An International Journal of Optimization and Control: Theories & Applications
ISSN: 2146-0957 eISSN: 2146-5703
Vol.16, No.2, pp.450-461 (2026)
https://doi.org/10.36922/IJOCTA025400167

RESEARCH ARTICLE

Tube-based stochastic model predictive control with flexible state initialization

Fei Li[1], Chuanfeng Li[1*], Chao Wu[1,2*], Shuzhen Wang[1*], Pengcheng Han[1,2], and Yan Wang[1]

[1] School of Power Electrical Engineering, Luoyang Institute of Science and Technology, Luoyang, Henan, China
[2] Henan International Joint Laboratory of Testing and Control Technology for Lithium-ion Batteries, Luoyang, Henan, China
lifei@lit.edu.cn, lcf@lit.edu.cn, wuchao@lit.edu.cn, Wangsz@lit.edu.cn, birdhpc@my.swjtu.edu.cn, wangyan@lit.edu.cn

ARTICLE INFO

Article History:
Received: October 3, 2025
Revised: November 19, 2025
Accepted: December 11, 2025
Published Online: January 8, 2026

Keywords:
Model predictive control
Stochastic model predictive control
Tube model predictive control
Constrained control
Chance constraint

ABSTRACT

The robust model predictive control does not exploit the potentially existing statistical properties of system uncertainties, which may result in overly conservative control solutions. To address this issue, this paper proposes a novel approach of stochastic model predictive control, specifically tailored for linear time-invariant systems that are confronted with bounded additive uncertainties. The proposed method is established within the robust tube-based model predictive control framework, where chance constraints are transformed into deterministic ones. In particular, by leveraging the propagation characteristics of uncertainties, an algorithm of time-varying tube-based stochastic model predictive control is devised through computing tightened constraints along the prediction horizons. Furthermore, utilizing the infinite-horizon propagation property of uncertainties, a constant tube-based stochastic model predictive control method is derived by implementing conservatively constant tightened constraints throughout the entire prediction horizons. The feasibility and closed-loop stability results are rigorously developed, and a numerical example is provided to demonstrate the efficacy of the proposed method.

1. Introduction

Model predictive control (MPC) is an effective receding horizon control strategy, widely adopted for its capability to handle uncertain dynamics, achieve optimal control performance, and satisfy state and input constraints. [1–3] When model uncertainties are assumed to be bounded, robust MPC approaches address such uncertainties by considering worst-case scenarios. However, these frameworks typically neglect the potential statistical properties of uncertainties, despite the fact that statistical information is often available in practical applications. [4–7] Consequently, robust methods may exhibit excessive conservatism. To mitigate this limitation, stochastic MPC (SMPC) leverages the stochastic characteristics of uncertainties by incorporating chance constraints, which enable a permissible degree of constraint violation to occur within a probabilistic framework. [8–11] This approach reduces conservatism in constraint satisfaction and improves control performance, as worst-case scenarios are rarely encountered in practice. Generally, SMPC methods can be categorized into two primary classes: [12–14] analytic approximation methods and scenario-based methods. The former reformulates probabilistic constraints into deterministic approximations via constraint tightening, whereas the latter ensures satisfaction of chance constraints by generating

*Corresponding Author

a sufficient number of randomly sampled realizations.

