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RESEARCH ARTICLE

Optimizing subgroup selection in petrochemical industries: A robust data envelopment analysis approach for uncertainty management

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ABSTRACT

This study addresses the critical challenge faced by organizations in selecting an optimal subgroup of decision-making units (DMUs). Such a selection procedure can significantly influence efficiency, profitability, and strategic development. Recognizing the limitations of existing methods in handling inexact data and incorporating managerial preferences, this study proposes a novel framework that integrates data envelopment analysis (DEA) with binary linear programming models. The model applies belief-degree–based representations of uncertainty to capture imprecise inputs and outputs. For this model, two solution approaches—namely, chance-constrained programming and expected value approaches—were developed. These approaches are suitable for real-world applications using standard optimization software. The effectiveness of the proposed method was validated through a case study in Iran's petrochemical industry, where it successfully identified the optimal technology for a new refinery unit while balancing efficiency and profitability under uncertainty. This work is the first study in the literature to combine DEA and binary linear programming under belief-degree–based uncertainty for DMU selection, offering a systematic, practical, and computationally efficient solution, with recommendations for future research to explore alternative uncertainty modeling techniques and broader industrial applications.

1. Introduction

1.1. Conceptual background and literature review

Efficiency is a crucial factor in the success of any organization, and data envelopment analysis (DEA) provides an effective method for measuring and improving efficiency.[1] In fact, DEA is a widely used method for determining the efficiency of decision-making units (DMUs) in the presence of multiple inputs and outputs.[2] It was initially introduced by Farrell in 1957 to evaluate and compute efficiency.[3] Later, in 1978, Charnes et al.[4] extended Farrell's idea to incorporate units with multiple inputs and outputs, and this model became known as the Charnes–Cooper–Rhodes model. Subsequently, Banker et al.[5] introduced another model, the Banker–Charnes–Cooper model, which disregards the principle of constant returns to scale in the production possibility set. Additionally, in classical DEA models, each decision-maker can select appropriate

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weights according to their inputs and outputs to maximize efficiency. In some cases, to compare DMUs, all units must be measured using a model with a common set of weights while optimizing their efficiency.[6] DEA models have been applied in numerous fields—such as finance, healthcare, and manufacturing—to evaluate the efficiency of DMUs.[7]

An application of DEA in real-world problems is the selection of an optimal group from several units under evaluation. For instance, DEA has been applied to supplier selection in supply chain management,[8] portfolio optimization in finance, and the development of efficient healthcare facilities through the selection of clinical units. This selection challenge can be expressed as a combinatorial optimization problem aimed at identifying the group of DMUs that achieves the maximum aggregate efficiency. However, these problems are usually Non-deterministic polynomial time (NP-hard) problems, and traditional methods cannot be easily solved.[9,10] To select a subgroup from the available options, Contreras et al.[11] applied the DEA model to determine the most efficient group. The proposed model was then used to rank the best distribution among hydrogen fuel stations. However, the formulation presented by Contreras et al.[11] only allows the selection of units that lack inputs. This is a very restrictive condition, making the approach unsuitable for many applications.

Another important limitation that should be considered is that input and output data are often uncertain due to multiple factors. Uncertainty commonly arises from measurement errors, incomplete data, and environmental changes. Given the prevalence of imprecise and uncertain information in real-world contexts, it is often impractical to use precise (crisp) input–output data in DEA models. Therefore, classical DEA models have been extended to handle uncertain data.

To overcome this limitation, several types of DEA models have been proposed, including stochastic DEA,[12,13] fuzzy DEA,[14–16] and robust DEA.[17,18] Considering uncertainty in DEA applications provides an appealing framework for generating more precise and informative results for decision-makers. This enables policymakers to incorporate uncertainty into the inputs and outputs of DMUs, thereby facilitating more realistic and robust assessments of efficiency. Additionally, these variables may be treated as random variables when sufficient historical data are available, allowing their probability distributions to be estimated.

In contrast, in certain situations—such as adverse weather conditions, sudden accidents, or crowding—historical data may be unavailable, and the probability distributions of such parameters are therefore inaccessible. In these cases, experts use degrees of belief to describe the likelihood of events. To address this, Liu[19] proposed the uncertain theory, which can represent the degree of belief. This theory has been used in multiple fields, including material requirements planning,[20] supply chain network optimization and assembly line balancing,[21,22] supply chain management,[23,24] and DEA.[17,25–27]

The application of DEA under belief-degree–based uncertainty is a growing research area that deals with DEA models involving uncertain inputs and outputs. For instance, Kiani et al.[28] applied uncertain theory to the additive DEA model to evaluate efficiency over time. Salehi et al.[29] analyzed the additive DEA model to examine its sensitivity and stability. Additionally, Wen et al.[30] proposed new models and several ranking indicators by incorporating uncertainty into the inputs and outputs of DEA models. Mohmmad Nejad and Ghaffari-Hadigheh[31] developed a model designed to achieve the highest degree of belief that a DMU is efficient. Although the inputs and outputs are of uncertain type, this model is highly sensitive to small changes in the data; therefore, it is not suitable for practical applications. Lio and Liu[32] developed a DEA model under uncertainty and conducted stability and sensitivity analyses for the uncertain DEA model. Jamshidi and co-workers[17,25–27] investigated different uncertain DEA models based on belief degrees and applied them to the Iranian banking system. Pourmahmoud and Bagheri[33] employed the Malmquist productivity index to evaluate healthcare systems during the COVID-19 pandemic in an uncertain environment based on belief degrees. Mahmoodirad et al.[34] studied the slacks-based measure model of DEA in an uncertain environment, where inputs and outputs were defined based on belief degrees.

Although there is a great deal of research on developing new models and addressing the limitations of traditional DEA models, some researchers have also investigated the application of DEA models in different industries in addition to model development. In recent years, many studies have employed DEA models to evaluate the performance of different industries, including the petrochemical industry. Considering that the petrochemical industry plays a crucial role in the economies of many countries, the following section reviews several studies conducted in this field. For example, Han and Geng[35] introduced an integrated DEA–analytic hierarchy process method to assess ethylene production processes in China. Chung and Heshmati[36] focused on pollutant emissions in several industries, including petrochemicals, in Korea. They used the Malmquist–Luenberger Metafrontier model to account for undesirable outputs and non-homogeneous production groups. Their DEA-based performance evaluation revealed that groups with higher labor productivity exhibited greater performance growth than other competitors.

According to expert opinions, although supplier selection is an important issue in most organizations, it is particularly critical in the petrochemical industry, especially in some countries, and must be performed with greater precision. Therefore, Bafrooei et al.[37] examined supplier evaluation in the petrochemical industry using a DEA–multi-criteria decision analysis approach that incorporates both quantitative and qualitative indicators. In another study, Assaf et al.[38] used DEA models to measure the performance of employees and maintenance units in Saudi Arabia's petrochemical industry. Through this analysis, they identified the existing gaps to improve the performance of the petrochemical sector.

Ramazankhani et al.[39] considered the importance of petroleum and petrochemical products in Iran and evaluated the performance of the districts of Iran for hydrogen production by water electrolysis using generated electricity. Therefore, for ranking and prioritizing the ability of different districts in the field, DEA models were applied, and the obtained outcomes were compared with the literature. Gilsa et al.[40] analyzed the performance of second-generation petrochemical companies by considering changes in technology and capital. For this purpose, the efficiency of these companies was calculated using DEA models, and technical changes and their impact on performance were analyzed using the Malmquist index.

