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RESEARCH ARTICLE

# Dynamic modeling and optimal control of bank balance sheets under capital adequacy constraints

Moch. Fandi Ansori*

Department of Mathematics, Faculty of Science and Mathematics, Universitas Diponegoro, Semarang, Indonesia
*mochfandiansori@lecturer.undip.ac.id*

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ABSTRACT

The aim of this study is to examine the dynamics of deposits, loans, and equity on the balance sheet of the bank under capital adequacy constraints. The intention is to provide a tractable framework for assessing solvency and regulatory policies. Therefore, a nonlinear continuous-time model was developed with logistic growth, credit risk, and capital adequacy conditions. The model was calibrated using monthly data for Indonesian commercial banks from 2022 to 2024, and the parameters were estimated using particle swarm optimization. The results showed that the model replicates observed trajectories with mean absolute percentage errors below 2.1% to confirm its empirical validity. The simulations showed that stricter capital requirements slowed equity growth while moderate requirements supported long-run capitalization. A time-varying capital adequacy policy was formulated as an optimal control problem, and the Pontryagin maximum principle was applied to derive an optimal regulatory path. The results showed that adaptive regulation stabilized capitalization while limiting policy costs. The trend reflected the value of the continuous-time control theory in financial regulation.



## 1. Introduction

The aim of this study is to develop a continuous-time mathematical model for the balance sheet of the bank to capture the joint dynamics of deposits, loans, and equity under capital adequacy constraints. Another aim is to evaluate the regulatory implications through stability analysis and optimal control. The stability and performance of the banking institution critically depend on the dynamic interplay between its core balance sheet components, including deposits, loans, and equity. These aggregates shape liquidity creation, solvency, and the capacity of the bank to absorb systemic shocks while remaining compliant with the prudential regulations. Banks are required to provide liquidity by transforming illiquid assets (loans) into liquid liabilities (deposits), which are important for economic growth.[1,2] However, the process exposes banks to liquidity risk due to the need to maintain sufficient reserves to meet unexpected deposit withdrawals.[1,3] The liquidity risk is an important concern, but this study focuses on solvency through capital adequacy rather than liquidity regulation. The role of liquidity is only acknowledged as part of the broader context, not explicitly modeled, and left for future extensions of the framework. This is because capital or equity serves as a buffer against potential losses and assists banks in absorbing shocks, but also impacts liquidity creation.[4,5] Higher capital requirements and liquidity levels can enhance financial stability but are capable of potentially limiting liquidity creation and affecting bank profitability.[6] Therefore, regulators are required to balance prudential regulations with the ability of the banking system to provide liquidity to the economy. The impact of these factors can vary depending on bank size, market conditions, and institutional quality. The situation emphasizes the

*Corresponding Author

importance of context-specific methods for banking regulations.[2,7,8]

The global financial crisis motivated international standards such as Basel III to tighten capital and liquidity requirements. This further led to a surge in studies related to the propagation of regulatory constraints through bank balance sheets.[9,10] Studies showed that higher regulatory capital ratios could reduce the costs of financial intermediation and increase bank profitability.[11] Recent post-mortem analyses of the 2023 banking turmoil further underscored that inadequate capital buffers and poorly calibrated liquidity outflow assumptions could precipitate systemic stress even in advanced financial systems.[12] Therefore, policymakers and central banks increasingly depend on forward-looking balance sheet-based stress-testing frameworks to evaluate policy trade-offs and the resilience of individual institutions.[13] These developments show the need for tractable but sufficiently rich dynamic models that can capture the feedback loops between funding, credit risk, and capital adequacy. The feedback loops are associated with the fact that deposit inflows or funding determine lending capacity, loan performance affects nonperforming loans (NPL) and credit risk, while regulatory capital ratios constrain lending activity and equity growth. This subsequently shows that the constraints influence future funding and risk exposure to create an interdependent cycle.

Empirical evidence confirmed that macroprudential instruments developed based on capital and liquidity ratios exerted state- dependent effects on lending, funding structures, and systemic risk.[14,15] Panel studies covering both advanced and emerging markets showed that tighter capital requirements increased the common-equity-tier-1 ratio, altered asset composition, and curbed procyclical credit growth.[16] Complementary studies also reported that market stigma and deposit outflows often hampered capital buffer usability during downturns to reinforce the importance of coupled deposit-loan-equity dynamics in strategic balance sheet management.[10,17] The integration of the mechanisms into a mathematical framework allows the contribution of this study to the strand of mathematical models of banking dynamics that bridge micro-prudential behavior and stability thresholds.

This study models dynamics of the bank's balance sheet using a deterministic continuous-time mathematical framework, in which deposits, loans, and equity change as time-dependent state variables governed by nonlinear differential equations. The evolution of deposits was modeled using a logistic growth process with a natural saturation level adjusted for customer withdrawals in line with previous studies.[18–21] Loans were also modeled by logistic growth but with reductions due to NPLs and repayment according to previous studies.[18–21] In previous studies, equity was modeled with dynamics proportional to bank profits without incorporating regulatory mechanisms. Meanwhile, this study used equity as a safeguard against insolvency and regulatory violations. This was in line with a modified logistic-type equation based on retained earnings and was subjected to a dynamic capital adequacy condition. The concept represented a novel contribution in addition to the assessment of capital adequacy controls.

The remainder of this study is organized by several sections. **Section 2** reviews the related literature. **Section 3** develops the bank balance sheet dynamic model along with an analysis of the positivity and boundedness of the solutions, equilibrium points, their local stability, and the design of a capital adequacy control strategy. **Section 4** presents numerical simulations based on Indonesian banking data to show model dynamics and policy effects. **Section 5** discusses the results and economic implications, while **Section 6** concludes and outlines the directions for future studies.

## 2. Literature review

An increasing number of studies used continuous-time mathematical models to capture the dynamics of bank balance sheet components. For example, authors in[18] modeled the evolution of deposits and loans using a logistic growth differential equation. The framework was later extended by incorporating equity dynamics driven by profit and NPL factors together with stochastic shocks in withdrawals and NPL realizations.[19] More recently, macroprudential instruments such as loan-to-deposit-ratio-based reserve requirements have been introduced into the models with the aim of offering insights into the regulatory impacts on banking stability.[20,21]

Another complementary line of studies applied discrete-time models to assess the dynamics of the main asset variables owned by banks, with a specific focus on loans in several regulatory environments. The framework was used to analyze the role of capital adequacy requirements and macroprudential regulations using both monopoly and duopoly settings.[22–27] These contributions emphasized the non-linearities arising from the feedback between bank profits, lending, and regulatory constraints.