Several representative SMPC methods have been developed to handle chance constraints while ensuring recursive feasibility. Li et al. [15] proposed a sophisticated SMPC strategy tailored for linear time-invariant (LTI) systems, which are subjected to additive uncertainties following a truncated normal distribution, by leveraging the one-step-ahead constraint. Cannon et al. [16] introduced an SMPC framework that incorporates time-varying constraint tightening throughout the prediction horizon. This framework is grounded in tubes characterized by fixed cross-sectional shapes and variable scaling factors, which are precomputed offline based on the permissible probability of constraint violation. Kouvaritakis et al. [17] constructed tubes offline directly by explicitly utilizing the distribution characteristics of additive disturbances. Leveraging the scenario method to enable offline computation of probabilistic reachable sets, Hewing and Zeilinger [18] proposed an SMPC framework for linear discrete-time systems with unbounded and correlated additive disturbances. Lorenzen et al. [19] computed constraint tightening via an offline sampling strategy, aiming to accelerate online computation while guaranteeing robust recursive feasibility. Meanwhile, another recursively feasible SMPC scheme put forward, which involves the imposition of supplementary constraints specifically on the initial predicted. step [20] Tsai and Malak [21] addressed control codesign problems under probabilistic disturbances by formulating a finite-horizon optimal control problem with chance constraints. By combining tube-based SMPC with constraint-tightening techniques, this approach achieves a balanced trade-off between performance and robustness, while mitigating the risks of instability and infeasibility. Farina et al. [22] designed an SMPC method that selects the nominal state initialization online between an open-loop strategy and a closed-loop strategy. However, regardless of the varying assumptions made about uncertainties, the nominal state initializations in all the aforementioned works are primarily chosen as the measured actual state. This could potentially result in inaccuracies when using the open-loop nominal state as an approximation of the actual closed-loop state, thereby requiring more intricate demonstrations to establish recursive feasibility and ensure stability. [23]

In this paper, we investigate the MPC problem for LTI systems subject to bounded additive uncertainties. To address this problem, we propose a tube-based SMPC scheme that belongs to the analytic approximation category, which is developed within the framework of a typical robust tube-based MPC (RTMPC). [24] Specifically, the proposed scheme, termed time-varying tube-based SMPC, is formulated by constructing time-varying tightened constraints that account for the propagation of uncertainties over the prediction horizon. A special case, referred to as constant tube-based SMPC (CTSMPC), is derived by imposing infinitely tightened constraints with constant bounds on all predicted nominal states across the entire horizon. The novelty of this work lies in the adoption of the RTMPC framework, which enables accurate prediction of nominal states through flexible state initializations. This not only results in simple and standard MPC algorithms but also facilitates the guarantee of recursive feasibility and stability.

The remainder of this paper is structured as follows. Section 2 elaborates on the problem under investigation and outlines the fundamental preliminaries pertaining to the RTMPC algorithm. Section 3 presents the proposed SMPC framework, commencing with the formulation of the time-varying tube-based SMPC (TTSMPC), proceeding to the construction of its specialized variant, and culminating in a comparative analysis of their feasible regions. In Section 4, numerical examples and comparative studies are provided to validate the proposed approach, while Section 5 draws together the key findings and concludes the paper.

Notations: The symbol $\mathbb{R}$ represents the set encompassing all real numbers, $\mathbb{N}_i$ denotes the set of all integers that are greater than or equal to i, and $\mathbb{N}_i^j$ signifies the set composed of all consecutive integers within the range $\{i, \cdots, j\}$. The value of a variable x at a specific time instant k is denoted by x_k, whereas the k-step-ahead prediction of x at time t is represented as $x_{k|t}$. We define the weighted norm as $\|x\|_Q^2 = x^T Q x$, where the notation $Q \succ 0$ is used to indicate that Q is a positive definite matrix. The probability of an event A taking place is denoted by $\mathbf{Pr}(A)$. When a random variable x adheres to the distribution $\mathcal{Q}^x$, it is expressed as $x \sim \mathcal{Q}^x$. A Gaussian distribution characterized by a mean of μ and a variance of Σ is denoted by $\mathcal{N}(\mu, \Sigma)$. The Minkowski sum is represented by the notation $A \oplus B$, which is defined as the set $\{a + b | a \in A, b \in B\}$. The Pontryagin set difference is denoted by $A \ominus B$, and it is defined as the set $\{a \in A | a + b \in A, \forall b \in B\}$.

2. Problem statement and preliminaries

2.1. Problem statement

Consider a discrete-time LTI system subject to additive uncertainties:

$$x_{k+1} = Ax_k + Bu_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $w_k \in \mathbb{R}^n$ are the system state, control input, and uncertainty at time k, respectively. It is assumed that all the system states are measured directly, and the bounded uncertainty $w_k \sim \mathcal{Q}^w$ is independently and identically distributed and lies in a polytope $\mathbb{W}$, i.e., $w_k \in \mathbb{W}$. Moreover, the system in Equation (1) is subject to the following constraints on states and inputs:

$$x_k \in \mathbb{X}, k \in \mathbb{N}_0, \tag{2}$$

$$u_k \in \mathbb{U}, k \in \mathbb{N}_0, \tag{3}$$

where $\mathbb{X} \subset \mathbb{R}^n$, and $\mathbb{U} \subset \mathbb{R}^m$ are compact and each set contains the origin in its interior.

