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RESEARCH ARTICLE

A fuzzy–digital twin optimization framework for simultaneous management of waste and energy consumption in sustainable manufacturing

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ABSTRACT

Sustainable manufacturing systems require intelligent methods to balance economic performance with environmental responsibility. This research presents a digital twin-fuzzy multi-objective optimization framework for simultaneously managing cost, energy consumption, and waste in sustainable manufacturing. In this framework, fuzzy logic is used to model data uncertainty, a digital twin is used to obtain real-time data from the manufacturing process, and the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is used to generate a Pareto front and analyze the relationships between economic and environmental objectives. The proposed model was tested in 10 simulated scenarios based on digital twin data. The results showed that the proposed framework maintained the service level above 95%, reduced the total cost by 14% and the amount of waste by 18% compared to the baseline. Pareto front analysis also showed that although there is a relative conflict between economic and environmental objectives, this conflict is controllable. Also, sensitivity analysis revealed that energy ceiling and machinery efficiency have the greatest impact on the sustainability and profitability of the system. Overall, the proposed framework provides a reliable, quantitative decision-making tool for managers and policymakers on the path to green and sustainable production.

1. Introduction

In recent decades, industrial development and increased competition in global markets have led organizations to pay more attention to efficiency, flexibility, and sustainability in production. Pressures from rising energy costs, environmental requirements, and supply chain complexities have forced managers to adopt new decision-making approaches. [1] In the meantime, the concept of sustainable production has been proposed as a macro approach that simultaneously emphasizes the three main dimensions of economy, environment, and society. Achieving these goals requires advanced tools for data analysis, simulation of real-world conditions, and provision of optimal solutions.

One emerging tool in this field is the digital twin, which acts as a digital representation of physical systems and enables real-time monitoring, behavior prediction, and decision-making. [2] With the expansion of Internet of Things (IoT) technologies, smart sensors, and data infrastructures, the digital twin has become one of the main pillars on the path to Industry 4.0. At the same time, the use of fuzzy logic as an efficient tool for modeling uncertainty and data ambiguity enhances the accuracy of forecasts and decisions. [3]

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The combination of these two approaches can create conditions in which both information is received in real time and with sufficient detail, and decisions are made in complex, high-risk environments by relying on human and mathematical logic.

On the other hand, multi-objective optimization is a fundamental challenge in production management. In most cases, managers must balance conflicting goals, such as reducing costs, increasing productivity, reducing energy consumption, and improving environmental performance. In these circumstances, single-objective optimization loses its effectiveness, and the use of multi-objective methods that generate Pareto fronts and examine relationships between objectives is essential. [4] Especially in biological and sustainable production systems that face demand fluctuations, resource constraints, and legal pressures, the use of multi-objective algorithms can play a decisive role in selecting the best strategies.

The main innovation of this study lies in the simultaneous combination of three approaches: fuzzy, digital twin, and multi-objective optimization. First, fuzzy logic enables the structured inclusion of uncertain and variable data, such as demand forecasts or energy efficiency, in the model. Second, the digital twin enables real-time data from production processes, ensuring the model remains in sync with the actual state of the factory. Third, the use of the NSGA-II algorithm enables the framework to generate a diverse set of Pareto solutions from which managers can choose based on their economic or environmental priorities. [5]

In addition, another innovation of the present study is the simultaneous management of energy consumption and waste generation. Most previous studies have addressed cost optimization or waste reduction separately, whereas this study shows that both dimensions can be considered together and under a single framework. Such an approach will lead to greater coherence in production decisions and simultaneously meet the expectations of regulatory bodies and customers in the field of sustainability. [6]

In addition, this study used the General Algebraic Modeling System (GAMS) software to validate the model and evaluate the accuracy and reliability of the results obtained from the meta-heuristic algorithm used. Comparison between the results obtained from NSGA-II and the outputs of the exact GAMS models showed that the proposed framework is not only capable of producing near-optimal solutions but also offers significant advantages in terms of computational time and flexibility in handling big data. [7]

This study also contains important messages for managerial and policy-making perspectives. The findings showed that changes in parameters, such as shortage penalty, energy cap, and waste restriction, can directly affect the position of the Pareto front. Therefore, managers and policymakers can use the results of this framework to regulate energy and the environment in a way that balances economic goals and environmental sustainability. One of the key recommendations of this study is to invest in improving machine efficiency, thereby simultaneously increasing profitability and reducing environmental impacts. [8]

From a methodological perspective, the current study is structured into four main steps. In the first step, production process data is collected and simulated using the digital twin to represent the actual state of the system. In the second step, the uncertainty in the demand data, energy consumption, and machinery efficiency is modeled using fuzzy logic. In the third step, a multi-objective mathematical model is formulated, including economic (cost reduction and profit increase) and environmental (waste reduction and energy consumption) objectives. Finally, in the fourth step, the model is solved using the NSGA-II meta-heuristic algorithm, and the results are compared to the exact model in the GAMS environment for validation. This step-by-step structure forms the methodological pillar of the research and explains the relationship among the fuzzy layers, the digital twin, and the optimization algorithm in a coherent manner.

The innovation of this research can be summarized in three main dimensions:

(i) Providing an integrated and innovative framework that combines three approaches of fuzzy logic, digital twin, and multi-objective optimization simultaneously and in the form of a decision-support system. Such a combination has rarely been observed in the sustainable manufacturing literature in a practical and simultaneous manner.

(ii) This framework enables dynamic decision-making under uncertainty conditions by real-time synchronization of real data from the manufacturing environment with the fuzzy model in the digital twin layer.

(iii) Finally, the model results have been validated not only with the meta-heuristic algorithm NSGA-II but also with the exact mathematical model in the GAMS environment to quantitatively prove the reliability and accuracy of the proposed framework.

The remainder of the paper is structured as follows: the next section describes the theoretical foundations and the proposed framework. In the following, the mathematical model and solution method are introduced, then the simulation results and comparative and sensitivity analyses are presented, and finally, conclusions and suggestions for future research are presented.