Across both continuous- and discrete-time methods, the models reproduced several key stylized facts of banking dynamics, including S-shaped deposit expansion, loan contraction driven by defaults, and equity accumulation associated with profitability. The extensions further showed liquidity risks through deposit convexity effects [17] and capital feedback from NPL shocks. [28] The combination of the studies provides a rigorous analytical platform for simulating shock transmission, identifying tipping points, and evaluating the effectiveness of corrective capital policies.

The contributions provide valuable insights into the interplay between banking activities and regulatory measures. However, most existing models treat equity dynamics in a simplified manner and do not consider regulatory control as an active mechanism. Therefore, this study is motivated to develop a continuous-time balance sheet model with explicit equity dynamics under capital adequacy regulations.

## 3. Bank balance sheet dynamic model

A mathematical model was proposed to describe the continuous-time dynamics of bank deposits ($D$), loans ($L$), and equity ($E$) using a system of nonlinear differential equations. Realistic financial mechanisms such as logistic growth, portfolio deterioration through NPL, capital adequacy constraints, and repayment dynamics were incorporated into the model. The proposed system is presented as follows:

$$\frac{dD}{dt} = \alpha_D \, D\left(1 - \frac{D}{K_D}\right) - wD \tag{1}$$

$$\frac{dL}{dt} = \alpha_L \, L\left(1 - \frac{L}{K_L}\right) - \eta L - \delta(1-\eta)L \tag{2}$$

$$\frac{dE}{dt} = \alpha_E \, E\left(1 - \frac{E}{K_E}\right)\left(\frac{E}{L} - \kappa\right) - \eta L \tag{3}$$

All the parameters in the model are assumed to be positive and have economically meaningful interpretations. For example, the constants $\alpha_{\mathrm{D}}, \alpha_{\mathrm{L}}$, and $\alpha_{\mathrm{E}}$ represent the intrinsic growth rates of deposits, loans, and equity, respectively, while $K_D$, $K_L$, and $K_E$ are the corresponding carrying capacities, which reflect market saturation levels or institutional limits. $w$ is the withdrawal rate of deposits, which captures customer liquidity behavior, and $\eta$ is the proportion of loans that become non-performing to represent credit risk in the loan portfolio. Moreover, $\delta$ is the loan repayment rate and $\kappa$ denotes the required capital adequacy ratio (CAR), which is a regulatory constraint to ensure banks maintain sufficient equity relative to loan exposure.

Equation (1) presents depositor behavior with logistic growth tempered by withdrawal. Equation (2) extends logistic loan growth by subtracting both NPL and scheduled repayments, while Equation (3) connects equity growth to retained earnings, which is scaled by the capital-adequacy factor $(E/L - \kappa)$. An important observation is that equity also absorbs the losses from NPLs $(-\eta L)$.

### 3.1. Positivity and boundedness of the solutions

The balance sheet models in Equations ((1)-(3)) were considered to present the following remarks and theorems regarding the positivity and boundedness of the model.

**Remark 1.** *The deposit model* (1) *and loan model* (2) *exhibit generalized logistic-type dynamics with linear-decay terms. Given strictly positive initial conditions* $D(0) > 0$ *and* $L(0) > 0$ *and that all parameters are positive, the solutions* $D(t)$ *and* $L(t)$ *remain positive and bounded for all* $t \geq 0$. *These properties directly follow from the standard results of the logistic differential equations.*

The qualitative behavior of the equity variable $E(t)$ was subsequently investigated. The first result shows that the non-negative equity is not forward invariant.

**Proposition 1.** *Assume* $\eta > 0$ *and* $L(t) > 0$ *for all* $t \geq 0$. *Then, the set* $\{E \geq 0\}$ *is not forward invariant for* (3). *In particular, if there exists* $\ell > 0$ *with* $L(t) \geq \ell$ *on* $[0, T]$, *any solution with* $E(0) > 0$ *crosses* $E = 0$ *no later than time* $E(0)/(\eta\ell)$.

***Proof.*** The evaluation of (3) at $E = 0$ produces $\frac{dE}{dt} = -\eta L(t) < 0$. This shows that the vector field at the boundary points toward $E < 0$ and the invariance fails. If $L(t) \geq \ell > 0$ on $[0, T]$, then there is $dE/dt \leq -\eta L(t) \leq -\eta\ell$ for $E \in [0, \kappa L(t)]$. This shows $E(t) \leq E(0) - \eta\ell t$ and $E$ hits 0 within time $E(0)/(\eta\ell)$.

The proposition reflects a possible crossing of equity into negative values. The next remark interprets this outcome in economic terms and suggests a simple modification to preserve the non-negativity.

**Remark 2.** *A negative $E$ corresponds to insolvency, which is economically meaningful. If one prefers enforcing $E(t) \geq 0$ by construction, the loss term can be regularized, e.g.*

$-\eta L \cdot \mathbf{1}_{\{E>0\}}$ with

$$\mathbf{1}_{\{E>0\}} = \begin{cases} 1, & \text{if } E > 0 \\ 0, & \text{if } E \leq 0 \end{cases} \tag{4}$$

This ensures $\frac{dE}{dt} E\big|_{E=0} = 0$ and $\{E \geq 0\}$ becomes forward invariant.

The issue of non-negativity was addressed, and the process was followed by studying the long-term growth of equity. The following theorem establishes that equity cannot grow without bounds.

**Theorem 1.** *Assume $L(t)$ is positive and bounded: $0 < L(t) \leq L_{\max}$ for all $t \geq 0$. Then, any solution $E(t)$ of (3) is bounded above on $[0, \infty]$.*

***Proof.*** Fix $M \geq 2\max\{K_E, \kappa L_{\max}\}$. For $E \geq M$ this leads to $1 - \frac{E}{K_E} \leq -\frac{E}{2K_E}$ and $\frac{E}{L} - \kappa \geq \frac{E}{2L_{\max}}$ Therefore,

$$\begin{aligned} \frac{dE}{dt} &= \alpha_E E\Big(1 - \tfrac{E}{K_E}\Big)\Big(\tfrac{E}{L} - \kappa\Big) - \eta L \\ &\leq -\frac{\alpha_E}{4K_E L_{\max}} E^3 \;\; =: \; -c\, E^3 \end{aligned} \tag{5}$$

The comparison with $\dot{y} = -cy^3$ shows a decrease in the solutions with $E(0) \geq M$ , which enter $[0, M]$ in finite time. Meanwhile, those with $E(0) \leq M$ remain $\leq M$ by continuity, which shows $E(t)$ is bounded above.