In light of the stochastic characteristics inherent in the system dynamics, chance constraints, which permitting violations with a probability not exceeding $\varepsilon \in (0, 1)$, are employed to mitigate conservatism across all predicted states within the future horizon. The selection of the violation ratio is determined by specific engineering requirements. Generally, a greater violation ratio implies increased optimization opportunities in the enlarged feasible region, whereas a lower violation ratio (i.e., higher satisfaction probabilities) results in more conservative solutions.

The primary objective of this paper is to devise an SMPC algorithm that stabilizes the system in Equation (1) while satisfying the following constraints:

$$\mathbf{Pr}(x_{k+1} \in \mathbb{X}) \geq 1 - \varepsilon, k \in \mathbb{N}_0, \tag{4}$$

$$u_k \in \mathbb{U}, k \in \mathbb{N}_0. \tag{5}$$

Remark 1. *The deterministic constraints on control represent the actuator limitations. In addition, since system uncertainty is bounded, the deterministic treatment is used to ensures the feasibility of the algorithm.*

2.2. Preliminaries

In this subsection, we first recapitulate the RTMPC framework for the system described by (1), which serves as a foundational basis for the subsequent algorithmic design. To streamline the presentation, we begin by introducing several key definitions.

Definition 1. *Robust positively invariant set [24] A set Z is defined as a robust positively invariant set for the system described by the equation:*

$$x_{k+1} = f(x_k, w_k), \tag{6}$$

where $w_k \in \mathbb{W}$, and $f(\cdot)$ represents the system's dynamic function. This set Z satisfies the condition that for every $x \in \mathcal{Z}$ and every $w \in \mathbb{W}$, the state x_{k+1} remains within the set Z.

Definition 2. *Confidence Region [25]. A set $\mathcal{E}_p$ is said to be a confidence region of probability level p for a random variable $x \sim \mathcal{Q}^x$, if $\boldsymbol{Pr}(x \in \mathcal{E}_p) \geqslant p$.*

The system dynamics presented in Equation (1) can be decomposed into two distinct components: a nominal dynamics and an error dynamics, as follows:

$$s_{k+1} = As_k + Bv_k, \tag{7}$$

$$e_{k+1} = A_{cl}e_k + w_k, \tag{8}$$

where the nominal state is denoted by $s_k \in \mathbb{R}^n$ and the nominal input is denoted by $v_k \in \mathbb{R}^m$. The closed-loop system matrix $A_{cl} = A + BK$ exhibits stability, where the feedback gain matrix K is derived by solving the linear quadratic regulator problem associated with the nominal dynamics in Equation (7). The discrepancy between the observed state x_k and the nominal state s_k is formally defined as the error e_k, which can be expressed mathematically as:

$$e_k = x_k - s_k. \tag{9}$$

The state feedback control law is:

$$u_k = Ke_k + v_k. \tag{10}$$

Let $\mathcal{Z}$ denote the minimal robust positively invariant (mRPI) set corresponding to the error dynamics in Equation (8), taking into account the uncertainty $w_k \in \mathbb{W}$, which is defined as:

$$\mathcal{Z} \triangleq \sum_{i=0}^{\infty} \oplus A_{cl}^i \mathbb{W}. \tag{11}$$

Construct the robustly tightened state constraint set as:

$$\mathcal{C} \triangleq \mathbb{X} \ominus \mathcal{Z}. \tag{12}$$

Then the satisfaction of state constraint Equation (2) can be guaranteed by Equation (12), i.e., if $s \in \mathcal{C}$, $e \in \mathcal{Z}$, then $x = s + e \in \mathbb{X}$. And the satisfaction of input constraint Equation (3) can be guaranteed by imposing:

$$\mathcal{V} \triangleq \mathbb{U} \ominus K\mathcal{Z}, \tag{13}$$

i.e., provided that the control input component v is an element of the set $\mathcal{V}$ and the error e lies within the set $\mathcal{Z}$, it follows that the actual control input $u = v + Ke \in \mathbb{U}$.