2. Literature review

The issue of sustainable production has become a major research area in production management and industrial engineering over the past two decades. Many early studies focused more on reducing costs and increasing productivity, while environmental or social considerations were marginalized. [9] With the spread of concerns related to climate change and excessive consumption of resources, the scientific literature has moved toward integrating economic goals with environmental requirements. This development has led to several concepts, such as green optimization, life-cycle management, and clean production, gaining a special place in the research literature. [10]

In recent years, the use of multi-objective optimization has become increasingly popular to address complex production problems. These methods, by creating a Pareto front, allow for the analysis and comparison of conflicting goals, such as cost, energy, and waste. Several studies have shown that metaheuristic algorithms, especially the evolutionary algorithm family that can extract diverse and near-optimal solutions to large-scale problems. [11] However, a significant part of these studies has focused only on economic aspects and often investigated energy and waste management independently or as secondary objectives. [12]

The existing literature also shows that fuzzy logic is among the most widely used tools for handling uncertainty in input data to optimization models. From demand forecasting to machine efficiency estimation, data is always subject to errors and ambiguity, and deterministic models cannot accurately reflect reality. [13] For this reason, combining fuzzy logic with multi-objective optimization models has attracted the attention of researchers in recent years. However, most of these studies have been limited to the theoretical level or tested on a small scale, and few have applied such a framework in a data-driven environment that aligns with the real process. [14]

One emerging trend that has received special attention in the recent literature is the digital twin. This technology combines simulation, real-time data, and advanced modeling to enable monitoring and predicting the performance of physical systems. [15] A review of studies shows that digital twins have been mostly used in areas such as predictive maintenance, equipment condition monitoring, and product life-cycle management. [16] However, only a few studies have investigated the use of digital twins in conjunction with multi-objective optimization for manufacturing decisions, leaving a significant gap in this field. [17]

Some studies have attempted to apply the combination of digital twins and evolutionary algorithms to solve operational problems. These studies have shown that using real-time data can significantly improve the quality and efficiency of optimal solutions. [18] However, many of these studies have pursued only a specific objective, such as production scheduling or downtime reduction, and have rarely addressed combined objectives, such as cost, energy, and waste. [19] Therefore, the need for a framework that can simultaneously address several key objectives with a data-driven and adaptive approach remains strong.

From an innovation perspective, the present study has three main differences from previous literature. First, unlike studies that have focused on cost reduction or waste reduction separately, this study models both dimensions simultaneously in a single framework, along with energy management. Such a combination is rarely seen in the literature and is an important added value for the field of sustainable manufacturing. [20] Second, the use of fuzzy logic to handle uncertain data, combined with the digital twin, is another innovation that distinguishes the present framework from many previous studies. While most studies have used either fuzzy logic alone or digital twin alone, this study has used both approaches in an integrated manner. [21] Third, in this study, the NSGA-II algorithm was used to extract the Pareto front, and the results were validated using GAMS software. This dualization between the meta-heuristic method and the exact mathematical method ensures the accuracy and reliability of the results and fills the gap that existed in many previous studies.

A review of previous studies shows that although the application of digital twins in smart manufacturing is rapidly expanding, most studies have addressed cost and energy management separately, and their relationship with waste management under uncertainty has received less attention. Also, most existing fuzzy models operate without connecting to real-time data and neglect data synchronization between the model and the actual process.

In the field of multi-objective optimization, a significant part of the research focuses only on the analysis of cost-energy relationships and does not use hybrid frameworks. Therefore, the main gap in the literature is the lack of an integrated framework that simultaneously leverages the three components of digital twins, fuzzy logic, and multi-objective optimization for real-time decision-making in sustainable manufacturing. The present study is designed precisely to fill this gap.

3. Proposed framework

The proposed framework in this research integrates digital twin capabilities and multi-objective fuzzy optimization methods. The main idea is that real-time data of the production process is transmitted to the digital twin through sensors and data collection systems. This twin provides a virtual, dynamic picture of production status through real-time simulation and allows monitoring of key indicators, such as energy consumption and waste volume. In the next step, this data is entered into the fuzzy optimization layer to make balanced decisions in the presence of uncertainty and conflicting objectives.

The interactive connection between the digital twin and the fuzzy optimization engine creates a feedback loop, during which the optimization results are returned to the twin and applied to the real process. Thus, the proposed framework not only manages energy consumption and waste generation simultaneously, but also has the flexibility to respond to instantaneous changes in the production environment. The overall structure of this framework is shown in Figure 1, which clearly displays the main components and their relationships.

As shown in Figure 1, the proposed framework consists of three main parts: a digital twin as a data and simulation platform, a fuzzy optimization engine for simultaneous energy and waste management, and a feedback loop for applying the results to the real environment. This structure provides the necessary context for defining the mathematical model. The mathematical model will be as follows (Table 1, Equations (1)–(15)).

The mathematical model formulation comprised objective functions Equations (1)–(4), fuzzy aggregation Equation (5), final fuzzy model Equation (6), and constraints Equations (7)–(15).

(i) Objective functions:

$$Min\ f_1 = \sum_t \Bigg[\sum_{p,m,r} c^{\text{prod}}_{pmr}\, x_{pmrt} + \sum_{p,m} c^{\text{setup}}_{pm}\, y_{pmt} + \sum_e \pi^E_{et}\, E_{et} + \sum_k \psi_k\, W_{kt} + \sum_p \pi^{\text{late}}_p\, s_{pt} \Bigg] \tag{1}$$

$$Min\ f_2 = \sum_t \sum_e \phi_e\, E_{et} \tag{2}$$

$$Min_3 = \sum_t \sum_k W_{kt} \tag{3}$$

$$Max\ f_4 = \sum_t \sum_p \gamma_p\,(d_{pt} - s_{pt}) - \theta \sum_t \sum_m \Big(1 - \hat{Q}_{mt}(x,u)\Big) \tag{4}$$

where θ is a penalty weight for reduced OEE.

(ii) Fuzzy aggregations: For each objective f_q with aspiration level G_q and tolerance Δ_q, the membership function is:

$$\mu_q(f_q) = \begin{cases} 1, & f_q \le G_q, \\ \dfrac{G_q + \Delta_q - f_q}{\Delta_q}, & G_q < f_q < G_q + \Delta_q, \\ 0, & f_q \ge G_q + \Delta_q. \end{cases} \tag{5}$$

(iii) Final fuzzy model:

$$Max\, \lambda \quad s.t.: \quad \mu_q(f_q) \ge \lambda,\ \forall q \tag{6}$$

(iv) Constrains:

$$\sum_{m,r} x_{pmrt} + I_{p,t-1} - I_{pt} + s_{p,t-1} - s_{pt} = d_{pt}, \quad \forall p, t \tag{7}$$

If no inventory I is considered:

$$\sum_{m,r} (x_{pmrt} + s_{pt}) \ge d_{pt}, \quad \sum_{p,r} \frac{\tau_{pmr}}{\eta_m}\, x_{pmrt} \le \bar{C}_{mt}\, z_{mt}, \quad \forall m, t \tag{8}$$

$$x_{pmrt} \le U_{pmr}\, y_{pmt}, \qquad \forall p, m, r, t \tag{9}$$

$$y_{pmt} - y_{pm,t-1} \le v_{pmt}, \quad \sum_p v_{pmt} \le V^{\max}_m, \quad \forall m, t \tag{10}$$