Considering the fact that in some countries, including Saudi Arabia, the economy is highly dependent on oil and oil products, Alidrisi et al.[41] evaluated and ranked petrochemical and oil companies in Saudi Arabia using DEA. To reduce costs and focus on the production of petrochemical products in Vietnam, which often have high economic value, and to select the efficient suppliers in this industry, Wang et al.[42] used DEA and the analytic hierarchy process as suitable tools for evaluating and analyzing suppliers in used-oil projects and for providing appropriate strategies. Iran is another country where the petrochemical industry plays an important role in its economy. Keivani et al.[43] analyzed the monthly performance of the ZPC Company using DEA.

Over the past few decades, due to the increase in greenhouse gases, the analysis of the performance of oil, gas, and petrochemical industries aimed at reducing greenhouse gases has become one of the important goals of policymakers. Thus, unlike other common DEA models, Mozaffari et al.[44] proposed a new model for a two-stage process and evaluated the performance of petrochemicals in Iran using a DEA network model, considering internal relationships in petrochemicals and undesirable outputs such as greenhouse gases in the production of ammonia in the petrochemical sector.

Additionally, Izadikhah et al.[45] noted that sustainability and the consideration of risk factors are influential issues in petrochemicals. Therefore, to manage risk in petrochemicals, they evaluated the stability of suppliers related to petrochemicals in Iran. Their proposed method using DEA models was compared and validated with other techniques.

Bazargan et al.[46] investigated the performance of Iran's oil supply chain and determined its progress and regression. The performance was evaluated using network DEA models and the Malmquist productivity index. Zhu and Zhang[47] evaluated the use of DEA models to increase petrochemical capacity and joint innovation platforms, enhance cooperation between innovative institutions, and strengthen the relationship between industry and academia in China, addressing innovation gaps in the joint structure between the petrochemical industry and universities.

1.2. Novelties of this study

The selection of an optimal subgroup of DMUs is an important problem in many applications of DEA. However, only a limited number of studies have been conducted in this field. To address this limitation, we propose a novel model for selecting the optimal subset of DMUs using DEA. The proposed model is based on the concept of optimal subset selection and aims to identify the best subset of DMUs that achieves the highest efficiency. The model is first described using crisp input and output data and then extended to an uncertain environment with uncertain data based on the belief degrees of experts.

Table 1. Relevant studies on optimal subgroup selection and merging of decision-making units

Authors	Type of merging	Type of data	Solution method	Area
Gattoufi et al.[48]	Combination of their	Crisp	IDEA	Banking
Amin and Boamah[49]	inputs and outputs	Crisp	IDEA	Banking
Zeinodin and Ghobadi[50]	to create a new DMU	Crisp	IDEA	Educational
Soltanifar et al.[51]		Crisp	IDEA	Banking
Thomas et al.[52]	Independent DMUs	Crisp	BILP–DEA	Facility location
Klimberg and Ratick[53]	to create a	Crisp	DEA–ILP	Facility location
Moheb-Alizadeh et al.[54]	new optimal group	Fuzzy	DEA–ILP	optimal location
Contreras et al.[48]		Crisp	BILP–DEA	Hydrogen fueling
Current study		Belief degree	DEA–ILP	Petrochemical

Abbreviations: BILP: Binary integer linear programming; DEA: Data envelopment analysis;
DMU: Decision-making unit; IDEA: Inverse data envelopment analysis; ILP: Integer linear programming.

Table 1 summarizes the studies conducted on the optimal subgroup selection problem using DEA.

1.3. Structure of the article

The rest of this article is organized into several sections. Section 2 introduces the uncertainty theory relevant to this study. The proposed model for optimal subgroup selection using DEA in both crisp and uncertain environments is presented in Section 3. Section 4 describes the solution methodology. Since the petrochemical industry is a complex and dynamic industry that is exposed to various sources of uncertainty, such as fluctuating market demand, changing regulations, and unpredictable natural events, to demonstrate the applicability of the proposed model for selecting the optimal subset of DMUs under uncertain conditions, the petrochemical industry was chosen as the case study and is discussed in Section 5. The managerial insights derived from the analysis are also presented in this section. Finally, Section 6 discusses the results and suggestions for future research.

2. Uncertainty theory

Definition 1. [19]*Assume Γ is a non-empty set and L is a σ-algebra of Γ. $M\{\Lambda\}$ is a function that calculates the belief degree (uncertainty value) of the event Λ ($\Lambda \in L$).*

$M\{\Lambda\}$ is considered as the belief degree function for the occurrence of the uncertain event Λ (where $\Lambda \in L$). This function should satisfy the following conditions:

(i) $M\{\Gamma\} = 1$.
(ii) $M\{\Lambda\}+M\{\Lambda'\} = 1$ (Λ' is the complement of Λ).
(iii) For a countable sequence $\{\Lambda_i\}$ with $i = 1, 2, \ldots, \infty$, $M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}$.
(iii) If $\{\Gamma_k, L_k, M_k\}$ be the k-th uncertainty space from an infinite set of uncertainty spaces, then $M\{\prod_{k=1}^{\infty} \Lambda_k\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\}$.

Definition 2. [19]*The uncertainty distribution of Φ from the uncertain variable ξ is defined as $\Phi(x) = M\{\xi \leq x\}$, $\forall x \in \mathbb{R}$.*

Definition 3. [19]*Suppose ξ is an uncertain variable, its expected value is determined as $E[\xi] = \int_0^{\infty} M\{\xi \geq r\}\, dr - \int_{-\infty}^{0} M\{\xi \leq r\}\, dr$, where one of the integrals must be finite.*

Definition 4. [19]*Uncertain variable ξ defined on its related uncertainty space $\{\Gamma, L, M\}$, is called a positive uncertain variable, if $M\{\xi \leq 0\} = 0$.*

Theorem 1. [19]*Suppose that $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing in $x_1, x_2, \ldots, x_m$ and strictly decreasing in $x_{m+1}, x_{m+2}, \ldots, x_n$, then $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has the inverse uncertainty distribution:*

$$\Psi^{-1}(\alpha) = f\big(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\big).$$

Definition 5. [19]*Suppose that ξ is the uncertain variable with a regular uncertainty distribution $\Phi(x)$. If the expected value exists, then $E[\xi] = \int_0^1 \phi^{-1}(\alpha)\, d\alpha.$, where $\varphi^{-1}(\alpha)$ is the inverse of the uncertainty distribution of ξ.*

Theorem 2. [19]*Assume that $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If the function $f(x_1, x_2, \ldots, x_n)$ is strictly increasing in $x_1, x_2, \ldots, x_m$ and strictly decreasing in $x_{m+1}, x_{m+2}, \ldots, x_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has the expected value of*

$$E[\xi] = \int_0^1 f\big(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha),$$
$$\Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha)\big)\, d\alpha.$$

Theorem 3. [19]*For independent uncertain variables ξ and η, and real numbers a and b, the expected value satisfies $E[a\xi + b\eta] = aE[\xi] + bE[\eta]$.*

Definition 6. [19]*An uncertain variable ξ is linear (denoted by $L[a,b]$ with real numbers a and b, where $a < b$) if its distribution has the following linear form*

$$\Phi(x) = \begin{cases} 0, & x \le a, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ 1, & x \ge b. \end{cases}$$

Hence, the inverse uncertainty distribution of the linear uncertain variable $L(a,b)$ is $\Phi^{-1}(\alpha) = (1-\alpha)a + \alpha b$ and its expected value is $E[\xi] = \frac{a+b}{2}$.