### 3.2. Equilibrium points

The equilibrium points of the system Equations (1)-(3) were determined by solving for $(D^*, L^*, E^*)$ as follows:

$$\frac{dD}{dt} = 0, \quad \frac{dL}{dt} = 0, \quad \frac{dE}{dt} = 0 \tag{6}$$

From equation (1), there is

$$\begin{aligned} \frac{dD}{dt} &= \alpha_D D\left(1 - \frac{D}{K_D}\right) - wD \\ &= D\left[\alpha_D\left(1 - \frac{D}{K_D}\right) - w\right] = 0 \end{aligned} \tag{7}$$

The relationship shows that the equilibrium values for $D$ are $D = 0$ and $D^* = K_D\left(1 - \frac{w}{\alpha_D}\right)$ provided that $\alpha_D > w$.

Let, $r = \eta + \delta(1-\eta)$ represents the effective loan decay rate. This shows the possibility of writing Equation (2) as follows:

$$\begin{aligned} \frac{dL}{dt} &= \alpha_L L\left(1 - \frac{L}{K_L}\right) - rL \\ &= L\left[\alpha_L\left(1 - \frac{L}{K_L}\right) - r\right] = 0 \end{aligned} \tag{8}$$

The equilibrium values for $L$ are $L = 0$ and $L^* = K_L\left(1 - \frac{r}{\alpha_L}\right)$ provided that $\alpha_L > r$.

The next stage was to determine the equilibrium by considering the following two cases.

Case 1: $L = 0$. In this case, the term $\frac{E}{L}$ is undefined unless $E = 0$. Therefore, there is a need to have

$$L = 0 \quad \Rightarrow \quad E = 0 \tag{9}$$

Case 2: $L = L^* > 0$. Substituting into equation (3) leads to the following:

$$\frac{dE}{dt} = \alpha_E E\left(1 - \frac{E}{K_E}\right)\left(\frac{E}{L^*} - \kappa\right) - \eta L^* = 0 \tag{10}$$

This is further reduced to a cubic in $E$ as follows:

$$E^3 - (K_E + \kappa L^*)E^2 + \kappa K_E L^* E + \frac{\eta K_E (L^*)^2}{\alpha_E} = 0. \tag{11}$$

Let $A = K_E + \kappa L^*$, $B = \kappa K_E L^*$, and $C = \frac{\eta K_E (L^*)^2}{\alpha_E}$. This shows the possibility of writing Equation (11) in the standard form $E^3 - AE^2 + BE + C = 0$. Moreover, the transformation $E = y + A/3$ was used to determine the depressed cubic form of $y^3 + py + q = 0$ with

$$p = B - \tfrac{A^2}{3} \qquad q = -\tfrac{2A^3}{27} + \tfrac{AB}{3} + C \tag{12}$$

The discriminant $\Delta = (q/2)^2 + (p/3)^3$ determines the number of real solutions. This shows that if $\Delta > 0$, there is exactly one real root, while $\Delta < 0$ represents three distinct real roots. Only positive real roots satisfying $0 < E^* < K_E$ are economically admissible. The roots are computed numerically using numerical methods during the application process.

The number and location of admissible roots provide insights into the possible steady-state configurations associated with the equity level of the bank and the sensitivity to structural and regulatory parameters. The summary of the equilibrium points is presented as follows.

i. Trivial equilibrium (always exists):

$$(D^*, L^*, E^*) = (0, 0, 0) \tag{13}$$

ii. Deposit-only equilibrium (exists if $\alpha_D > w$):

$$(D^*, L^*, E^*) = \left(K_D\left(1 - \frac{w}{\alpha_D}\right), 0, 0\right) \tag{14}$$

iii. Fully active equilibria (exist if $\alpha_D > w$, $\alpha_L > r$, and Equation (11) has positive solutions):

$$(D^*, L^*, E^*) = (D_0, L^*, E^*) \tag{15}$$

where

$$D_0 \in \left\{0,\ K_D\left(1 - \frac{w}{\alpha_D}\right)\right\} \tag{16}$$

$$L^* = K_L\left(1 - \frac{r}{\alpha_L}\right) \tag{17}$$

and $E^*$ is the positive real root of the cubic Equation (11).

The trivial equilibrium in Equation (13) economically corresponds to a collapsed banking system with no activity. Meanwhile, the deposit-only equilibrium in Equation (14) represents a passive bank that collects deposits but does not issue loans or accumulate equity. The fully active equilibria in Equation (15) describe the functioning banks where deposits, loans, and equity coexist, with equity levels determined by the solutions of cubic Equation (11). The equilibria obtained are consistent with those in previous studies.[19,21]

### 3.3. Local stability of equilibrium points

The local stability of the equilibrium points was analyzed by examining the Jacobian matrix of the system at each steady state. This method linearized the nonlinear dynamics around each equilibrium to determine the local behavior of trajectories from the eigenvalues of the Jacobian. The negative real parts of the eigenvalues corresponded to locally asymptotically stable equilibria, while positive values showed instability.

The partially decoupled structure of the model allowed the Jacobian matrix to take a lower block triangular form. The evolution of equity, explicitly depended on both loans and equity, while the dynamics of deposits and loans changed independently of equity. The Jacobian matrix is presented as follows.

$$J(D, L, E) = \begin{bmatrix} \alpha_D\left(1 - \frac{2D}{K_D}\right) - w & 0 & 0 \\ 0 & \alpha_L\left(1 - \frac{2L}{K_L}\right) - r & 0 \\ 0 & \frac{\partial \dot{E}}{\partial L} & \frac{\partial \dot{E}}{\partial E} \end{bmatrix} \tag{18}$$

The partial derivatives in the third row represent nonlinear equity dynamics. This is specifically presented as follows:

$$\frac{\partial \dot{E}}{\partial L} = \alpha_E E\left(1 - \frac{E}{K_E}\right)\left(-\frac{E}{L^2}\right) - \eta \tag{19}$$

$$\frac{\partial \dot{E}}{\partial E} = \alpha_E\left[\left(1 - \frac{2E}{K_E}\right)\left(\frac{E}{L} - \kappa\right) + \left(1 - \frac{E}{K_E}\right)\frac{1}{L}\right] \tag{20}$$