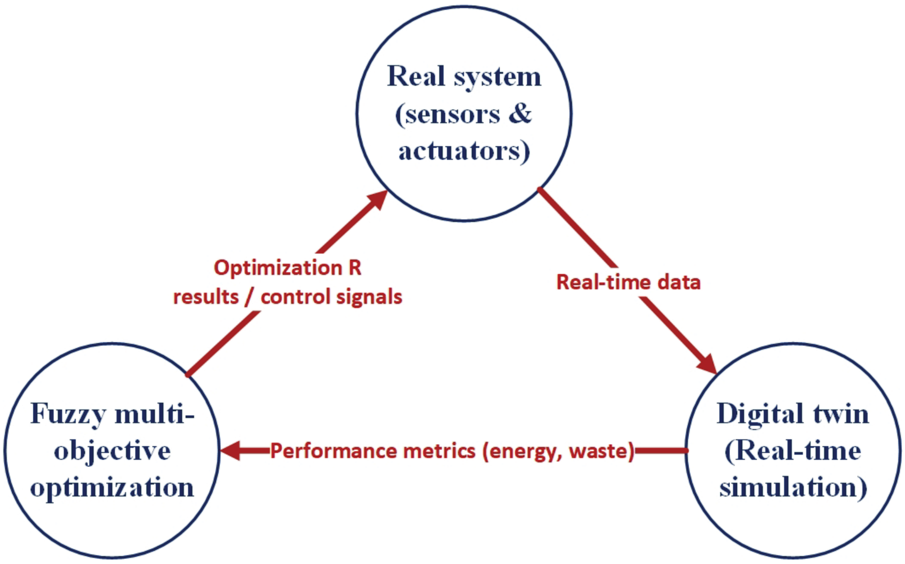

Figure 1. Proposed fuzzy–digital twin optimization framework

$$E_{et} \geq \sum_{p,m,r} \left(a^{E}_{pmre}\, x_{pmrt} + \alpha^{E}_{me}\, z_{mt}\right) + \beta^{E}_{me}\, u_{mt}, \quad \forall e,t \sum_{e} E_{et} \approx \hat{E}_t(x,u) \tag{11}$$

$$W_{kt} \geq \sum_{p,m,r} a^{W}_{pmrk}\, x_{pmrt}, \quad \forall k,t, \qquad W_{kt} \approx \hat{W}_{kt}(x,u), \quad \forall k,t \tag{12}$$

$$\sum_{e} E_t \leq \bar{E}_t, \quad W_{kt} \leq \bar{W}_{kt}, \quad \forall k,t \tag{13}$$

$$u^{\min}_{m}\, z_{mt} \;\leq\; u_{mt} \;\leq\; u^{\max}_{m}\, z_{mt}, \quad \forall m,t \tag{14}$$

For each $q \in \{1,2,3,4\}$:

$$\mu_q(f_q) \geq \lambda, \quad 0 < \lambda < 1; \quad \mu^{E}_{t}\left(\sum_{e} E_{et}\right) \geq \lambda, \quad \mu^{W}_{kt}(W_{kt}) \geq \lambda \tag{15}$$

Equation (1) shows how the total system costs, including production, start-up, energy, waste management, and shortage penalties, should be minimized. This function actually considers the overall financial burden of the production process and plays a pivotal role in reducing operating costs. Equation (2) focuses on minimizing the carbon footprint from energy consumption and uses emission factors for calculation. This function highlights the environmental importance of production decisions and directly connects to pollutant reduction policies. Equation (3) is dedicated to minimizing production waste and covers all types of waste across different time periods. This function is defined in line with sustainable production and tries to reduce negative environmental impacts. Equation (4) maximizes profit and efficiency of the system, considering the added value of products and the penalty for machine efficiency loss. This function balances economic benefits and operational efficiency. Equation (7) ensures that the production and shortage levels can meet the demand for each product in each period. This constraint means maintaining a balance between supply and demand in the system. Equation (8) controls the capacity of each machine and prevents its usage time from exceeding the actual capacity in each period. This constraint prevents unrealistic scheduling and excessive pressure on resources. Equation (9) specifies the relationship between line activity and production quantity, and specifies that production is only possible when it is started. This constraint ensures the logical and technical consistency of the model. Equation (10) manages the conditions for product change and production line replacement and limits the number of changes allowed in each period. This constraint helps reduce the time loss and costs caused by frequent changes. Equation (11) establishes an energy balance based on digital twin data and specifies the actual consumption of energy resources at the machine level. This constraint provides a more accurate understanding of the energy flow in the entire system. Equation (12) defines the waste balance equation and matches the amount of waste generated with the digital twin predictions. This constraint helps

Table 1. The components of the mathematical models

Component	**Description**
Sets	
P	Set of products
M	Set of machines/stations
R	Set of processes/operations
T	Discrete planning horizon
K	Types of waste
E	Energy resources (electricity, gas, etc.)
Parameters	
d_{pt}	Demand for product p in period t
c_{pmr}^{prod}	Unit production cost of product p on machine m in operation r
c_{pm}^{setup}	Setup/changeover cost for product p on machine m
π_p^{late}	Penalty cost for one unit of shortage/delay of product p
a_{pmre}^{E}	Energy consumption coefficient of resource e per unit of product p on m in r
a_{pmrk}^{W}	Waste generation coefficient of type k per unit of product p on m in r
$\alpha_{me}^{E},\ \beta_{me}^{E}$	Base and slope coefficients of energy consumption of machine m from source e (function of control variable, e.g., speed)
ϕ_e	Carbon emission factor of energy source e
ψ_k	Cost/environmental impact of disposing/recycling waste type k
$\bar{C}_{mt}$	Available capacity of machine m in period t (man-hours or machine-hours)
τ_{pmr}	Unit processing time of product p on machine m in operation r
η_m	Effective efficiency of machine m
$\bar{E}_t$	Maximum total energy consumption allowed in period t (if policy exists)
$\bar{W}_{kt}$	Maximum waste allowed of type k in period t
γ_p	Unit added value/profit of product p
U_{pmr}	Maximum production (or processing) capacity of product $p \in P$ on machine $m \in M$ Using resource $r \in R$ during period $t \in T$.
V^{max}	Maximum allowable number of setups/changeovers over the planning horizon.
Outputs of the digital twin (updated and data-driven)	
$\hat{E}_t(x, u)$	Predicted total energy consumption in period t
$\hat{W}_{kt}(x, u)$	Predicted waste of type k in period t
$\hat{Q}_{mt}(x, u)$	Predicted effective capacity/OEE of machine m in period t
Decision variables	
$x_{pmrt} \geq 0$	Production quantity of product p on machine m, operation r, and period t
$y_{pmt} \in \{0, 1\}$	Setup/allocation decision for p on machine m in t
$z_{mt} \in \{0, 1\}$	On/off status of machine m in t
$u_{mt} \in [\,u_m^{\min},\, u_m^{\max}\,]$	Continuous control variable of machine m (e.g., speed)
$s_{pt} \geq 0$	Shortage/backlog of product p in t
$E_{et} \geq 0$	Energy consumed from resource e in t
$W_{kt} \geq 0$	Waste of type k in t
v_{pmt}	Binary decision variable, equal to 1 if a new setup for product p on machine m occurs at period t, and 0 otherwise.
Fuzzy variable	
$\lambda \in [0, 1]$	Level of fuzzy satisfaction (Max–Min approach)
$G_t^E, \quad \Delta_t^E \quad for\ energy$ $G_{kt}^W, \quad \Delta_{kt}^W \quad for\ waste$ $G^{\text{cost}}, \quad \Delta^{cost} \quad for\ cost$ $G^{\text{srv}}, \quad \Delta^{\text{srv}} \quad for\ service\ level/productivity$	Goal values and tolerances