Theorem 4. [19]*For an uncertain variable ξ with continuous uncertainty distribution Φ and real number x, the following relations hold:*

$$M\{\xi \le x\} = \Phi(x).$$

$$M\{\xi \ge x\} = 1 - \Phi(x)$$

Lemma 1. *For positive real numbers a and t, and independent uncertain variable ξ with continuous uncertainty distribution Φ, $M\{\, a - \xi \ge t \,\} \ge r$ is held if $a - \Phi^{-1}(r) \ge t$.*

3. Optimal subgroup selection problem using data envelopment analysis

In real-world scenarios, organizations and individuals often face complex decisions in which they must select a subset from a larger group based on specific criteria or managerial goals. The selection process involves multiple factors such as profitability, cost, and other measures. This study focuses on a particular case where the efficiency of the selected group plays a significant role. In other words, the decision-makers aim to select a group consisting of homogeneous DMUs that produce the same outputs, ultimately leading to the highest efficiency compared to other groups. In this study, we examine different categories of DMUs to explore the selection of optimal groups. Several key considerations must be taken into account regarding the selected groups:

(i) The number of members in each category is predetermined by the manager. Therefore, $\binom{\mathrm{n}}{\mathrm{k}}$ categories must be examined and compared, where n is the total number of units and k is the number of units selected for the group.
(ii) It is important to emphasize that within each category, the operation of individual units is independent, and there is no provision to exchange resources between them.
(iii) Although the decision-maker supervises the selected units, they cannot modify the inputs and outputs of these units. In other words, each unit must be accepted with all its weaknesses and strengths.
(iv) It is essential to note that the aggregation of inputs and outputs of DMUs is not permitted.

3.1. Proposed data envelopment analysis model for optimal subgroup selection

Suppose that we have n homogeneous DMUs, represented by $j = 1, 2, \ldots, n$, such that DMU_j produces the output vector $y_j = (y_{1j}, \ldots, y_{sj})^T$ by consuming the input vector $x_j = (x_{1j}, \ldots, x_{mj})^T$. The aim is to choose a subset containing k members from the set of n members ($k > n$), so that the efficiency of the obtained group is higher than that of other possible groups.

Suppose that $v_j = (v_{1j}, \ldots, v_{mj})$ and $u_j = (u_{1j}, \ldots, u_{sj})$ are the weight vectors corresponding to the input and output of DMU_j respectively. In this case, the efficiency of DMU_j is defined as follows:

$$e_j = \frac{u_j y_j}{v_j x_j} = \frac{\sum_{r=1}^{s} u_{rj} y_{rj}}{\sum_{i=1}^{m} v_{ij} x_{ij}}, \quad j = 1, \ldots, n \tag{1}$$

and the total efficiency of the selected subgroup is determined by a convex combination of the efficiency value of the selected units, as shown in Model (2).

$$e^{Total} = \sum_{j \in \{j_1, \ldots, j_k\}} w_j e_j = \sum_{j \in \{j_1, \ldots, j_k\}} w_j \frac{\sum_{r=1}^{s} u_{rj} y_{rj}}{\sum_{i=1}^{m} v_{ij} x_{ij}},$$
$$\sum_{j \in \{j_1, \ldots, j_k\}} w_j = 1, \qquad w_j \ge 0. \tag{2}$$

As mentioned earlier, the decision-maker seeks to determine a group of DMUs such that the efficiency of the selected category is maximized.

Henceforth, this group is referred to as the optimal subgroup. Therefore, to specify the members of the optimal group, Model (3) is proposed:

$$Max \sum_{j=1}^{n} t_j w_j \frac{u_j y_j}{v_j x_j}$$

$$subject\ to$$

$$\frac{u_j y_j}{v_j x_j} \le 1, \quad j = 1, \ldots, n \tag{3}$$

$$\sum_{j=1}^{n} t_j = k$$

$$\sum_{j=1}^{n} w_j = 1$$

$$v_j \ge 0, \quad u_j \ge 0, \quad j = 1, \ldots, n$$

$$t_j \in \{0, 1\}, \quad j = 1, \ldots, n$$

The selected units in the optimal group are identified using the binary variable t_j. Specifically, $t_p = 1$ indicates that DMU_p is selected, whereas $t_p = 0$ indicates that DMU_p is not selected. Additionally, the constraint $\sum_{j=1}^{n} t_j = k$ ensures the selection of exactly k units in the optimal group.

Determining the value of w_j is particularly crucial for assessing the performance of each subgroup. Thus, the weight w_p is defined as the ratio of the inputs consumed by DMU_p to the total inputs used in each subgroup. Accordingly,

$$w_p = \frac{v_p x_p}{\sum_{j \in \{j_1, \ldots, j_k\}} v_j x_j} = \frac{v_p x_p}{\sum_{j=1}^{n} t_j v_j x_j} \tag{4}$$

As a result, by applying the Charnes–Cooper transformation, Model (3) can be reformulated as follows:

$$Max \sum_{j=1}^{n} t_j u_j y_j$$

$$subject\ to$$

$$\sum_{j=1}^{n} t_j v_j x_j = 1$$

$$u_j y_j - v_j x_j \le 0, \quad j = 1, \ldots, n \tag{5}$$

$$\sum_{j=1}^{n} t_j = k$$

$$v_j \ge 0, \quad u_j \ge 0, \quad j = 1, \ldots, n,$$

$$t_j \in \{0, 1\}, \quad j = 1, \ldots, n$$

The above model can be reformulated as Model (6). Therefore, Models (5) and (6) are equivalent.

$$Max \sum_{j=1}^{n} u_j y_j$$

$$subject\ to$$

$$\sum_{j=1}^{n} v_j x_j = 1,$$

$$u_j y_j - v_j x_j \le 0, \quad j = 1, \ldots, n,$$

$$u_j \le M \times t_j, \quad j = 1, \ldots, n, \tag{6}$$

$$v_j \le M \times t_j, \quad j = 1, \ldots, n,$$

$$\sum_{j=1}^{n} t_j = k,$$

$$v_j \ge 0,\ u_j \ge 0, \quad j = 1, \ldots, n,$$

$$t_j \in \{0, 1\}, \quad j = 1, \ldots, n.$$

In Model (6), M is a very large positive number. Thus, if $t_p = 1, u_p \le M \times t_p$ and $v_p \le M \times t_p$ are redundant constraints. Additionally, if $t_p = 0$, then $u_p = 0$ and $v_p = 0$ Therefore, the performance of DMU_p does not influence the objective function. Thus, only a binary programming problem determines the optimal group.

Theorem 5. *Model (6) is feasible.*

Proof. We know that $x_j \neq 0$. Without loss of generality, suppose that $x_{1j} \neq 0$. We define $K = \{j_1, j_2, \ldots, j_k\}$. It is easy to show that,$u_j = (0, \ldots, 0)$, $v_{1j} = \frac{1}{x_{1j}}$, $v_{ij} = 0, i \neq 1$, $j = 1, \ldots, n$, $t_j = 1$, $j \in K$ and $t_j = 0$, $j \in K$ is feasible for Model (6).

It is important to note that when dealing with practical problems, various constraints and conditions need to be considered. Additionally, internal policies may impose additional constraints on the proposed model. For example, let us consider a scenario where both units p and q must be included in the optimal subgroup simultaneously. In this case, the model should include the constraint $t_p + t_q = 2$ to represent this condition. Similarly, $t_p + t_q = 1$ is incorporated into Model (6) when exactly one of the two units, p or q, must be selected, and it is not possible to choose both alternatives. Additionally, applying the constraint $t_p + t_q \le 1$ indicates that at most one of the two units p and q can be selected in the optimal group. Finally, in some cases, according to organizational requirements, if unit p is selected to be in the optimal group, then unit q should also be selected. In such a case, adding the constraint $t_p \leq t_q$ will ensure this condition.

