "white" is not mentioned, but "the object that is white" is mentioned. Thus, by saying "white is wood", it is said that "the object that is white is wood". Accordingly, we have a substance that is an object, and this substance is taken as "white" in the subject. The reason why this substance is taken as "white" is the attribution of "white" to the substance. Afterwards, the subject of this substance is taken as "white" and wood is predicated. In that case, it is actually about the object said to be wood.

Aristotle says here that "white" as "the one who is white" expresses another being. In the expression "man is white", he says that man does not express another being. In other words, by being taken in the subject, the human shows the human, and by being taken in the white subject, it shows the human who is white. If the subject expresses another entity, it is a non-absolute predication, and if it does not express another entity, it is an absolute predication. It is understood that if a substance is called and predicated by utilising its accidents, this predication is a non-absolute predication.

It should be noted here that the traditionally accepted interpretation that the subject substance denotes absolute predication is not appropriate. In our example, "that which is white" indicates substance, whereas "white" does not refer to quality. In non-absolute predication, the subject need not denote substance, but must still denote some other entity. Commentators traditionally hold the view that predicates whose subject is substance are absolute predicates, whereas those without substance are accidental predicates (Barnes, 1993; Hamlyn, 1961, p. 121). Philoponus looks at this interpretation from a broader perspective and uses it for those in the same category. Thus, he argues that "white is colour" would also be an absolute predication (Philoponi, 1898, ll. 219.7-10; 235.29-236.8). However, it should be noted that Hamlyn says that the predication of an accident is not an accidental predication (Hamlyn, 1961, p. 118). Ebert says that accidents are analysed in this way by taking them as accidental (Ebert, 1998, p. 133).



Bronstein says that Aristotle gives a complete account of absolute predication and non-absolute predication in Second Analytics 1.19 and 1.22. Thus, he opposes traditional interpretations. He says that the predication of the definition to be obtained from here due to an accidental connection between the subject and the predicate would be absolute predication, while the predication without such a connection would be non-absolute predication. Thus, the traditional orthodoxy holds that absolute predication is determined according to the subject, whereas the more recent and especially Bronstein's assessment is that it is determined depending on the relationship between the subject and the predicate (Bronstein, 2016).

The expressions "that <white thing> is walking", "the big <thing> is wood" are non-absolute predicates, and their absolute form is "man is walking", "wood is big". In the expression "That white thing is wood", that white is not a wood because it is white, but because the wood is white. Aristotle does not mention "white" in the accidental predication; he says that white is not the subject for wood. Here, "the object that is white" is mentioned. Thus, with the expression "white is wood", it is actually said "wood that is white". In that case, we have a substance that is an object, and the subject of this substance is white. Then the subject of this substance is said to be white for wood. Then it is actually about the object that is said to be wood. Here, Aristotle said that white as white expresses another entity. In the expression "man is white", he says that man does not express another being. Thus, absolute predication is determined only according to the subject, not depending on the type of predication. If the subject expresses another entity, it is non-absolute predication, and if it does not express another entity, it is absolute predication. In other words, if the subject is taken as an accident of another subject and used for this other subject, it is non-absolute predication.

Substantiation

In our daily life and in the world of science, most of our concepts are about abstract objects. Do these objects actually exist or are they a product of our minds? Whatever the answer to this question, we can see that these entities have become a common acceptance. Mathematical objects are explicitly recognised by mathematicians, institutions by societies, and other abstract objects by groups.



It is often the case that the naming is realised by non-absolute predication. The name of the accident used in place of the substance is itself the name of the substance. In this case, the accident replaces the substance and becomes a substance itself instead of denoting a substance. In this way, accidents used instead of substance with non-absolute predication can be substantiated.

Until the 12th century, we do not come across such a structure as a university. Nevertheless, it is accepted that these universities have been providing education since 10th-11th centuries. The first known examples of universities are seen in cities such as Bologna, Paris and Oxford. Although there were institutions that provided education before, vocational training was personally received from certain professors. The student would create his own curriculum and attend the lectures of the teachers. It is common for tradesmen in cities to gather in a certain area and create a market place. Hodjas also opened classrooms in a part of the city and gave lectures. Later on, the teachers in this region united against interventions from the authorities and formed a lodge. This structure, which emerged as a region and developed as a lodge, has been referred to as a university until today (Le Goff, 2010). In other words, an accident used for a certain region and the substances in it developed with a non-absolute predication instead of the substances in this region and started to act as a substance in time.

In the expression "university students", a compound predication is realised. In other words, the university is predicated of the student and a group of students is predicated of university students. Thus, the university is attributed to each of the elements such as people, buildings, regions, vehicles, etc. as accidents. Concepts such as university, state, department are accidents. With non-absolute predication, they begin to behave like substance. However, none of these substantiated accidents are actually substances. When we say university, we do not talk about students, buildings, and professors, yet we cannot think of it separately from these because we cannot find a substance that we can place in its place. There are clusters that exist in this way but cannot accept any substance. The accidents substantiated in this way are actually empty sets. The set of empty sets is not empty, so we can form a set of universities. Enrolling, studying, graduating from a university is related to reality thanks to the substance that the



university is predicated of as accidents. Thus, we turn the predications about the university into reality. The university is not a substance, it is an accident. Thus, it is known through substances, i.e. people, buildings, students, academics, classes, etc. through the substances.

To summarise this process, the general accident is attributed to substances. Then, with non-absolute predication, the general accident begins to behave like an individual substance. Thus, it becomes a general substance thanks to the non-absolute predicates taken instead of different substances. Finally, the accidents attributed to this substantialised accident are determined.