to control the environmental impacts more precisely. Equation (13) considers the policy ceiling for energy and waste and ensures that actual consumption is within the allowed range. This constraint reflects legal requirements or environmental standards. Equation (14) specifies the range of control variables and the logic of whether the machines are on or off. This constraint ensures that the technical controls of the machines remain within acceptable operational boundaries. Equation (15) expresses the fuzzy conditions of satisfaction with the objectives and relates the degree of achievement of different objectives to the satisfaction level Î». This constraint provides a flexible framework for decision-making under uncertainty.

The digital twin model in this research is designed as an interface layer between the real system and the fuzzy optimization engine. This model continuously collects real-time data on production, energy consumption, and waste amount using a network of sensors and actuators. After initial processing and cleaning, the data is transferred to a central database and presented as indicators for the decision-making model. This process creates an accurate virtual image of the current state of the production lines that always matches the real conditions.

The digital twin is connected to the optimization model through a feedback loop. In this way, the values from the sensors are fed into the mathematical model, and parameters such as predicted energy consumption, waste volume, and effective capacity of the machines are entered into the constraints and objectives as dynamic functions. The optimization engine, taking into account these updated inputs, generates optimal solutions for adjusting machine speeds, allocating production, and managing resources. The results are then fed back to the digital twin, which then transfers them to the real system. This two-way interaction allows decision-making to be based not only on historical data but also on the current situation, providing flexibility and rapid response to environmental changes.

4. Methodology and solution methods

This study is classified as applied-developmental research. The main objective is to present and test a hybrid framework that can simultaneously optimize several conflicting objectives, including cost reduction, energy consumption reduction, and waste management, under uncertainty based on fuzzy logic and digital twin. The fuzzy multi-objective approach was chosen because the complexity and uncertainty in contemporary manufacturing systems pose serious limitations to the use of deterministic models. The digital twin, as a data-driven platform, also enables real-time synchronization between the physical process and the virtual model, thus improving operational decision-making in real conditions.

In this study, fuzzy logic is used to model uncertainty in key parameters of the production process. The main variables include demand rate, machine efficiency, and energy intensity, each with three linguistic levels: low, medium, and high. These variables are quantified from digital twin data and transformed into corresponding linguistic values through a fuzzification process. In the next step, fuzzy inference rules of the "if–then" type are defined for the relationship between input and output variables; for example, when demand is high and machine efficiency is high, the system performance is at the desired level, while the combination of low demand and low efficiency indicates poor performance.

The fuzzy inference is based on the Mamdani method, in which rule combination is based on minimization for the simultaneous operator (AND) and maximization for the disjunctive operator (OR). The final output of this process is converted to a corresponding numerical value using the center of gravity method so that it can be used in the multi-objective optimization model. In this way, the fuzzy system continuously interacts with the digital twin data, and its outputs are transferred as dynamic inputs to the optimization model. This mechanism allows decisions to remain sensitive and adaptive to environmental changes, demand fluctuations, and energy consumption.

The theoretical foundations of this research are based on three main pillars: digital twin, fuzzy logic, and multi-objective optimization. In the digital twin layer, real-time data of the production process is collected from sensors and control systems and dynamically updates the virtual model of the process. This layer allows for real-time simulation and analysis of system performance. The second layer is based on fuzzy logic to model the uncertainty in demand data, energy consumption, and machine efficiency. In this section, linguistic variables are transformed into fuzzy membership functions, and if–then rules are used to evaluate different scenarios. Finally, the third layer consists of the NSGA-II algorithm, which acts as a multi-objective optimization engine and extracts the Pareto front between economic (cost reduction) and environmental (waste and energy consumption reduction) objectives.

These three layers work simultaneously and interactively; updated data from the digital twin is sent to the fuzzy system, the fuzzy output is transferred to the optimization model, and the results are fed back to the virtual and real models. This three-layer structure forms the theoretical and logical foundation of the present research methodology.

To solve the developed mathematical model, the NSGA-II meta-heuristic algorithm was used. This algorithm is considered one of the most reliable tools for solving multi-objective optimization problems due to its strong ability to generate diverse Pareto fronts, appropriate exploration

Table 2. Parameter settings of the Non-dominated Sorting Genetic Algorithm II algorithm

Parameter	Value	Description
Population size	100	Number of individuals in each generation to ensure solution diversity
Number of generations	200	Maximum iterations until convergence
Crossover probability (Pc)	0.9	High probability to explore the search space through recombination
Mutation probability (Pm)	0.05	Mutation rate to prevent premature convergence
Crossover operator	Simulated binary crossover (SBX)	Suitable for continuous and multi-objective problems
Mutation operator	Polynomial mutation	Creates diversity in the population and enhances Pareto front quality
Selection mechanism	Tournament Selection (size = 2)	Selects superior individuals based on Pareto rank and crowding distance
Elitism strategy	Preserving the best solutions	Transfers the best solutions to the next generation to ensure gradual improvement

power in the search space, and prevention of premature convergence. In this study, after testing several different configurations, a set of key algorithm parameters was selected to achieve a balance between solution quality and computation time. The details of these settings are presented in Table 2.

The results of the NSGA-II algorithm were obtained using the Python programming language and standard metaheuristic libraries. GAMS software was also used to validate the model and ensure the correctness of the answers, and the results of the two approaches were compared. This comparison showed that the proposed framework, in addition to high accuracy, can solve complex problems with larger dimensions and under uncertainty conditions.

In terms of infrastructure, a combination of several complementary tools was used to implement the framework. Python was used as the primary environment for developing the model and implementing the NSGA-II algorithm, and libraries, such as NumPy and Matplotlib, were used to analyze the results. GAMS software was used as a comparative reference to accurately solve the mathematical model and control the accuracy of the results. Also, the digital twin module enabled the collection of real-time data and simulation of real-world conditions, allowing the modeling results to be directly compared to the actual state of the production system. This integration of simulation data, optimization models, and analysis tools turns the proposed framework into an efficient system for decision-making toward green and sustainable production.