3.2. Optimal subgroup selection problem with uncertain data

In classical DEA models, such as Model (6), the input and output vectors contain deterministic values. However, this is not always the case in real-world studies. Some inputs and outputs may be uncertain parameters due to factors such as the unavailability of historical data, weather conditions, sudden incidents, or congestion. When historical data are available and their probability distributions can be estimated, the uncertain parameters are considered random variables.

On the other hand, in cases where there are no historical data, determining a probability distribution is impossible. To solve this problem, experts' experience is required to characterize the probabilities of uncertain parameters. Thus, Liu[19] proposed an uncertainty theory using the belief degree concept, providing a theoretical framework for modeling uncertain data. The uncertain DEA approach enables the incorporation of subjective expert knowledge and judgments into the modeling process. This also makes it possible to model uncertainty originating from different sources using domain experts' knowledge.

Section 3.1 presents a novel DEA model for selecting optimal subgroups, where all inputs and outputs are considered deterministic. However, in real-world cases, due to the challenge of obtaining complete and accurate data, we often encounter situations where not all inputs and outputs are certain. To overcome this uncertainty, this study presents a novel model in which the inputs and outputs are considered uncertain and characterized by a degree of belief. Subjective judgments and expert assessments can be incorporated into the modeling process.

When inputs and outputs behave as uncertain numbers with a certain degree of belief, uncertainty theory provides an effective framework for solving the optimal selection problem. Thus, the definition of the optimal selection problem is based on the concept of uncertainty, expressed by the degree of belief in both input and output variables and the system's inherent uncertainty. Therefore, by applying uncertainty theory and the uncertain parameters defined in Table 2, the deterministic Model (6) is transformed into an uncertain Model (7).

$$
\begin{aligned}
&\text{Max} f(u,\eta) = \sum_{j=1}^{n} u_j \eta_j \quad 7.1\\
&\textit{subject to}\\
&\sum_{j=1}^{n} v_j \xi_j = 1 \quad 7.2\\
&u_j\eta_j - v_j\xi_j \leq 0, \quad j=1,\ldots,n \quad 7.3\\
&u_j \leq M \times t_j, \quad j=1,\ldots,n \quad 7.4\\
&v_j \leq M \times t_j, \quad j=1,\ldots,n \quad 7.5\\
&\sum_{j=1}^{n} t_j = k \quad 7.6\\
&v_j \geq 0,\ u_j \geq 0, \quad j=1,\ldots,n \quad 7.7\\
&t_j \in \{0,1\}, \quad j=1,\ldots,n \quad 7.8
\end{aligned}
\tag{7}
$$

We can account for the inherent imprecision in the input and output variables by introducing uncertainty based on the degree of belief. This allows us to make more informed and reliable decisions even when accurate data are difficult to obtain or are unreliable. This section further examines the approach and methods used to address the uncertain optimal selection problem and evaluates how uncertainty theory and the level of confidence in inputs and outputs can improve decision-making in uncertain situations and provide deeper insights. Before continuing the discussion, Lemma 1 is presented below.

Lemma 2. *Considering Model (8) as follow, it has same optimal solution as Model (7).*

$$
\begin{aligned}
&\text{Max} f(u,\eta) = \sum_{j=1}^{n} u_j \eta_j\\
&\textit{subject to}\\
&\sum_{j=1}^{n} v_j \xi_j \leq 1,\\
&u_j\eta_j - v_j\xi_j \leq 0, j=1,\ldots,n,\\
&\textit{Constraints} \quad (7.4)-(7.8)
\end{aligned}
\tag{8}
$$

Proof. (Proof by contradiction). Suppose that for each j, $(\bar{u}_j, \bar{v}_j, \bar{t}_j) \geq 0$ is optimal for $Model(8)$, and $\sum_{j=1}^{n} \bar{v}_j\, \xi_j < 1$. Now, we define $(\hat{u}_j, \hat{v}_j) = \left(\frac{\bar{u}_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}, \frac{\bar{v}_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}\right) \geq 0, \text{and} \hat{t}_j = \bar{t}_j \in \{0,1\}$.

Table 2. Symbols used for uncertain parameters

		Range
$\xi_j = (\xi_{1j}, \ldots, \xi_{mj})$	A vector of independent uncertainty inputs	$j = 1, \ldots, n$
$\eta_j = (\eta_{1j}, \ldots, \eta_{sj})$	A vector of independent uncertainty outputs	$j = 1, \ldots, n$
$\phi_{ij}(x)$	Uncertainty distribution ξ_{ij}	$i = 1, \ldots, m, \quad j = 1, \ldots, n$
$\varphi_{ij}(x)$	Uncertainty distribution η_{rj}	$r = 1, \ldots, s, \quad j = 1, \ldots, n$
$\xi_{ij} = \left(a_{\xi_{ij}}, b_{\xi_{ij}}\right)$	Linear input uncertain variable	$i = 1, \ldots, m, \quad j = 1, \ldots, n$
$\eta_{rj} = \left(a_{\eta_{rj}}, b_{\eta_{rj}}\right)$	Linear output uncertain variable	$r = 1, \ldots, s, \quad j = 1, \ldots, n$

Hence, $(\hat{u}_j, \hat{v}_j) \geq 0$ and $\hat{t}_j \in \{0,1\}$ is a feasible solution for Model (8), because $\sum_{j=1}^{n} \hat{v}_j\, \xi_j = \sum_{j=1}^{n} \frac{\bar{v}_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}\, \xi_j = \frac{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j} = 1 \leq 1$, and $\hat{u}_j\eta_j - \hat{v}_j\xi_j = \frac{\bar{u}_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}\eta_j - \frac{\bar{v}_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}\xi_j = \frac{\bar{u}_j\eta_j - \bar{v}_j\xi_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j} \leq 0$. Additionally, the value of the objective function of Model (8) is obtained as $(\hat{u}, \eta) = \sum_{j=1}^{n} \hat{u}_j\eta_j = \frac{\sum_{j=1}^{n} \bar{u}_j\eta_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j}$. Therefore, it can be easily shown that $f(\hat{u}, \eta) = \sum_{j=1}^{n} \hat{u}_j\eta_j = \frac{\sum_{j=1}^{n} \bar{u}_j\eta_j}{\sum_{j=1}^{n} \bar{v}_j\, \xi_j} > \sum_{j=1}^{n} \bar{u}_j\eta_j = f(\bar{u}, \eta)$. This is a contradiction, as we assume that $(\bar{u}_j, \bar{v}_j, \bar{t}_j)$ is optimal for Model (8). As a result, for each optimal solution of Model (8), $\sum_{j=1}^{n} \bar{v}_j\, \xi_j = 1$. Hence, Models (7) and (8) share the same optimal solution, and the proof is complete.

4. Proposed methodology for solving the model

Model (8) cannot be solved directly due to its inherent uncertainty. Therefore, it must first be converted into a crisp form, after which an appropriate optimization solver can be applied to obtain the solution. To convert Model (8) into a crisp model, the following two approaches were used:

(i) Expected value model (EVM).
(ii) Chance-constrained model (CCM).