The eigenvalues of the Jacobian determine the local behavior of the trajectories near each equilibrium. Moreover, the triangular structure shows that the eigenvalues are the diagonal elements of $J(D^*, L^*, E^*)$ and are presented as follows:

$$\lambda_1 = \alpha_D\left(1 - \frac{2D^*}{K_D}\right) - w \tag{21}$$

$$\lambda_2 = \alpha_L\left(1 - \frac{2L^*}{K_L}\right) - r \tag{22}$$

$$\lambda_3 = \left.\frac{\partial \dot{E}}{\partial E}\right|_{(D^*, L^*, E^*)} \tag{23}$$

The equilibrium point is locally asymptotically stable if and only if all eigenvalues have strictly negative real parts. The conditions $\lambda_1 < 0$ and $\lambda_2 < 0$ specifically require that the equilibrium levels of deposits and loans lie in the upper half of the respective logistic growth curves. Moreover, the sign of $\lambda_3$ is determined by the interplay between the nonlinear capital adequacy feedback term $(E/L - \kappa)$ and the convexity induced by the logistic equity growth.

The complexity of fully active equilibria in Equation (15) led to the analysis of only the local stability of the trivial equilibrium in Equation (13) and the deposit-only equilibrium in Equation (14).

**Theorem 2.** *Consider the bank balance sheet system in (1)-(3), where $r = \eta + \delta(1 - \eta)$ represents the effective loan decay rate. The trivial equilibrium point $(D^*, L^*, E^*) = (0, 0, 0)$ is locally asymptotically stable if $\alpha_D < w$ and $\alpha_L < r$. If either inequality is violated, the trivial equilibrium is unstable.*

***Proof.*** The Jacobian matrix of the system was evaluated at its origin. The Jacobian has a lower triangular structure, and its entries at $(0, 0, 0)$ are stated as follows:

$$J(0,0,0) = \begin{bmatrix} \alpha_D - w & 0 & 0 \\ 0 & \alpha_L - r & 0 \\ 0 & -\eta & \left.\dfrac{\partial \dot{E}}{\partial E}\right|_{(0,0,0)} \end{bmatrix} \tag{24}$$

The third diagonal entry was evaluated by considering the equity Equation (3). Moreover, the limit was computed as follows because both $E \to 0$ and $L \to 0$.

$$\lim_{(E,L)\to(0,0)} \left(\frac{E}{L} - \kappa\right) = -\kappa. \tag{25}$$

This changes the derivatives as follows:

$$\left.\frac{\partial \dot{E}}{\partial E}\right|_{(0,0,0)} = \alpha_E(1-0)(-\kappa) = -\alpha_E\kappa. \tag{26}$$

The eigenvalues of the Jacobian are stated as follows:

$$\lambda_1 = \alpha_D - w, \quad \lambda_2 = \alpha_L - r, \quad \lambda_3 = -\alpha_E\kappa. \tag{27}$$

$\lambda_3 < 0$ is often the same for $\alpha_E, \kappa > 0$, and this leads to the determination of the asymptotic stability using the signs of $\lambda_1$ and $\lambda_2$ for the first and second eigenvalues, respectively. Therefore, the equilibrium is locally stable if $\alpha_D < w$ and $\alpha_L < r$.

If either $\alpha_D > w$ or $\alpha_L > r$, the corresponding eigenvalue is positive and the equilibrium loses stability.

**Theorem 3.** *Suppose $\alpha_D > w$ so that the deposit-only equilibrium*

$(D^*, L^*, E^*) = \left(K_D\left(1 - \frac{w}{\alpha_D}\right),\ 0,\ 0\right)$ exists. This shows that the equilibrium is locally asymptotically stable if $\alpha_L < \eta + \delta(1-\eta)$.

***Proof.*** The Jacobian matrix was evaluated at the point $(D^*, 0, 0)$. The change in the status of $L^* = 0$ and $E^* = 0$ makes the Jacobian to become the following:

$$J(D^*,0,0) = \begin{bmatrix} \alpha_D\left(1 - \dfrac{2D^*}{K_D}\right) - w & 0 & 0 \\ 0 & \alpha_L - r & 0 \\ 0 & -\eta & \left.\dfrac{\partial \dot{E}}{\partial E}\right|_{L=0} \end{bmatrix} \tag{28}$$

The first eigenvalue is

$$\begin{aligned} \lambda_1 &= \alpha_D\left(1 - \frac{2D^*}{K_D}\right) - w \\ &= \alpha_D\left(1 - 2\left(1 - \frac{w}{\alpha_D}\right)\right) - w \\ &= w - \alpha_D < 0, \end{aligned} \tag{29}$$

because $\alpha_D > w$

The second eigenvalue is $\lambda_2 = \alpha_L - r$ and the third eigenvalue depends on the behavior of $\partial\dot{E}/\partial E$ at $L = 0$. This was evaluated by considering the equity equilibrium in Equation (3). The $L = 0$ was fixed, and $E > O$ was varied compared to the method used in the trivial equilibrium. This shows that the $\frac{E}{L} \to \infty$ as $L \to 0^+$, and the derivative is states as follows:

$$\left.\frac{\partial \dot{E}}{\partial E}\right|_{L=0} = \alpha_E\Bigg[\left(1 - \frac{2E}{K_E}\right)\left(\frac{E}{L} - \kappa\right) + \left(1 - \frac{E}{K_E}\right)\left(\frac{1}{L}\right)\Bigg] \to -\infty. \tag{30}$$

This divergence reflects a strong restoring force that effectively maintains the equity component at zero. Therefore, the direction was interpreted as being strongly stable. The eigenvalues are presented as follows:

$$\lambda_1 = w - \alpha_D < 0, \quad \lambda_2 = \alpha_L - r, \quad \lambda_3 = -\infty \tag{31}$$

The deposit-only equilibrium was locally asymptotically stable if $\lambda_2 < 0$ and this showed $\alpha_L < r = \eta + \delta(1-\eta)$.

### 3.4. Capital adequacy control strategy

This section was used to investigate a regulatory control strategy aimed at maintaining financial stability by adjusting capital adequacy requirements. The dynamic bank balance sheet models (1)-(3) was considered with the regulatory CAR $\kappa$ treated as a time-dependent control function.