Select the stage cost function as $\ell(s, v) = \|s\|_Q^2 + \|v\|_R^2$, and define the terminal cost as $V_f = \|s\|_P^2$. Here, the matrices $Q \succ 0$, $R \succ 0$, and the matrix P is obtained as the solution to the algebraic Lyapunov equation:

$$(A+BK)^T P(A+BK) - P = -Q - K^T RK. \tag{14}$$

The terminal constraint set is formulated as follows:

$$\mathcal{X}_f \triangleq \{s \in \mathbb{R}^n : s_k \in \mathcal{C}, Ks_k \in \mathcal{V}, k \in \mathbb{N}_0\}. \tag{15}$$

This set $\mathcal{X}_f$ represents the maximal output admissible set, as defined by Kolmanovsky and Gilbert, [26] for the autonomous system characterized by the state transition equation:

$$s_{k+1} = (A + BK)s_k. \tag{16}$$

Under these circumstances, $\mathcal{X}_f$ adheres to the axioms outlined by Mayne et al.: [27]

A1: $(A + BK)\mathcal{X}_f \subset \mathcal{X}_f, \mathcal{X}_f \subset \mathbb{X} \ominus \mathcal{Z}, K\mathcal{X}_f \subset \mathbb{U} \ominus K\mathcal{Z}$.

A2: $V_f(A_{cl}x) + \ell(x, Kx) \leq V_f(x), \forall x \in \mathcal{X}_f$.

Drawing upon the aforementioned components, the finite horizon optimal control problem $\mathbb{P}_N^{rtmpc}(x_t)$, as introduced in the study by Mayne et al., [24] which needs to be addressed at each time instant t, is formulated as follows:

$$\begin{aligned} \min_{s_{0|t}, \boldsymbol{v}_t} & \sum_{k=0}^{N-1} (\|s_{k|t}\|_Q^2 + \|v_{k|t}\|_R^2) + \|s_{N|t}\|_P^2 \\ s.t.\quad & s_{k+1|t} = As_{k|t} + Bv_{k|t}, \\ & s_{k|t} \in \mathcal{C}, k \in \mathbb{N}_1^{N-1}, \\ & v_{k|t} \in \mathcal{V}, k \in \mathbb{N}_0^{N-1}, \\ & x_t - s_{0|t} \in \mathcal{Z}, \\ & s_{N|t} \in \mathcal{X}_f. \end{aligned} \tag{17}$$

3. Tube-based stochastic model predictive control

3.1. Time-varying tube-based stochastic model predictive control

Suppose that a polytope $\mathcal{E}_{1-\varepsilon} \subset \mathbb{W}$ is a confidence region of probability level $1-\varepsilon$ for the uncertainty w, where $\varepsilon \in (0, 1)$. Then:

$$\mathbf{Pr}(w \in \mathcal{E}_{1-\varepsilon}) \geq 1 - \varepsilon \tag{18}$$

follows from Definition 2. Various methodologies for formulating confidence regions of uncertainty, including the scenario generation method, [12] Chebyshev inequality method, [22] box-shaped method, and ellipsoidal method, [25] can be incorporated into the proposed tube-based SMPC framework.

Especially, suppose that the uncertainty w follows the truncated normal distribution, i.e., $w \sim \mathcal{N}(0, \sigma^2)$, and $w \in \mathbb{W}$. Let $1 - \varepsilon$, where $\varepsilon \in (0, 1)$, be the confidence coefficient of w, then the quantile value corresponding to the confidence coefficient is:

$$\alpha = \Phi^{-1}(1 - \varepsilon), \tag{19}$$

where Φ^{-1} denotes the quantile function of the standard normal distribution. Consequently, the confidence region for the variable w, which corresponds to the confidence coefficient $1 - \varepsilon$, can be precisely defined as:

$$\mathcal{E}_{1-\varepsilon} \triangleq \{w : -\alpha\sigma \leq w \leq \alpha\sigma\}. \tag{20}$$

Given the computational challenges associated with evaluating probability integrals, we propose a strategy to reformulate the chance constraints in Equation (4) into time-varying, tightened deterministic approximate constraints across the prediction horizons, as detailed below.