Based on these two methods, we propose two new models for addressing the optimal selection problem within the framework of uncertainty theory to obtain their crisp forms. The rest of this section provides detailed descriptions of these models.

4.1. Expected value model

Liu[19] introduced the expected value operator for an uncertain variable. Based on this operator, an uncertain model was converted into an equivalent crisp EVM. By applying this operator, the expected value of an uncertain objective function in a model was optimized under the expected value of the uncertain constraints. Here, by applying the expected value operator, the uncertain Model (7) was replaced by the following formulation:

$$\text{Max } E[f(u,\eta)] = E\Big[\sum_{j=1}^{n} u_j\eta_j\Big]$$

subject to

$$E\Big[\sum_{j=1}^{n} v_j\, \xi_j\Big] \leq 1, \tag{9}$$

$$E[u_j\eta_j - v_j\xi_j] \leq 0, \quad \forall j,$$

$$\textit{Constraints} \quad (7.4) - (7.8).$$

Definition 7. *The vector (u_j, v_j, t_j) is a feasible solution for the expected value Model (9), if it satisfies the following constraints:*

$$E\Big[\sum_{j=1}^{n} v_j\, \xi_j\Big] \leq 1,$$

$$E[u_j\eta_j - v_j\xi_j] \leq 0, \quad \forall j \tag{10}$$

$$\text{Constraints} \quad (7.4) - (7.8).$$

Definition 8. *Suppose (u_j, v_j, t_j) is an arbitrarily feasible solution of the Model (9). As a result, $\left(u_j^*, v_j^*, t_j^*\right)$ is an optimal solution of Model (9) if and only if:*

(i) $\left(u_j^*, v_j^*, t_j^*\right)$ is a feasible solution of Model (9).
(ii) $E\left[f\left(u^*, \eta\right)\right] \geq E\left[f(u, \eta)\right].$

Theorem 6. *Assume that ξ_j and η_j are independent uncertainty variables with regular uncertain $\varphi_{ij}(x)$ and $\phi_{ij}(x)$, respectively. In this case, Model (9) is equivalent to the following model:*

$$\text{Max } \; E[f(u,\eta)] = \sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj} \int_0^1 \varphi_{rj}^{-1}(\alpha)\, d\alpha$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij} \int_0^1 \phi_{ij}^{-1}(\alpha)\, d\alpha \le 1 \quad (11)$$

$$\sum_{r=1}^{s} u_{rj} \int_0^1 \varphi_{rj}^{-1}(\alpha)\, d\alpha - \sum_{i=1}^{m} v_{ij} \int_0^1 \phi_{ij}^{-1}(\alpha)\, d\alpha \le 0, \quad \forall j \quad (12)$$

$$Constraints (7.4) - (7.8)$$

Proof. Based on **Definition 5** and **Theorems 2** and **3**, the proof is evident.

Corollary 1. *According to Table 1 and* ***Definition 6****, Model (11) converts to the deterministic Model (12).*

$$MaxE\left[f(u,\eta)\right] = \sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj} \times \frac{a_{\eta_{rj}} + b_{\eta_{rj}}}{2}$$

$$subject\ to$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij} \times \frac{a_{\xi_{ij}} + b_{\xi_{ij}}}{2} \le 1 \quad (13)$$

$$\sum_{r=1}^{s} u_{rj} \times \frac{a_{\eta_{rj}} + b_{\eta_{rj}}}{2} - \sum_{r=1}^{s} v_{rj} \times \frac{a_{\xi_{ij}} + b_{\xi_{ij}}}{2} \le 0, \forall j$$

$$Constraints \quad (7.4) - (7.7).$$

4.2. Chance-constrained model

Chance-constrained programming is another method of obtaining the optimal solution to uncertainty problems.[19] The main idea of this method is that the decision-maker expects to achieve the lowest value $\overline{f}$ such that uncertain variable f(u,η) is less than or equal to $\overline{f}$, with confidence level α and $\alpha \in [0,1]$. With the CCM approach, the Model (8) is written as Model (13).

$$Max\overline{f}$$

$$subject\ to$$

$$M\left\{\sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\eta_{rj} \le \overline{f}\right\} \ge \alpha_1$$

$$M\left\{\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\xi_{ij} \le 1\right\} \ge \alpha_2 \quad (14)$$

$$M\left\{\sum_{r=1}^{s} u_{rj}\ \eta_{rj} - \sum_{i=1}^{m} v_{ij}\xi_{ij} \le 0\right\} \ge \beta_j$$

$$Constraints \quad (7.4) - (7.8).$$

Definition 9. *The vector (u_j, v_j, t_j) is a feasible solution of Model (13), if it satisfies the following constraints:*

$$M\left\{\sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\eta_{rj} \le \overline{f}\right\} \ge \alpha_1$$

$$M\left\{\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\xi_{ij} \le 1\right\} \ge \alpha_2$$

$$M\left\{\sum_{r=1}^{s} u_{rj}\ \eta_{rj} - \sum_{i=1}^{m} v_{ij}\xi_{ij} \le 0\right\} \ge \beta_j \forall j$$

$$Constraints \quad (7.4) - (7.8).$$

Definition 10. *Suppose (u_j, v_j, t_j) is an arbitrary feasible solution of the Model (13). As a result, $\left(u_j^*, v_j^*, t_j^*\right)$ is an optimal solution of Model (13) if and only if:*

(i) $\left(u_j^*, v_j^*, t_j^*\right)$ is a feasible solution of Model (12).

(ii) $Max\left\{\overline{f}|M\left\{f\left(u^*,\eta\right) \le \overline{f}\right\} \ge \alpha\right\} \ge Max\left\{\overline{f}|M\left\{f(u,\eta) \le \overline{f}\right\} \ge \alpha\right\}$, that $\alpha \in [0,1]$ is the predefined confidence level.

Theorem 7. *The uncertainty Model (13) is equivalent to the deterministic Model (15).*

$$Max\sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\phi_{rj}^{-1}\left(\alpha_1\right)$$

$$subject\ to$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\varphi_{ij}^{-1}\left(\alpha_2\right) \le 1 \quad (15)$$

$$\sum_{r=1}^{s} u_{rj}\ \phi_{rj}^{-1}\left(\beta_j\right) - \sum_{i=1}^{m} v_{ij}\ \varphi_{ij}^{-1}\left(1-\beta_j\right) \le 0\ \forall j$$

$$Constraints (7.4) - (7.8).$$

Proof. Consider the constraint $M\left\{\sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\eta_{rj} \le \overline{f}\right\} \ge \alpha_1$. Since $u_{rj}\eta_{rj}, j = 1,\ldots,n$ is an uncertain variable and an increasing variable on η_{rj}, the $M\left\{\sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\eta_{rj} \le \overline{f}\right\} \ge \alpha_1 \iff \sum_{r=1}^{s}\sum_{j=1}^{n} u_{rj}\phi_{rj}^{-1}\left(\alpha_1\right) \le \overline{f}$. In a similar way, $\left\{\sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\xi_{ij} \le 1\right\} \ge \alpha_2 \iff \sum_{i=1}^{m}\sum_{j=1}^{n} v_{ij}\varphi_{ij}^{-1}\left(\alpha_2\right) \le 1$, and $M\left\{\sum_{r=1}^{s} u_{rj}\ \eta_{rj} - \sum_{i=1}^{m} v_{ij}\xi_{ij} \le 0\right\} \ge \beta_j \iff \sum_{r=1}^{s} u_{rj}\ \phi_{rj}^{-1}\left(\beta_j\right) - \sum_{i=1}^{m} v_{ij}\ \varphi_{ij}^{-1}\left(1-\beta_j\right) \le 0$.