The original system of nonlinear differential equations was modified by allowing $\kappa$ to vary dynamically over time as a control variable $u(t)$. Therefore, controlled system changes to the following:

$$\frac{dD}{dt} = \alpha_D\, D\left(1 - \frac{D}{K_D}\right) - wD \tag{32a}$$

$$\frac{dL}{dt} = \alpha_L\, L\left(1 - \frac{L}{K_L}\right) - \eta L - \delta(1-\eta)L \tag{32b}$$

$$\frac{dE}{dt} = \alpha_E\, E\left(1 - \frac{E}{K_E}\right)\left(\frac{E}{L} - u(t)\right) - \eta L \tag{32c}$$

The control variable $u(t)$ represents the dynamically imposed capital adequacy requirement by the regulator, bounded within an admissible set $\mathcal{U}$ as follows:

$$\mathcal{U} = \{u \in \mathcal{L}^\infty(0,T) : u_{\min} \le u(t) \le u_{\max}\} \tag{33}$$

where, $u_{\min}$ and $u_{\max}$ are the minimum and maximum enforceable capital ratios. The $\mathcal{L}^\infty(0,T)$

represents the space of essentially bounded measurable functions on the interval $[0, T]$. This shows the functions $u(t)$ with the essential supremum identified to be finite.

The regulator was used to enforce capital adequacy while minimizing intervention costs. Therefore, the quadratic objective function was defined as follows:

$$J[u] = \int_0^T \left[ C_1 \left( \frac{E}{L} - \kappa^* \right)^2 + C_2 u(t)^2 \right] dt \quad (34)$$

where, $\kappa^*$ is the target CAR and $C_1, C_2 > 0$ are weighting constants that reflect the priority between regulatory alignment and the cost of control.

The optimal control problem was applied to determine $u^*(t) \in \mathcal{U}$ at

$$J[u^*] = \min_{u \in \mathcal{U}} J[u] \quad (35)$$

The relationship is subject to a dynamic system in Equation (32)-(34), initial conditions $(D(0), L(0), E(0))$, and the constraints on $u(t)$.

The Pontryagin maximum principle (PMP) was applied to derive the necessary conditions for optimality. The PMP is a cornerstone result in optimal control theory that provides a framework to determine an optimal control strategy for continuous-time dynamic systems. It introduces adjoint variables and a Hamiltonian function. Moreover, the principle states that optimal control maximizes or minimizes the Hamiltonian along the optimal trajectory for standard control problems.[29] The preference for the principle was based on the specific suitability for nonlinear differential equation models, such as the bank balance sheet framework without explicit closed-form solutions. From an economic perspective, the PMP enables the design of a time-varying regulatory policy that achieves capital adequacy targets while minimizing intervention costs. It has been previously used in optimal control problems in other fields.[30,31]

The formulation of the PMP was achieved by defining the Hamiltonian function $\mathcal{H}$. The function integrated the integrand of the objective functional and inner products of the adjoint variables $(\lambda_D(t), \lambda_L(t), \lambda_E(t))$ with the respective right-hand sides of the system dynamics. The Hamiltonian is subsequently presented as follows:

$$\begin{aligned} \mathcal{H} &= C_1 \left( \frac{E}{L} - \kappa^* \right)^2 + C_2 u^2 \\ &+ \lambda_D \left[ \alpha_D D \left( 1 - \frac{D}{K_D} \right) - wD \right] \\ &+ \lambda_L \left[ \alpha_L L \left( 1 - \frac{L}{K_L} \right) - \eta L - \delta(1-\eta) L \right] \\ &+ \lambda_E \left[ \alpha_E E \left( 1 - \frac{E}{K_E} \right) \left( \frac{E}{L} - u \right) - \eta L \right] \end{aligned} \quad (36)$$

The dynamics of the adjoint variables were determined by the negative partial derivatives of the Hamiltonian with respect to the state variables.

$$\frac{d\lambda_D}{dt} = -\frac{\partial \mathcal{H}}{\partial D} \quad (37)$$

$$\frac{d\lambda_L}{dt} = -\frac{\partial \mathcal{H}}{\partial L} \quad (38)$$

$$\frac{d\lambda_E}{dt} = -\frac{\partial \mathcal{H}}{\partial E} \quad (39)$$

The transversality conditions were imposed at the following terminal time $T$:

$$\lambda_D(T) = \lambda_L(T) = \lambda_E(T) = 0 \quad (40)$$

The optimal control $u^*(t)$ was determined by minimizing the Hamiltonian with respect to $u$ subject to the constraint $u \in [u_{\min}, u_{\max}]$. The optimality condition produces the following:

$$u^*(t) = \mathrm{Proj}_{[u_{\min}, u_{\max}]} \left( \frac{\lambda_E \alpha_E E \left( 1 - \frac{E}{K_E} \right)}{2C_2} \right) \quad (41)$$

Where, $\mathrm{Proj}_{[a,b]}(x)$ is the projection of $x$ onto the closed interval $[a, b]$ to ensure optimal control respects the regulatory bounds.

## 4. Model calibration and numerical results

The numerical study of the bank balance sheet dynamics governed by the nonlinear differential system described in Equation (1)-(3) is presented in this section. Empirical relevance was achieved by estimating the parameters of each sub-model, including deposits, loans, and equity, using monthly aggregate data of Indonesian commercial banks over a 3-year period from January 2022 to December 2024 with values in IDR billion. This horizon was selected due to the ability to provide consistent and complete reporting for all three balance sheet components in the official statistics of the Indonesian Financial Services Authority (Otoritas Jasa Keuangan, OJK) and is accessible at https://ojk.go.id/en/kanal/perbankan/data-dan-statistik/statistik-perbankan-indonesia/Default.aspx. Moreover, the use of monthly frequency allowed the capturing of medium-term balance sheet dynamics while avoiding the noise of higher-frequency series not available at the aggregate banking sector level. Earlier data were also excluded to avoid structural breaks associated with pre-pandemic conditions and major regulatory changes.

The particle swarm optimization (PSO) algorithm was adopted to estimate the unknown parameters in each equation. This algorithm is a population-based stochastic optimization method inspired by the social dynamics of swarming behavior. The trend is based on collective patterns observed in bird flocks or fish schools where movements are adapted to both individual experience and the behavior of neighboring agents.[32] It has been applied in several studies, including to solve parameter estimation problems related to nonlinear models.[33,34] The PSO was used in this study through the `particleswarm` routine in MATLAB, which allowed the customization of the swarm size, boundary constraints, and convergence criteria to ensure efficient exploration of the parameter space.