5. Case study/simulation

To evaluate the proposed framework, a case study was designed in which input data were generated through manufacturing simulations derived from the digital twin. Due to limited access to real industrial data at this stage, the data were generated based on common patterns of energy consumption and waste generation in process industries (such as food and pharmaceutical industries). These data are consistent with the real data in terms of structure and scale and follow valid distributions for energy consumption, waste generation, and demand requirements. In this way, the simulated data not only reflect operational realities but also allow for the implementation of diverse scenarios and the examination of the model's performance under different conditions.

To ensure transparency and reproducibility of the results, Table 3 presents the baseline data that underlie all subsequent calculations and analyses. This table presents ten scenarios in which changes in product demand, energy consumption, and waste generation levels are considered. The scenarios are selected to cover a range of optimal, typical, and critical conditions to test the reliability of the proposed framework in diverse environments.

As shown in Table 3, the baseline data covers a wide range of operating conditions, from low demand and low energy consumption scenarios to high demand and high waste scenarios. The aim of this design is to test the proposed framework across a variety of environments and to fully evaluate its performance against key changes, such

Table 3. Baseline data used for simulation

Scenario	Product demand (units)	Energy consumption (kWh)	Waste generation (kg)	Machine capacity (h)	Productivity level (%)	Unit production cost	Shortage penalty	Energy limit (kWh)	Waste limit (kg)	Unit profit
1	500	1200	80	400	85	50,000	10,000	1500	100	70,000
2	600	1350	95	420	82	52,000	10,500	1600	110	72,000
3	550	1280	88	410	84	49,500	9800	1550	105	71,500
4	650	1450	100	430	80	51,800	11,000	1650	120	73,000
5	700	1500	120	440	78	53,000	11,200	1700	125	74,000
6	480	1100	75	390	87	48,500	9500	1450	95	69,000
7	800	1600	130	450	76	54,000	11,500	1750	130	75,000
8	750	1580	125	445	77	52,800	11,300	1720	128	74,500
9	620	1400	110	425	81	51,200	10,800	1620	115	72,500
10	580	1250	85	405	83	50,200	10,200	–	–	–

as demand fluctuations, energy constraints, and increased waste.

These data are used as direct inputs to the optimization model, and all subsequent calculations will be based on them. In fact, each scenario serves as a starting point for the model implementation, and the optimization results will demonstrate the efficiency of the framework in simultaneously managing waste and energy. In this way, the relationship between the simulated data and the model outputs is clearly established, providing a basis for analyzing the results and comparing the scenarios in the following sections of the paper.

6. Results and discussion

This section presents the results of the model implementation. The baseline data presented in Table 3 were used to evaluate the performance of the proposed framework under various conditions using the fuzzy–digital twin optimization model. The model outputs included the optimal production values for each scenario, the total system cost, energy consumption, the level of waste generated, and the profitability of the production operation.

The main emphasis in this section is that all results are directly extracted from the combined fuzzy–digital twin framework, and no model-independent simplifying assumptions are applied in the calculations. The comparison across different scenarios not only demonstrated the model's flexibility to different conditions but also confirmed the validity of the proposed framework in achieving both economic and environmental goals. This paves the way for more in-depth analyses in the following sections.

To begin the analysis, Scenario 1 was selected as the baseline. This scenario represented conditions close to normal production conditions and served as the basis for comparing other scenarios. The input data in this scenario included a demand of 500 units of product, energy consumption of 1200 kWh, waste generation of 80 kg, and machine capacity of 400 h. An efficiency level of 85%, a unit production cost of 50,000 USD, and a deficiency penalty of 10,000 USD per unit were considered. An energy ceiling of 1500 kWh, a waste ceiling of 100 kg, and a profit per unit of product of 70,000 USD were also determined.

The model output showed that in this scenario, demand was fully met and no shortages occurred. Energy consumption was stabilized below the permitted ceiling, and the amount of waste remained below the permitted values. The total system cost was calculated by including production, energy, and waste management costs, and the resulting net profitability was positive and significant. To more accurately display the results, Figure 2 shows the trend of changes in energy consumption and waste generation compared to the permitted limits over the time horizon. These curves confirm that the proposed framework can balance economic and environmental objectives and ensure the sustainability of operations.

Figure 2 shows the trend of changes in energy consumption and waste generation in the reference scenario. The energy consumption values remain below the permissible limit of 1500 kWh in all time periods, and the amount of waste remains below the ceiling of 100 kg. This indicates that the proposed framework simultaneously meets environmental and economic criteria in the reference conditions.

After examining the reference scenario, this section makes a comprehensive comparison between the ten scenarios presented in Table 3. The aim is to show how the proposed framework performs under various production and demand conditions and to what extent it balances economic and environmental criteria. The optimization results for each scenario are calculated and presented in the form of main indicators, including total cost, energy consumption, waste amount, service level, and profitability.

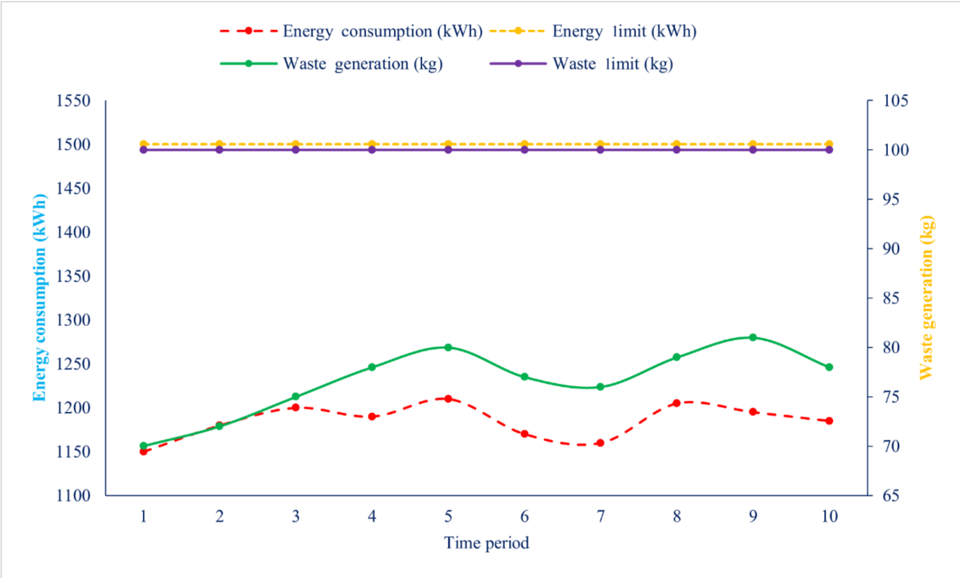

Figure 2. Baseline scenario trends of energy consumption and waste generation

The main outputs of the model are shown in Table 4. This table shows that with increasing demand, total cost and energy consumption increase. However, the framework maintains the service level above the target value in all scenarios and prevents shortages. Also, in scenarios with heavy production loads, waste has increased, but it remains below the permissible ceiling. On the other hand, profitability is the highest in scenarios with high productivity and moderate demand, indicating the role of the optimal combination of capacity, energy, and demand in achieving sustainable profits.