Corollary 2. *Based on Table 2, the uncertain Model (15) becomes the deterministic Model (16).*

$$Max \sum_{r=1}^{s} \sum_{j=1}^{n} u_{rj} \ \left((1-\alpha_1)\, a_{\eta_{rj}} + \alpha_1 b_{\eta_{rj}} \right)$$

$$subject\ to$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij} \left((1-\alpha_2)\, a_{\xi_{ij}} + \alpha_2 \ b_{\xi_{ij}} \right) \leq 1 \qquad (16)$$

$$\sum_{r=1}^{s} u_{rj} \ \left((1-\beta_j)\, a_{\eta_{rj}} + \beta_j b_{\eta \ {rj}} \right) - \sum_{i=1}^{m} v_{ij} \left(\beta_j \ a_{\xi_{ij}} + (1-\beta_j) \ b_{\xi_{ij}} \right) \leq 0 \quad \forall j$$

$$Constraints \quad (7.4) - (7.8).$$

The Model (16) is an equivalent CCM of the uncertain Model (13) under different confidence level values. This model is a deterministic form, and it can be easily solved to obtain the optimal solution.

5. Case study

Today's global market is highly competitive, and a successful organization must be able to sustain its mission in such a market. Appropriate technology can help businesses gain a competitive edge by simplifying processes and reducing manual labor. Choosing the right technology is important in any industry, as it can strongly impact a company's productivity, profitability, and efficiency.[55,56] However, choosing the wrong technology may result in missed opportunities, reduced production, and resource waste.

Considering that the petrochemical industry is one of the most significant and influential industries in the economy of any country—and even the global economy—selecting the right technology in this industry plays an important role in improving productivity both domestically and globally

Another important point to note is that production lines in the petrochemical industry usually form a complex system consisting of multiple interconnected processes that convert primary raw materials into final products.[57,58] Hence, the use of appropriate technology in this industry helps to reduce costs caused by product defects and material waste. Additionally, since the raw materials used in this industry are often expensive and, in some cases, scarce—or their excessive extraction may cause environmental harm—the use of appropriate technology minimizes material waste, which is vital for environment preservation.

Therefore, selecting the right technology and integrating it effectively before making any productivity-related decisions in the petrochemical industry is essential.

For this purpose, the following steps should be followed.

(i) Determining the available technologies.
(ii) Determining the indexes (inputs and outputs).
(iii) Determining index values.
(iV) Solving the model and analyzing the results.
(V) Management insights.

5.1. Determining the technologies

There are many technologies in every industry, including the petrochemical industry, each with its own strengths and weaknesses. Therefore, it is important to choose the most appropriate technology that suits the desired industry before establishing and launching it.[59] For this purpose, the following steps are outlined.

(a) Step 1: Identifying the needs and problems in the petrochemical industry

The first step in choosing the right technology is to identify the petrochemical industry's needs and problems related to producing the desired product. For this purpose, the issues faced by current factories and refineries should be identified. For example, in one refinery, the problem may be the excessive waste of raw materials, while in another, the issue may be failure to meet production deadlines. Therefore, the desired type of technology should be selected based on the specified problems identified.

(b) Step 2: Researching available technologies

After identifying the problems and needs of the desired industry, the next step is to research the available technologies related to it. Although we may encounter several technologies at this stage, certain factors—such as geographical conditions, available financial resources for technology transfer, and political constraints (e.g., sanctions)—may limit access to all technologies at all times.

(c) Step 3: Using the opinions of experts in each field

Some problems and needs within the discussed industry, as well as the weaknesses and strengths of each technology, can only be identified by individuals who are directly involved in the industry under investigation. Thus, petrochemical experts and professionals active in the desired industry should be consulted at this stage.

(d) Step 4: Testing and evaluation

Considering the high costs associated with purchasing, installing, and producing any new technology, it is important to test and evaluate the desired technology on a small scale before making a full investment and implementing it in the target setting. This approach helps prevent unnecessary expenditure of money, time, and raw materials, which are often highly valuable and costly to replace.

Based on these four steps, eight common technologies typically used in the petrochemical industry are described below:

(i) Enhanced process control: This technique uses optimization methods to control the production at different stages of chemical processes and enhance overall efficiency.
(ii) Highly efficient catalysts: In this technique, high-performance catalysts accelerate production reactions. Choosing the right catalyst can help improve process performance.
(iii) Innovative synthesis techniques: Innovative synthesis techniques use new chemical synthesis methods that can enhance product quality.
(iv) Energy recuperation systems: Due to limitations in accessing renewable and non-renewable energy sources, optimizing energy consumption is a key challenge. Energy recovery technologies are therefore designed to reduce energy wastage.
(v) Water treatment and reuse: The petrochemical industry has high water consumption, and proper water resource management is required due to the presence of pollutants generated from chemical processes in petrochemical wastewater. Therefore, water treatment and recovery technologies were designed to address this problem.
(vi) Nitrogen recycling systems: Since nitrogen plays an important role in petrochemical processes, nitrogen recovery technologies can help enhance productivity.
(vii) Streamlined supply chain management: Supply chain management is one of the main components of process optimization and cost reduction in the petrochemical industry. It helps minimize production time and costs, reduce inventory levels, and improve customer satisfaction.
(viii) Internet of Things monitoring and control: The Internet of Things is an emerging technology that enables enhanced monitoring of various processes in the petrochemical industry. Thus, the production process can be optimized by promptly detecting equipment malfunctions or failures.

Therefore, by selecting appropriate technologies in the petrochemical industry, it is possible to simultaneously reduce the consumption of raw materials—particularly non-renewable resources—as well as cost and energy use, while increasing overall production output. As a result, the production process and efficiency can be optimized. These technologies are generally applicable to multiple sectors of the petrochemical industry. However, the suitability and effectiveness of specific technologies may vary depending on operational conditions, plant design, and other influencing factors. Therefore, it is essential to consult with experts and conduct a comprehensive evaluation that considers the specific requirements and constraints of the petrochemical facility to determine which technologies are most suitable for optimizing production while minimizing costs and resource consumption. In this study, we propose a combination of these technologies to achieve high performance in establishing a refinery. As a result, each technology is considered a DMU and is briefly defined in Table 3.

5.2. Determining the indexes

DEA is a method based on an optimization model for evaluating the efficiency of a DMU, which produces several outputs by consuming several inputs. Therefore, selecting appropriate inputs and outputs is essential in assessing DMUs. If this selection is not made correctly, the results obtained for the DMUs' performance will not be valid, as the selected inputs and outputs represent the system's objectives. Therefore, it is recommended to consider the following criteria when selecting inputs and outputs in the systems under evaluation.

(i) Relevance and consistency: Inputs and outputs must be related and consistent with the production process and system objectives.
(ii) Measurability: Inputs and outputs, whether qualitative or quantitative, must be measurable.
(iii) Non-redundancy: Avoid selecting inputs and outputs that describe the same characteristic or process.

Therefore, it is crucial to be careful when selecting indicators to measure the performance of each system and to categorize them into two categories: input and output.

Table 3. Decision-making units and their descriptions

DMU	Type of technology
DMU1	Enhanced process control
DMU2	Highly efficient catalysts
DMU3	Innovative synthesis techniques
DMU4	Energy recuperation systems
DMU5	Water treatment and reuse
DMU6	Nitrogen recycling systems
DMU7	Streamlined supply chain management
DMU8	Internet of Things monitoring and control

Abbreviation: DMU: Decision-making unit.