The estimation was conducted individually for each subcomponent, including the deposit dynamics in Equation (1), loan dynamics in (2), and equity dynamics in (3). The objective function used for minimization was the mean absolute percentage error (MAPE), which measured the percentage deviation between the simulated model trajectories and actual observed data. The minimization of the MAPE allowed the PSO algorithm to identify the parameters that best replicated the empirical financial behavior of Indonesian commercial banks over the observation window.

**Table 1.** Estimated parameters for the bank balance sheet model using particle swarm optimization(PSO).

| Model | Parameter | Estimated value |
|---|---|---|
| Deposits | $\alpha_D$ | 0.4494 |
| | $K_D$ | 87,611,692.61 |
| | $w$ | 0.3435 |
| | MAPE | 0.8% |
| Loans | $\alpha_L$ | 0.1831 |
| | $K_L$ | 49,962,745.85 |
| | $\eta$ | 0.0372 |
| | $\delta$ | 0.0138 |
| | MAPE | 0.85% |
| Equity | $\alpha_E$ | 1.4684 |
| | $K_E$ | 18,592,037.94 |
| | MAPE | 2.07% |

Note: k = 0.08 was used as the regulatory parameter.

The estimated parameters and associated MAPE values are summarized in **Table 1**. All three submodels showed a strong fit to the data, with MAPE values observed to be significantly lower than 2.1%. This supported the validity and applicability of the proposed dynamic framework for capturing real-world balance sheet movements. **Figure 1** shows the model-fitting results for the three principal components of the bank balance sheet compared with the observed data from Indonesian commercial banks between January 2022 and December 2024. It was observed that the model effectively replicated the upward trend in deposit accumulation as presented in panel (a) with minimal deviation. The trend showed the efficacy of the logistic growth formulation and the withdrawal adjustment. Panel (b) showed an excellent fit for loan dynamics, and this confirmed that the estimated parameters for loan growth, repayment, and credit risk accurately captured the credit market behavior. In panel (c), the equity component also showed a good fit, but with slightly larger deviations potentially due to unmodeled variations in retained earnings or regulatory capital adjustments. The close relationship between the model simulations and empirical data generally provided a practical manifestation of the ability of the model to capture important balance sheet dynamics and validated its use for policy experiments. The numerical results specifically established a strong empirical foundation for the subsequent analysis of regulatory scenarios such as the impact of alternative capital adequacy requirements and optimal control strategies.

The next stage was the presentation of numerical experiments to evaluate the impact of different CAR on equity dynamics. **Figure 2** shows the sensitivity of equity dynamics to varying values of the CAR parameter $\kappa$ over a 3-year period. The results showed that lower values of $\kappa$, such as 0.02 and 0.04, were associated with more rapid equity growth, while higher values in the form of 0.08 and 0.16 corresponded to a slower increase or even a decline. The trend was in line with the model structure, where higher capital requirements imposed greater constraints on equity accumulation by limiting the proportion of equity relative to loans. The simulation showed a critical trade-off in regulatory design where stricter capital requirements enhanced solvency buffers but also suppressed equity growth when not accompanied by a proportional increase in profitability or loan expansion. The results reflected that the choice of $\kappa$ significantly influenced long-term bank capitalization trajectories.

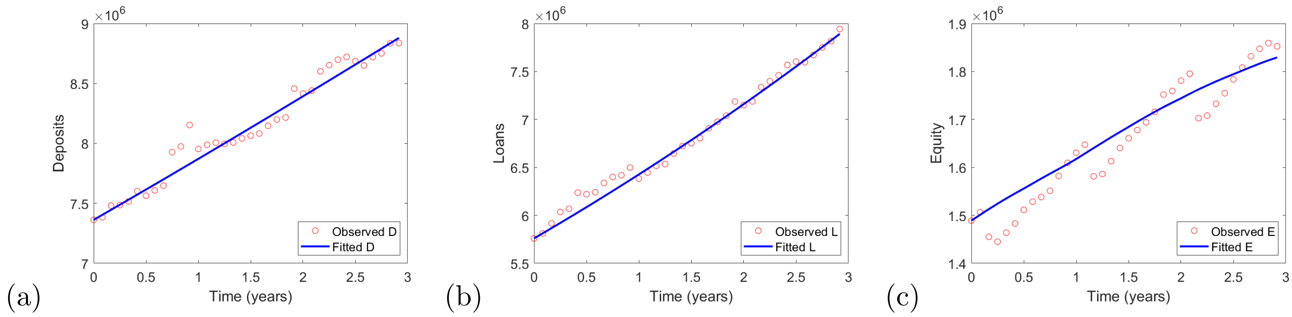


**Figure 1.** Model fitting results of the bank balance sheet components to Indonesian commercial banking data (in IDR Billion) from January 2022 to December 2024. Each sub-plot compares the observed data (red circles) with model simulations (blue lines) for **(a)** deposits, **(b)** loans, and **(c)** equity.