As shown in Table 4, as optimal production (demand) increases, total cost and energy consumption increase as well, while the framework maintains the service level above 95%. Also, the generated waste increases in high-load scenarios, but remains below the threshold. These results serve as the basis for the following comparative analyses.

To better explain these results, the changes in total cost and energy consumption in ten different scenarios are presented in Figure 3. With increasing demand, the costs and energy consumption have risen, but their growth rates are lower than the demand growth rate. This indicates the framework's ability to manage operational pressures.

It can be seen that with increasing demand, both indicators increase, but the growth rates of costs and energy are lower than that of demand. This indicates the ability of the proposed framework to control costs and energy consumption even under high demand.

The trend in waste changes compared to the permissible ceiling is shown in Figure 4. It can be seen that waste generation increases as demand rises.

Figure 4 shows that in most scenarios (demand < 650 units), the waste amount remains below the 100 kg limit. This result confirms that the proposed framework met certain demand levels while also satisfying environmental requirements.

Figure 5 compares service levels and profitability across different scenarios. The results show that the service level remains above 95% across all scenarios, while profitability fluctuates according to machine productivity and the amount of deficiency penalties. This indicates that the framework can strike a balance between economic and operational sustainability.

One of the most important aspects of multi-objective models is the examination of the relationships between conflicting objectives. In the present framework, the two main objectives, total cost reduction and waste reduction, are in relative conflict. In other words, decisions that lead to cost reduction are usually accompanied by increased waste, and vice versa, actions that lead to waste reduction often impose additional costs on the system.

To illustrate this relationship, Figure 6 shows the Pareto front obtained from solving the model. In this graph, each point represents an optimized scenario that offers a combination of cost and waste. The concave part of the Pareto front is the region where improving one objective is possible only by sacrificing the other. Analyzing this curve helps managers and decision-makers to choose a