In summary, in all industries—particularly the petrochemical industry—the issue of choosing appropriate indicators and categorizing them as inputs and outputs is addressed by reviewing the relevant research literature and consulting experts in the relevant industry, while considering the factory's objectives and available resources. Finally, the resulting performance scores provide the necessary guidelines for improving the performance of the systems under evaluation.

As natural gas is used in most of the petrochemical industry's production processes, it is considered an important input for producing products such as ammonia, urea, and industrial nitrates. Other inputs required in the production of these products include fuel, compressed air, water, and steam, which are considered inputs for production lines depending on the method used, resource availability, and other factors described above.

Another point that should be considered in input selection is the cost and manpower requirements, as some production technologies require greater investment in equipment and technology. Therefore, the cost-effectiveness of the available technologies and their alignment with available resources are important aspects that must be considered. Additionally, due to the complexity of the equipment, more skilled human resources are needed to operate the technology efficiently and to improve productivity in the selected process. Therefore, the availability of a trained workforce is an important factor that must be considered. In other words, different technologies may require varying levels of human resource investment, expertise, and training. Thus, the cost, capital, and availability of human resources should be carefully considered when selecting the indicators for each technology.

Different technologies produce different products in terms of output, such as ammonia, urea, and industrial and agricultural nitrates. However, one technology may be more suitable for producing urea, while another may be better suited for producing ammonia. Finally, these factors should be weighed by considering each technology's potential advantages and disadvantages to evaluate its efficiency. These aspects can be clarified by referring to scientific sources and documents available in each industry and by consulting experts familiar with the respective technologies.

In addition to the above-mentioned indicators, factors such as environmental impact, labor safety, and compliance with local laws can also be considered. Incorporating these indicators contributes to the development of sustainable technologies, which were not addressed in this study.

Therefore, the main issue is determining which technology should be used to maximize the technical efficiency of production. This study aims to identify the most appropriate technology or combination of technologies for establishing a petrochemical plant in Iran to enhance efficiency and productivity.

This study employs DEA to identify the most efficient technology or combination of technologies that maximizes the production of ammonia, urea, agricultural nitrate, and industrial nitrate, while minimizing the investment cost, human resources, natural gas feed, natural gas fuel, compressed air, water, and steam.

In other words, the indicators examined in this study, which were derived from expert opinions and supported by scientific sources, are as follows: investment cost, human resources, natural gas feed, natural gas fuel, compressed air, water, steam, ammonia, urea, agricultural nitrate, and industrial nitrate.

After determining the indicators and using expert opinions, as well as the definitions of input and output described at the beginning of this section, the identified indicators were classified into

Table 4. Classification of selected factors for the petrochemical industry

Input/output	Description	Type of index
Input	Investment cost	Input 1
	Human resources	Input 2
	Edible natural gas	Input 3
	Natural gas fuel	Input 4
	Compressed air	Input 5
	Water	Input 6
	Steam	Input 7
Output	Ammonia	Output 1
	Urea	Output 2
	Agricultural nitrate	Output 3
	Industrial nitrate	Output 4

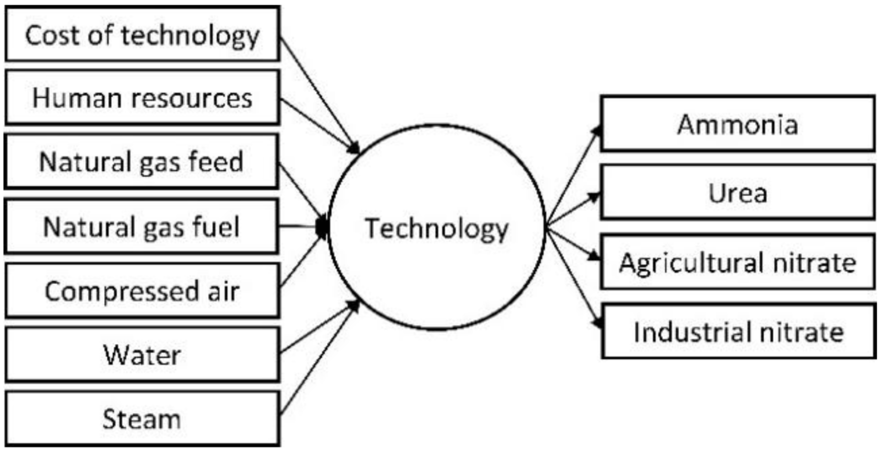

Figure 1. Inputs and outputs of the studied petrochemical industry

Table 5. The uncertain data for the input variables

DMU	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6	Input 7
DMU1	(540,650)	(590,800)	(24539320, 24669456)	(19843925, 19853431)	(3652760, 3652860)	(561870, 568421)	(268225, 269415)
DMU2	(590,670)	(700,820)	(24523322, 24574310)	(19854629, 19854929)	(3652655, 3652854)	(565865, 567892)	(269329, 269531)
DMU3	(600,680)	(715,850)	(24544321, 24563321)	(19853111, 19854928)	(3652654, 3654054)	(564864, 567421)	(269218, 269423)
DMU4	(610,700)	(618,790)	(24529319, 24589412)	(19853934, 19853994)	(3652664, 3652664)	(565474, 567831)	(268229, 269421)
DMU5	(650,720)	(590,810)	(24569329, 246540231)	(19853939, 19853939)	(3651677, 3652691)	(564510, 568280)	(269234, 269471)
DMU6	(640,725)	(630,750)	(24551320, 24569320)	(19853435, 19853971)	(3652665, 3652665)	(565375, 567812)	(269230, 269541)
DMU7	(680,719)	(650,770)	(26711341, 26801821)	(19888118, 19888797)	(3239361, 3274211)	(563215, 564575)	(269712, 269832)
DMU8	(650,720)	(710,880)	(25570832, 25623812)	(20206756, 20208556)	(3391957, 3393450)	(561120, 562280)	(270416, 270514)

Abbreviation: DMU: Decision-making unit.

Table 6. The uncertain data for the output variables

DMU	Output 1	Output 2	Output 3	Output 4
DMU1	(30800, 30920)	(138321, 138731)	(11700, 11510)	(1550, 1880)
DMU2	(33347, 33429)	(146480, 146480)	(11143, 11550)	(1741, 1908)
DMU3	(33426, 33576)	(145124, 146481)	(11208, 12210)	(1720, 1881)
DMU4	(33340, 33420)	(137200, 138666)	(12405, 13050)	(1522, 1678)
DMU5	(33112, 33422)	(136522, 138666)	(11125, 11435)	(1870, 1980)
DMU6	(33320, 33428)	(138667, 138667)	(11270, 11455)	(2009, 2200)
DMU7	(29571, 29890)	(61240, 67431)	(10710, 11987)	(2114, 2305)
DMU8	(30721, 30895)	(64250, 65300)	(12987, 13180)	(1710, 1987)

Abbreviation: DMU: Decision-making unit.

two categories: inputs and outputs. This classification is presented in Table 4 and shown in Figure 1. Furthermore, the overall procedure of the proposed methodology for evaluating the DMUs is illustrated in the flowchart shown in Figure 2.