Given that the error dynamics presented in Equation (8) is linear and the uncertainty w belongs to the set $\mathbb{W}$, the evolution of the propagation set of the uncertainty, where $e_k \in \mathcal{W}_k$, can be described by the following equation:

$$\mathcal{W}_{k+1} = A_{cl}\mathcal{W}_k \oplus \mathbb{W}, k \in \mathbb{N}_0, \tag{21}$$

where $\mathcal{W}_0 = \mathbb{W}$. Then, $\mathcal{W}_k = \sum_{i=0}^{k} \oplus A_{cl}^i \mathbb{W}$, $k \in \mathbb{N}_0$, can be induced by recursively expanding. $\mathcal{W}_k \subseteq \mathcal{W}_{k+1}$ follows. Consequently, considering Equation (11):

$$\lim_{k\to\infty} \mathcal{W}_k = \sum_{i=0}^{\infty} \oplus A_{cl}^i \mathbb{W} = \mathcal{Z}, \tag{22}$$

holds, and $\mathcal{W}_k \subseteq \mathcal{Z}$, and $k \in \mathbb{N}_0$ follows.

Construct the tightened propagation set of uncertainty as:

$$\mathcal{D}_k = A_{cl}\mathcal{W}_{k-1} \oplus \mathcal{E}_{1-\varepsilon}, k \in \mathbb{N}_1, \tag{23}$$

where $\mathcal{E}_{1-\varepsilon}$ is the confidence region of w corresponding to the confidence coefficient $1 - \varepsilon$ and $\mathcal{D}_0 = \mathcal{E}_{1-\varepsilon}$.

Then, $\mathcal{D}_k \subseteq \mathcal{D}_{k+1}$ follows directly. Then:

$$\mathbf{Pr}(e_k \in \mathcal{D}_k) \geq 1-\varepsilon, k \in \mathbb{N}_0, \tag{24}$$

follows from Equation (18).

Construct the time-varying tightened state constraint set as:

$$\mathcal{C}_k \triangleq \mathbb{X} \ominus \mathcal{D}_k, k \in \mathbb{N}_0. \tag{25}$$

Then, $\mathcal{C}_{k+1} \subseteq \mathcal{C}_k$ as a result of $\mathcal{D}_k \subseteq \mathcal{D}_{k+1}$, which will facilitate to ensure the recursive feasibility. Clearly, if the nominal state $s_k \in \mathcal{C}_k$, then the chance constraint in Equation (4) is satisfied, i.e., $\mathbf{Pr}(x_k = s_k + e_k \in \mathbb{X}) \geq 1-\varepsilon, k \in \mathbb{N}_1$, can be guaranteed by Equation (24), and the satisfaction of hard constraint in Equation (5) can be guaranteed by constructing the robust tightened input constraint set $\mathcal{V} \triangleq \mathbb{U} \ominus K\mathcal{Z}$ and letting $v_k \in \mathcal{V}$. Moreover, to ensure the feasibility, we construct the terminal constraint set as:

$$\tilde{\mathcal{X}}_f \triangleq \{s \in \mathbb{R}^n : s_k \in \mathcal{C}_k, Ks_k \in \mathcal{V}, k \in \mathbb{N}_0\}, \tag{26}$$

which satisfies the axioms **A1** and **A2**.

To leverage the time-varying constraint tightening strategy, the finite-horizon optimal control problem $\mathbb{P}_N^{ttsmpc}(x_t)$ that needs to be solved at each time instant t is formulated as follows:

$$\min_{s_{0|t},\boldsymbol{v}_t} \sum_{k=0}^{N-1} (\|s_{k|t}\|_Q^2 + \|v_{k|t}\|_R^2) + \|s_{N|t}\|_P^2 \tag{27}$$