Mathematical objects are also substantialised accidents. When mathematical objects are constituted by substances, I find the application of mathematics in the physical world. Just as we can build a perfectly functioning university system, we can build (at least try to build) a perfect mathematics. While Aristotle argues that mathematics is realised in the mind, we understand that he does not take it as a mere mind game. After all, how can a fiction in the mind have such useful and practical results? Substance is attributed "one" and is an accident. By replacing a substance, accidents are predicated by non-absolute predication. Its properties are determined according to the relationship between the accidents and substances to be predicated by substantiation. Similarly, mathematical objects are thus obtained by substantiation and are in accordance with reality thanks to the relation of these objects with substances. Thus, since the substantiated accidents are not substances, they will be the ones that are predicated about, although they do not exist as substances.

The mathematician's research is actually based on abstractions. The mathematician deals with the object by abstracting it from its physical qualities and analyses only this feature; he does not take other aspects into consideration (Aristoteles, 1831, l. 1061a24). This abstraction will be realised through substantialisation. However, the subject of mathematics is not physical objects (Aristoteles, 1831, ll. 997b35-998a5). In this context, as Lear states, Aristotle does not make mathematics the subject of study of the physical world; in other words, mathematics aims to study a field separate from physics (Lear, 1982, p. 192). Mathematicians, like physicists, work on physical objects, but unlike physicists, they work by making abstractions



through thought (Aristoteles, 1831, ll. 193b31-34). Mathematical objects have now begun to exhibit a substance structure and act as substance.

Since mathematics constructs the object by abstracting it from physical matter, it cannot perform such a process for beings without sensory matter. Therefore, mathematical certainty may not be possible in every field. This shows that the method of mathematics is not the same as the method of physical science (Aristoteles, 1831, ll. 995a15-19). Nevertheless, we think that the object of mathematics is not detached from the physical object in which it is substantiated. The reality of mathematical objects is determined by the physical substances in which they are embodied.

In 1077b31-34, Aristotle explains the idea that mathematical objects can be abstracted and analysed separately from physical objects. With the mathematical objects becoming substances, it is seen that they now act like substances and that new substantiations will occur from these new mathematical substances. Thus, Aristotle creates a legitimate basis for the abstraction of mathematical objects and provides a special approach by distinguishing ideal objects.

Basically, the mathematician begins to examine both substance and accidents starting from individual objects. Geometry focuses on the accidents of shape, while arithmetic focuses on numbers. These abstractions made by the mathematician determine different fields (Aristoteles, 1831, ll. 193b31-34). The study of non-absolute predicates between different categories on the quantities and qualities of the physical substance and the substantiated mathematical objects is carried out.

As the substantiation shows us, although we cannot perform a substantiation separate from physical objects, it is obvious that we are not obliged to have a substantiation on the accidents of physical objects. The mathematician can substantialise it by establishing a structure, and it does not have to have a substantial accident. However, the structure he will establish must definitely be connected with substances. It is possible that the substance here may be a physical substance as well as a substantiating mathematical substance. This explains to us how mathematics develops in a closed way. More precisely, it is the justification for the Platonic view that mathematicians call the mathematical world. However, contrary to the absolute existence of Plato's mathematical substances, a world that



substantiated and is not really a substance has been established. This world is not like a reflection of the world of ideas, on the contrary, it must be an image of the real world. Because it must be based on physical objects as a source.

Conclusion

Consequently, the mathematician's abstractions should not be limited to the accidents of matter, but should also take place at the levels of existence and definition (Studtmann Paul, 2002, p. 225). In this way, we can adopt the view that mathematical objects are preserved as certain qualities of accidents and that their mathematical properties are independent of accidents (Bäck, 2000, pp. 159-161). This will be given to us through substantiation.

We should also note that other systems established by Aristotle can also be evaluated about substantialisation. For example, the state system established by Aristotle in his work *Politics* is an example of this. Aristotle established and substantialised structures such as family, village and state based on individual essences. For this reason, it is seen that the concept of substantialisation, which we have defined, will be useful both in Aristotle studies and in other studies.

This mindset not only understands the structure of language, but also emphasises the abstract nature of concepts. While language is used as a tool to express abstract and concrete concepts, the ontological realities behind these tools are quite complex. The mereological perspective helps us to understand forms of existence in the real world, not limited to mathematical or logical thinking. Abstract expressions open doors that allow us to understand the depth of language and thought. Analysing the structures behind such complex expressions can deepen our understanding in both philosophy of language and ontology.

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Öz: Matematik, insan düşüncesinde soyut kavramlardan oluşan karmaşık bir evreni temsil eder. Ancak matematiksel nesnelerin ontolojisi, sadece fiziksel dünyayı değil, soyut dünyanın ötesini de anlamak için derin bir analiz gerektirir. Bu makale matematiksel nesnelerin temel niteliklerini ve bu varlıkları anlamak için Aristoteles'in cevher teorisini nasıl kullanabileceğimizi ele almaktadır. Aristoteles'in matematiksel nesnelere ilişkin açıklaması eksik olabilir, ancak onun cevher anlayışını kullanarak kendi değerlendirmemi Aristoteles'e en uygun şekilde sunmayı amaçlıyorum. Bu makale, Aristoteles'in doğrudan bir değerlendirmesi olmaktan ziyade, benim görüşümü mümkün olan en olumlu görüş olarak açıklamayı amaçlamaktadır.

Anahtar Kelimeler: Aristoteles, matematiksel nesneler, matematik felsefesi, cevher, cevherleştirme.