5.3. Determining index values

The traditional DEA models use deterministic inputs and outputs. However, in most real-world problems, the values of inputs and outputs may

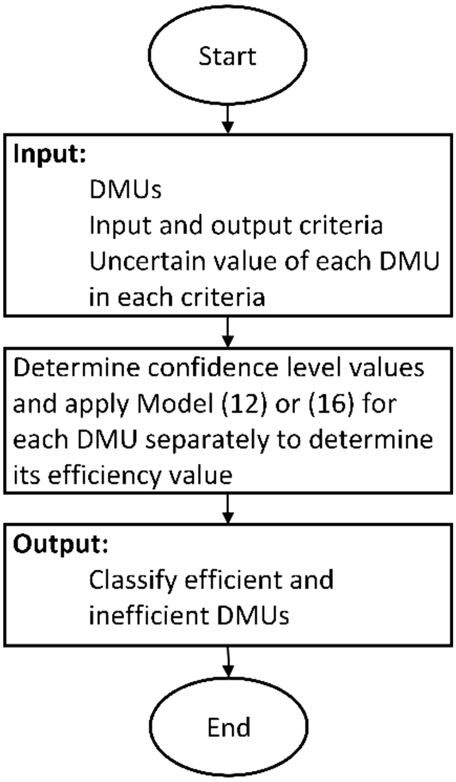

Figure 2. Flowchart of the proposed methodology for evaluating the DMUs
Abbreviation: DMU: Decision-making unit.

not be obtained precisely. Some factors, such as carbon emissions and social benefits of industries, can be among these inputs or outputs. Hence, in such situations, experts determine the values of the required indicators based on their experience and by applying belief theory. So far, the combination of different technologies reported in Table 3 has not been implemented to establish a refinery in the desired area; therefore, the indicator values are not available. Due to the unavailability of historical data, experts in this field were invited to estimate the input and output values. The experts' estimated input and output values are presented in Tables 5 and 6.

5.4. Obtained results

As explained in **Section 5**, two approaches were applied to solve the proposed model using uncertain data. Therefore, in this section, the process of solving the model and analyzing the results is presented from two perspectives.

Using the information in Tables 5 and 6 and the process explained in Section 5.1, the values of inputs and outputs were entered into Models (9) and (11). It was also assumed that only three technologies could be selected out of the eight available technologies. Then, the model was solved using the general algebraic modeling system software, and the extracted results are presented in Table 7. As shown in Table 7, the use of technologies 3–5 represents the best combination for establishing this industrial unit. With this combination of lines and technology, the highest technical efficiency is expected in producing finished products.

Table 7. Results of the expected value model

t_j	Value of t_j
t_1	0
t_2	0
t_3	1
t_4	1
t_5	1
t_6	0
t_7	0
t_8	0

As explained in the modeling process in Section 5.1, in the optimal solution, $t_j = 1$ indicates the selection of DMU_j in the optimal group, whereas $t_j = 0$ indicates the absence of DMU_j from the optimal group. Therefore, based on the results obtained in Table 7, the use of technology 3 (innovative synthesis techniques), technology 4 (energy recuperation systems), and technology 5 (water treatment and reuse) constitutes the optimal combination to achieve maximum efficiency in the mentioned factory.

Table 8. Results of different confidence levels using the chance-constrained model

Confidence level (%)	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
0	0	0	0	0	1	1	1	0
10	1	1	0	0	1	0	0	0
20	1	1	0	0	1	0	0	0
30	1	0	0	0	1	0	0	1
40	0	0	0	1	1	0	0	1
50	0	0	1	1	1	0	0	0
60	0	0	0	0	1	1	0	1
70	0	0	1	0	1	1	0	0
80	0	0	0	0	1	1	0	1
90	0	0	0	0	1	1	0	1
100	0	0	1	0	1	0	0	1

In the second approach, Models (12) and (14) were applied, and analyses were performed with different belief degrees. Belief degrees ranging from 0% to 100%, at 10% intervals, were analyzed, and the results of this analysis are presented in Table 8. With a belief degree of 0%, technologies 5–7 were selected. With a belief degree of 50%, technologies 3–5 were selected as appropriate combinations, which are consistent with the results of Model (9). Additionally, with a belief degree of 100%, technologies 3, 5, and 8 were selected.

According to the information in Table 8, technology 1 (enhanced process control) was selected at belief degrees of 10%, 20%, and 30%. Technology 2 (highly efficient catalysts) was selected at belief degrees of 10% and 20%. Technology 3 (innovative synthesis technique) was selected only at belief degrees of 50%, 70%, and 100%. Technology 4 (energy recuperation systems) was a candidate at belief degrees of 40% and 50%. Technology 5 (water treatment and reuse), which was selected at all belief degrees, demonstrates a significant advantage over the other options. After technology 5, the most frequently selected technology is technology 8 (Internet of Things monitoring and control), appearing six times at belief degrees of 30%, 40%, 60%, 80%, 90%, and 100%. Technology 7 was only selected at a 0% belief degree. Based on Table 8, technology 5 is the most suitable option, while technology 7 (streamlined supply chain management) is unsuitable for the design of this industrial unit.

5.5. Management insights

Survival in the global and competitive business environment requires transforming existing business processes into agile and customer-oriented structures, and technology, as a vital driving force, facilitates the achievement of performance goals and supports better business decision-making in a timely manner. Technology plays a crucial role in today's competitive economy, and it is clear that global competition strategies are increasingly driven by technologies that operate in highly dynamic, fast-paced, and volatile environments. Therefore, the selection and management of available technologies before launching an industrial unit are essential. The proposed model is particularly useful when complete information is unavailable. Based on the results of this study, organizational managers can make informed decisions by considering the future direction of the organization.

6. Conclusion

This research introduces a model that utilizes DEA to select optimal groups. In the context of evaluating a group of alternatives based on multiple inputs and outputs, this study presents a method for identifying the most suitable subgroup of units with the aim of optimizing the overall evaluation of the group. The proposed model has broad applicability in choosing subsets of alternatives and can be applied to various scenarios such as facility location, project portfolio selection, and supplier selection. An actual application of this model involves planning the establishment of a refinery in Iran. Survival in the global and competitive business environment requires transforming existing business processes into agile and customer-oriented structures, and technology, as a vital driving force, facilitates the achievement of performance goals and supports timely and effective business decision-making. Technology plays a crucial role in today's competitive economy, and it is clear that global competition strategies are increasingly driven by technologies that operate in highly dynamic, fast-paced, and volatile environments. Therefore, the selection and management of available technologies before launching an industrial unit are essential. The proposed model is particularly useful when complete information is not available. Based on the results of this study, organizational managers can make informed decisions by considering the future direction of the organization.

The aim of this study was to develop a new combination of eight technologies to improve the performance of the refinery. Since there was not enough information about the inputs and outputs of each DMU, expert opinions were used to estimate the required data, and belief theory was applied to extend the proposed model for uncertain data. In general, this study encountered several limitations, such as the presence of multiple criteria and the uncertain nature of the data. In future research, the proposed model can be customized for other objectives. For instance, budget constraints can be incorporated to limit the number of units selected, or equitable budget allocation can be considered. Moreover, the model can be further developed to accommodate fuzzy environments or interval and rough data.

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Conflict of interest

The authors certify that none of our interests is in conflict with one another. Each author attests to having contributed enough to the work to accept public accountability for its content, including assistance with the concept, design, analysis, writing, and editing of the text. All individuals who meet the standards for authorship are listed as authors. Additionally, each author attests that, prior to its publishing in the journal, none of the content in this or related works has been submitted to or published in any other magazine.

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Availability of data

The data supporting the findings of this study are included within this article. Additional information or datasets are available from the corresponding author upon reasonable request.

AI tools statement

All authors confirm that no AI tools were used in the preparation of this manuscript.

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