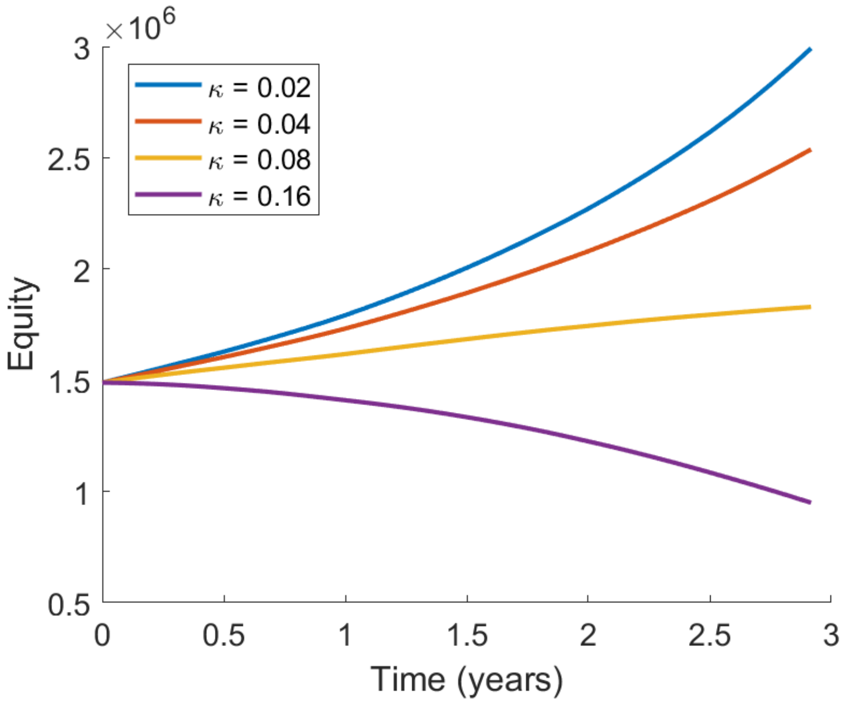


**Figure 2.** Sensitivity analysis of the equity (in IDR billion) trajectory with respect to different values of the capital adequacy ratio parameter $\kappa$. The simulation demonstrates how varying $\kappa$ affects the evolution of equity over a three-year period.

The next numerical experiments were conducted to compare uncontrolled and optimally controlled scenarios. These results serve as a practical application of the model to regulatory design. **Figure 3** shows the simulated dynamics of bank equity under both controlled and uncontrolled scenarios, along with the optimal capital adequacy control trajectory over time. In panel (a), the solid curve delineates the equity path under optimal regulatory control, while the dashed curve represents equity evolution under a constant CAR fixed at $u = 0.08$, in line with the standard in Indonesian commercial banking. The equity exhibited steady growth over time in the absence of control and initially declined under optimal control, but eventually stabilized. This controlled decline reflected a deliberate trade-off aimed at minimizing deviations from the target capital ratio while reducing intervention costs. Panel (b) shows the time-dependent optimal control $u(t)$ which starts at the upper bound of 0.2 and gradually declines to the lower bound of 0.05. This trajectory reflects a regulatory strategy that initially imposes stringent capital requirements before relaxing to support the recovery of equity. The result showed the capacity of the model to design dynamic and cost-effective regulatory interventions that balance capital adequacy targets with the evolution of internal financial dynamics within the bank.

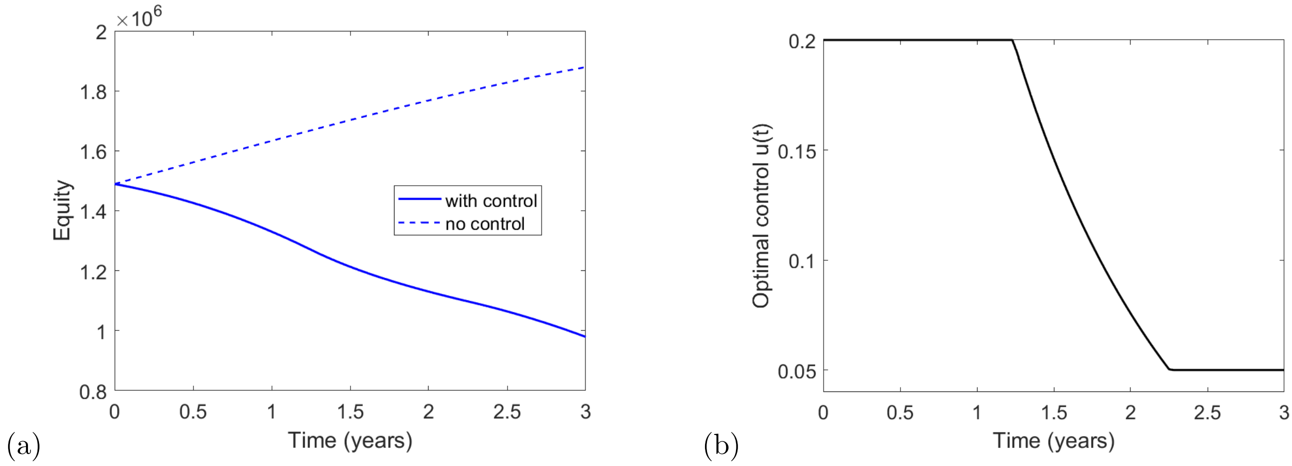


**Figure 3.** Simulation results for optimal regulatory control of bank equity. (a): Simulation of bank equity under two scenarios—uncontrolled ($u = 0.08$) and controlled using an optimally time-varying capital adequacy ratio. (b): The optimal control trajectory $u(t)$ representing a dynamically adjusted regulatory policy.

## 5. Discussion

The local stability conditions derived for trivial and deposit-only equilibria had significant economic implications for banking system dynamics. The trivial equilibrium represents a degenerate state in which the bank has no deposits, loans, or equity capital. Mathematically, this equilibrium is locally stable when the intrinsic growth rate of deposits $\alpha_D$ is less than the withdrawal rate $w$ and the growth rate of loans $\alpha_L$ is less than the effective decay rate $r = \eta + \delta(1 - \eta)$. The conditions economically imply that banks lack sufficient customer inflows or credit expansion to counterbalance outflows and portfolio deterioration. The scenario reflects a failing or inactive financial institution where liabilities and assets cannot grow from near-zero levels, and equity is continuously depleted due to loan losses. This shows the need for regulatory measures to address the situation. Kashif et al.[35] suggested that an increase in capital requirements for rapidly growing banks and the implementation of strong supervision could assist in preventing the adverse consequences of poor borrower selection and excessive risk-taking. Kandrac and Schlusche[36] also emphasized the importance of bank supervision in limiting risk-taking behavior and reducing the likelihood of bank failures.

The contrary observation in the deposit-only equilibrium showed that the bank successfully attracted and maintained a stable deposit base but did not engage in lending activities. This equilibrium is locally stable when the loan growth rate $\alpha_L$ is lower than the effective decay rate $r$. The condition shows that the loan expansion capacity is constrained by high credit risk or NPL ratio $\eta$ and frequent repayments captured by $\delta$ even though the bank can maintain deposit operations. In practice, this reflects a conservative or risk-averse bank that refrains from issuing loans when the market or borrower quality is low and prefers to operate passively or hold excess liquidity. An interesting observation is that banks exposed to higher liquidity risks during financial crises tend to contract credit supplies more sharply,[37] but the relationship is not straightforward. Some studies reported that banks with worse performance did not necessarily hoard liquidity during crises.[38] The expansion of peer-to-peer lending could also decrease the total risk of banks despite increasing insolvency risk by reducing the illiquidity risk.[39] However, the system becomes unstable in the loan direction, and the bank tends to transition toward a lending-active regime when $\alpha_L > r$. This leads to the movement toward a more complex, nontrivial steady state that includes all three balance sheet components.