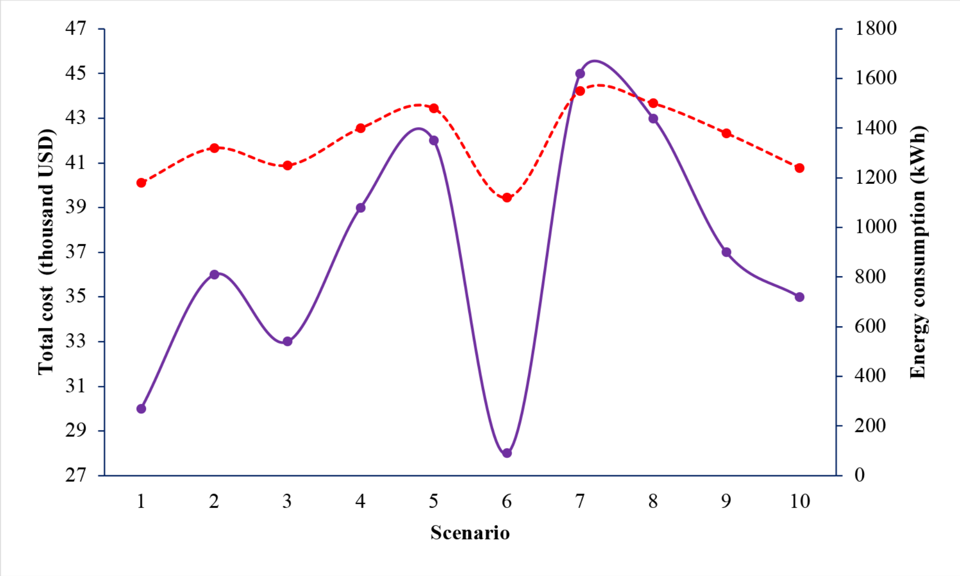

Figure 3. Comparative trends of total cost and energy consumption

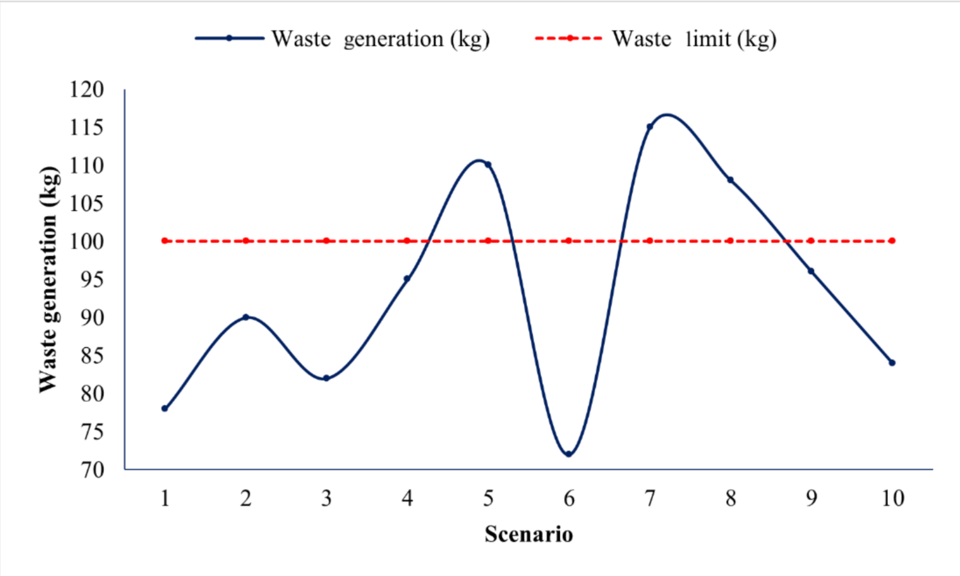

Figure 4. Comparative trends of waste generation and waste limits

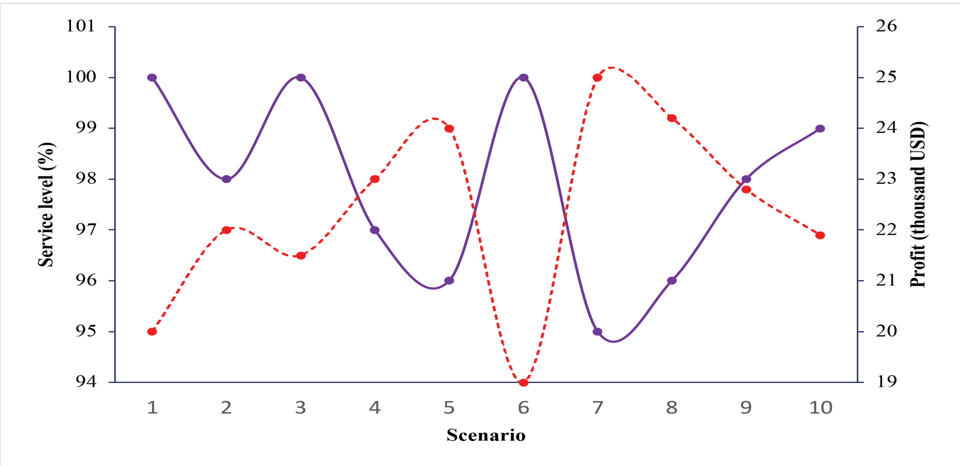

Figure 5. Service level and profitability across scenarios

Table 4. Optimized results for different scenarios

Scenario	Optimal production (units)	Total cost (thousand USD)	Energy consumption (kWh)	Waste generation (kg)	Service level (%)	Profit (thousand USD)
1	500	30	1180	78	100	20.0
2	600	36	1320	90	98	22.0
3	550	33	1250	82	100	21.5
4	650	39	1400	95	97	23.0
5	700	42	1480	110	96	24.0
6	480	28	1120	72	100	19.0
7	800	45	1550	115	95	25.0
8	750	43	1500	108	96	24.2
9	620	37	1380	96	98	22.8
10	580	35	1240	84	99	21.9

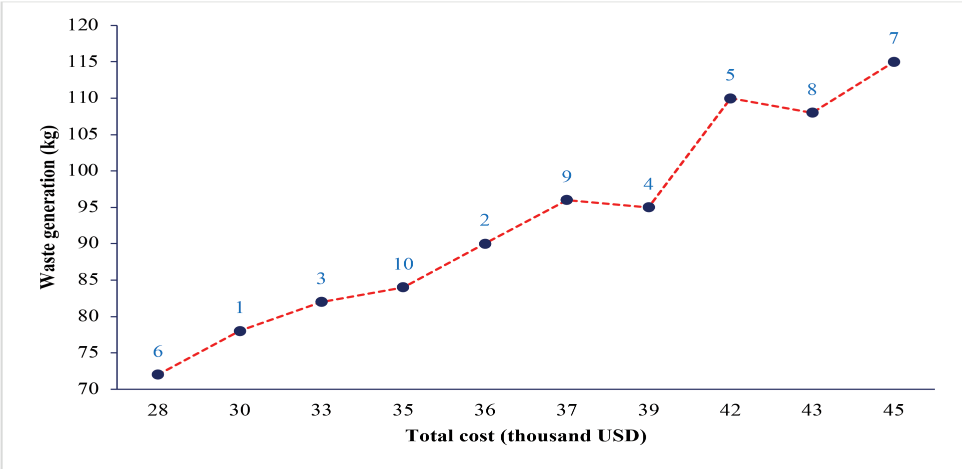

Figure 6. Pareto front of total cost vs. waste generation

balanced point between the two objectives, depending on policy priorities or environmental constraints.

The results show that waste generation and cost increase simultaneously. Therefore, the proposed framework not only allows for the identification of optimal solutions but also provides managers with a powerful tool for analyzing the sensitivity of decisions under real-world conditions.

To assess the stability of the proposed framework, a sensitivity analysis was conducted. In this analysis, key parameters, including deficiency penalty, energy cap, energy price, waste cap, and machine efficiency, were selected, and their changes were examined over different time intervals. The quantitative results of these changes are presented in Table 5.

The findings show that changes in the shortage penalty have the greatest effect on the service level and total cost. With increased penalties, service levels improve, but the costs rise as well. Reducing the penalty has the opposite effect, which decreases the service level. The pattern of these changes is shown in Figure 7. The energy cap also plays an important role. A low energy cap puts more pressure on the total cost and pushes the Pareto front toward more costly values, while increasing the energy cap simultaneously reduces costs and waste.

Changes in energy prices also have a dual effect. The increase in energy prices shifts the production policies toward reducing energy intensity, but the total cost increases. In contrast, the reduction in energy prices shifts the Pareto front toward lower cost and waste values.

In the case of the waste ceiling, tightening the constraint reduces waste but increases costs. Loosening the constraint reduces the total cost but leaves the waste level higher. The results of these changes are shown in Figure 8. These findings suggest that waste management policy requires a careful balance between economic and environmental goals.

Changes in machine efficiency directly impact the profitability and service levels. A decrease in efficiency results in higher costs and, in some cases, lower service levels, while an increase in efficiency leads to higher profits and enhanced performance. These effects are presented in Figure 9.

Table 5. Sensitivity analysis results and elasticity indices for key parameters

Parameter	Delta (%)	Total cost (million USD)	Energy (kWh)	Waste (kg)	Service level (%)	Profit (million USD)
Shortage penalty	−20	2.64	1128	83.2	97.4	2.226
Shortage penalty	−10	2.82	1164	81.6	98.2	2.163
Shortage penalty	0	3.00	1200	80.0	99.0	2.100
Shortage penalty	10	3.18	1236	78.4	99.8	2.037
Shortage penalty	20	3.36	1272	76.8	100.6	1.974
Energy cap	−20	3.12	1344	81.6	99.6	2.142
Energy cap	−10	3.06	1272	80.8	99.3	2.121
Energy cap	0	3.00	1200	80.0	99.0	2.100
Energy cap	10	2.94	1128	79.2	98.7	2.079
Energy cap	20	2.88	1056	78.4	98.4	2.058
Energy price	−20	2.52	1296	81.6	99.2	2.310
Energy price	−10	2.76	1248	80.8	99.1	2.205
Energy price	0	3.00	1200	80.0	99.0	2.100
Energy price	10	3.24	1152	79.2	98.9	1.995
Energy price	20	3.48	1104	78.4	98.8	1.890
Waste cap	−20	3.06	1200	96.0	99.3	2.184
Waste cap	−10	3.03	1200	88.0	99.2	2.142
Waste cap	0	3.00	1200	80.0	99.0	2.100
Waste cap	10	2.97	1200	72.0	98.8	2.058
Waste cap	20	2.94	1200	64.0	98.7	2.016
Machine productivity	−20	3.42	1320	84.8	98.0	1.722
Machine productivity	−10	3.21	1260	82.4	98.5	1.911
Machine productivity	0	3.00	1200	80.0	99.0	2.100
Machine productivity	10	2.79	1140	77.6	99.5	2.289
Machine productivity	20	2.58	1080	75.2	100.0	2.478