The general analytical results showed that the full system could exhibit qualitatively different stability regimes depending on whether the deposit and loan components dominated the respective carrying capacities and equity dynamics acted as restoring or destabilizing forces. These stability thresholds offer insights into the conditions when banks can maintain balance sheet resilience under regulatory constraints.

The numerical analysis results showed that the selection of the CAR policy parameter $\kappa$ had a substantial impact on the long-term trajectories of bank capitalization. Multiple previous studies also supported the opinion that CAR had implications for financial stability and performance. For example, the studies conducted on Turkish banks showed that loans, return on equity, and leverage negatively affected CAR, while loan loss reserves and return on assets had a positive influence.[40] A study of Ethiopian commercial banks similarly showed that the loan-to-deposit ratio and total deposit-to-total asset ratio positively impacted financial performance closely related to capitalization.[41] In the context of Islamic banks in the Gulf Cooperation Council, Basel III reforms are important for preserving financial sector stability by regulating dynamics between CAR and credit risk.[42] Pakistani banks also showed improvements in profitability, market returns, and CAR during the COVID-19 pandemic. The trend reflected the effectiveness of sustainable banking regulations in maintaining banking sector health, even during economic crises.[43] In summary, this study showed that CAR policies significantly influenced the long-term path of bank capitalization. Higher capital levels were associated with increased lending, liquidity creation, bank value, and improved survival rates during crises.[44] However, the relationship between CAR and bank performance is complex and could be influenced by several factors, such as competition, risk-taking behavior, and the broader economic environment.[45] The insights can also be perceived in light of current regulatory debates, which include the ability of Basel IV to strengthen capital adequacy rules by tightening risk-weighted asset calculations and leverage ratios. This is consistent with the results that stricter requirements can slow equity growth unless supported by profitability or better credit quality. From a macroprudential perspective, instruments such as countercyclical capital buffers and loan-to-deposit requirements can be represented in the model as dynamic controls. This shows the potential of the framework to assess ongoing regulatory reforms and the trade-offs between stability and lending capacity.

## 6. Conclusion

In conclusion, this study developed a continuous-time dynamic model of the bank balance sheet by integrating deposits, loans, and equity under capital adequacy constraints. The framework contributed to the literature by combining logistic-type growth for deposits and loans with equity dynamics explicitly connected to retained earnings and regulatory requirements. Indonesian commercial banking data from 2022 to 2024 were used to calibrate the model parameters and validate the empirical relevance. Furthermore, capital adequacy regulation was formulated as an optimal control problem, and the PMP was applied to show the ability of adaptive policies in stabilizing bank capitalization while limiting intervention costs.

This study contributed a tractable and empirically validated dynamic framework for analyzing the interactions between the deposits, loans, and equity under regulatory constraints. The explicit modeling of the feedback mechanisms associated with capital adequacy and credit risk allowed the model to enable a nuanced understanding of how capital requirements shaped the trajectory of equity accumulation and lending. Moreover, local stability analysis of the equilibrium points showed multiple dynamic regimes that depended on the intrinsic growth rates, credit quality, and regulatory settings.

Numerical simulations using Indonesian banking data showed that the model was closely related to the observed trends, particularly in capturing the saturation of deposits, decay of loans due to nonperforming exposures, and growth of equity tied to retained earnings. A sensitivity analysis of CAR also reflected the non-trivial trade-off between solvency requirements and equity performance. This was because higher regulatory capital thresholds tended to restrict equity growth, particularly when not accompanied by improvements in profitability or in asset quality.

The implementation of the optimal control framework showed the capacity of dynamic regulatory interventions in enhancing financial resilience while limiting policy costs. The PMP was also applied to derive time-dependent control laws that adapted CAR to the changing balance sheet conditions. This optimal policy was initiated with high regulatory pressure and gradually relaxed over time. The trend suggested that temporary stringency followed by easing could lead to the achievement of both prudential and developmental objectives. The insights contributed valuable implications for designing macroprudential tools in banking systems, particularly in emerging markets facing volatile credit cycles. Beyond the methodological contribution, the results provided concrete guidance for policymakers. Regulators could deploy time-varying capital adequacy requirements to curb excessive risk-taking during credit booms and also stimulate lending when the economy requires growth support. The flexibility was considered specifically relevant in jurisdictions where banking systems served as the primary channel of financial intermediation. The process was also considered useful where static, one-size-fits-all regulations could either stifle credit supply or fail to prevent instability. Therefore, this study showed the importance of adopting adaptive and context-sensitive prudential policies that balanced financial stability with economic development.

The model provides useful insights but has certain limitations. It focuses primarily on capital adequacy dynamics in determining dynamical systems, which can oversimplify the diversity and uncertainty observed in real financial systems. Future studies should explore several extensions of the model. First, the incorporation of stochastic shocks into a dynamic system can enable the assessment of regulatory policies under uncertainty and market volatility. Second, the model can be expanded to include additional banking variables such as liquidity buffers, risk-weighted assets, and interbank lending. Finally, calibrating the model to multiple banks in a network setting can enable the analysis of systemic risk propagation[46,47] and the design of coordinated macroprudential policies[48]. These directions offer promising avenues for enhancing the robustness and applicability of dynamic regulatory models in modern financial systems.

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## Conflict of interest

The author declares there are no competing interests.

## Author contributions

This is a single-authored article.

## Availability of data

The data is accessible at https://ojk.go.id/en/kanal/perbankan/data-dan-statistik/statistik-perbankan-indonesia/Default.aspx.

## AI tools statement

The author acknowledges the use of AI-assisted tools in the preparation of this manuscript. Specifically, ChatGPT by OpenAI and Paperpal by Editage were used to support language editing and formatting. All intellectual content, conceptual development, and analyses were solely the work of the author.

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***Moch. Fandi Ansori*** *earned a Doctoral degree in Mathematics from Institut Teknologi Bandung, Indonesia in 2021. He is currently an Assistant Professor in the Department of Mathematics at Universitas Diponegoro, Indonesia. His primary research interest lies in the mathematical modeling of banking dynamics using continuous and discrete-time models to analyze banking regulations.*

https://orcid.org/0000-0002-4588-3885

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