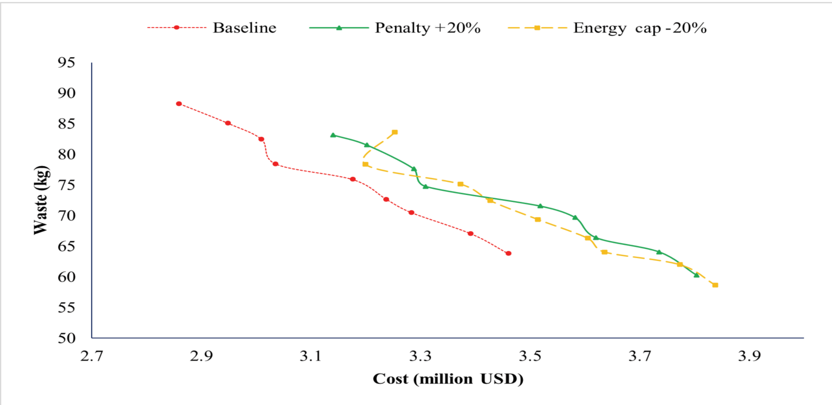

Figure 7. Pareto front under variations in shortage penalty and energy cap

Overall, the results of the sensitivity analysis indicate that the proposed framework remains stable against conventional parameter changes; only under severe changes, especially in the shortage penalty or energy and waste constraints, noticeable changes in the Pareto front are observed. These findings indicate the reliability and flexibility of the proposed framework in real operational environments.

The results of the digital twin-fuzzy optimization model indicate that this framework can serve as an effective decision-making tool for production managers. First, the analyses presented in the previous sections reveal that simultaneous management of cost, energy, and waste is possible, and the proposed framework can maintain high, stable service levels. This allows managers to make data-driven decisions with less risk in conditions of uncertainty and demand fluctuations.

From a policy perspective, the findings also show that changes in parameters, such as shortage penalties or energy caps, can have a direct impact

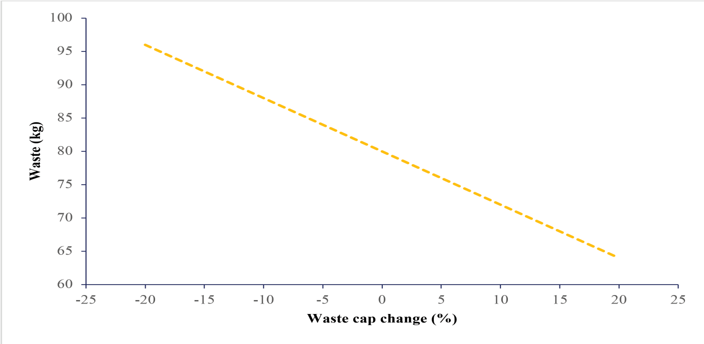

Figure 8. Waste generation trends under different waste cap scenarios

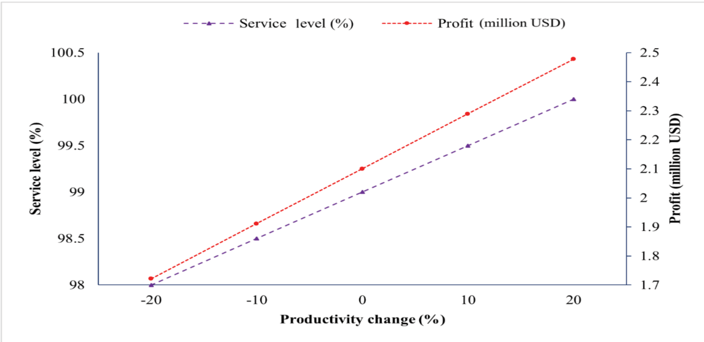

Figure 9. Impact of machine productivity variations on service level and profitability

on the shape of the Pareto front. In particular, stricter energy and waste restrictions increase total costs but reduce waste and improve environmental indicators. This suggests that policymakers should simultaneously consider both economic interests and environmental requirements of firms when formulating regulations. In addition, the results of the sensitivity analysis showed that machine efficiency plays a key role in maintaining profitability and service levels; therefore, investing in new technologies and preventive maintenance can be a key strategy to achieve sustainable production.

Overall, the proposed fuzzy–digital twin framework, by enabling simultaneous scenario analysis, Pareto fronts investigation, and parameter sensitivity assessment, is a valuable tool for managers and policymakers to identify and implement optimal paths between economic and environmental goals.

7. Conclusion

This research aims to develop and evaluate a novel framework based on fuzzy–digital twin multi-objective optimization for simultaneous management of cost, energy consumption, and waste in sustainable production. First, the proposed framework was introduced and its main components were described, including fuzzy modeling to handle uncertainty, a digital twin to receive and process real-time data, and a multi-objective optimization algorithm to create a balance between economic and environmental goals. Then, the mathematical model was fully developed, and simulation scenarios were defined to test the framework.

The findings showed that the proposed framework maintained service levels above 95% across different scenarios while simultaneously keeping the total cost and waste within acceptable limits. The analysis of the reference scenario indicated that even under normal operating load conditions, the framework could control energy consumption and waste while achieving significant profitability. Comparative analysis between scenarios showed that increasing demand naturally increased costs and energy consumption, but the growth rates of these indicators were lower than those of demand, indicating the framework's efficiency in controlling operational pressures.

The Pareto fronts clearly show a conflict between economic goals (cost reduction) and environmental goals (waste reduction). Waste reduction is possible only at the expense of increased cost and vice versa. This finding highlights the importance of using multi-objective analytical tools for managers and policymakers.

Sensitivity analysis also confirms that the proposed framework is stable to conventional parameter changes; only under drastic changes in the shortage penalty or in energy and waste restrictions is a noticeable shift observed in the results. This indicates the reliability and flexibility of the framework in real conditions.

From a managerial perspective, the developed framework can help managers make balanced decisions between economic efficiency and environmental goals. Also, from a policy perspective, the results of this research can inform regulations that cover both the economic interests and environmental requirements of industries.

Finally, it can be concluded that the proposed fuzzy–digital twin framework is an efficient tool to support green and sustainable manufacturing. Given its flexibility and generalizability, this framework has the potential to be applied in various industries. Future research directions include developing the model with real industrial data, directly connecting to IoT and programmable logic controller systems in manufacturing environments, and employing advanced machine learning algorithms to improve prediction and optimization. These steps can transform the current framework into a comprehensive decision-support system for moving toward smart and sustainable manufacturing.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Author contributions

Conceptualization: Hamed Nozari
Formal analysis: Hamed Nozari
Methodology: All authors
Investigation: All authors
Writing–original draft: Hamed Nozari
Writing–review & editing: Zornitsa Yordanova

Availability of data

All data generated or analyzed during this study are included in this published article.

AI tools statement

All authors confirm that no AI tools were used in the preparation of this manuscript.

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