$$s.t. \quad s_{k+1|t} = As_{k|t} + Bv_{k|t}, \tag{28}$$

$$s_{k|t} \in \mathcal{C}_{k+t}, k \in \mathbb{N}_1^{N-1}, \tag{29}$$

$$v_{k|t} \in \mathcal{V}, k \in \mathbb{N}_0^{N-1}, \tag{30}$$

$$x_t - s_{0|t} \in \mathcal{W}_t, \tag{31}$$

$$s_{N|t} \in \tilde{\mathcal{X}}_f. \tag{32}$$

The optimal solution to the problem $\mathbb{P}_N^{ttsmpc}(x_t)$ comprises two key components: the nominal state $s^*_{0|t}$ and input sequence $\boldsymbol{v}^*(x_t) = [v^*_{0|t}, v^*_{1|t}, \cdots, v^*_{N-1|t}]$. Correspondingly, the associated optimal state sequence for the nominal system is denoted as $\boldsymbol{s}^*(x_t) = [s^*_{0|t}, s^*_{1|t}, \cdots, s^*_{N|t}]$.

At each time instant t, the problem $\mathbb{P}_N^{ttsmpc}(x_t)$ is solved to generate the optimal input sequence $\boldsymbol{v}^*(x_t)$ and the initial nominal state $s^*_{0|t}$. Based on these results, the optimal control law is formulated as follows:

$$u^*(x_t) = K(x_t - s^*_{0|t}) + v^*_{0|t}. \tag{33}$$

The entire procedure of the time-varying tube-based SMPC is iteratively carried out for all $t \geq 0$. This iterative process gives rise to a receding horizon control strategy, which continuously updates the control inputs based on the current system state and the predicted future behavior.

Remark 2. *Owing to the time-varying characteristic of uncertainty propagation, the state constraint set $\mathcal{C}_k$ undergoes gradual tightening over the prediction horizon. It is noteworthy that all tightened sets can be computed offline; consequently, the computational burden of the resulting TTSMPC algorithm is comparable to that of the RTMPC algorithm.*

A property established in the following proposition will be employed to ensure recursive feasibility.

Proposition 1. *Let $\mathcal{W}_t$ be the propagation set of uncertainty defined in Equation (21) for the error dynamics in Equation (8) with $w_t \in \mathbb{W}$. For $x_{t+1} = Ax_t + Bu_t + w_t$, and $s_{k+1|t} = As_{k|t} + Bv_{k|t}$, if $x_t \in s_{0|t} \oplus \mathcal{W}_t$ and $u_t = K(x_t - s_{0|t}) + v_{0|t}$, then $x_{t+1} \in s_{1|t} \oplus \mathcal{W}_{t+1}$ for all $w_t \in \mathbb{W}$.*

Proof. In Equation (9) we have defined $e_t = x_t - s_{0|t}$ and $e_{t+1} = x_{t+1} - s_{1|t}$. Due to $x_t \in s_{0|t} \oplus \mathcal{W}_t$, it follows that $e_t = x_t - s_{0|t} \in \mathcal{W}_t$. Since, according to Equations (21) and (8), $e_{t+1} = A_{cl}e_t + w_t \in A_{cl}\mathcal{W}_t \oplus \mathbb{W} = \mathcal{W}_{t+1}$ holds, then $x_{t+1} = s_{1|t} + e_{t+1} \in s_{1|t} \oplus \mathcal{W}_{t+1}$ follows.

Building upon Proposition 1, the results concerning recursive feasibility and closed-loop chance constraint satisfaction are presented as follows.

Theorem 1. *Feasibility and constraint satisfaction. Considering the optimal control problem $\mathbb{P}_N^{ttsmpc}(x_t)$ in Equation (27) for the system dynamics in Equation (1) under the control law in Equation (33), then the following hold true:*

(i) If $\mathcal{M}_t = [s^*_{0|t}, v^*_{0|t}, v^*_{1|t}, \cdots, v^*_{N-1|t}]$ is feasible for the problem $\mathbb{P}_N^{ttsmpc}(x_t)$ at time instant t, then applying the control law in Equation (33) gives the recursive feasibility at time instant $t+1$, i.e., there exists a feasible $\mathcal{M}_{t+1}$ for x_{t+1}.

(ii) The closed-loop state chance constraint in Equation (4) and input hard constraint in Equation (5) are satisfied.

Proof. The proof of part (i) follows the similar line of reasoning as described in the work by Mayne et al. [24] (Proposition 3 in their work). Since $\mathcal{M}_t$ is feasible for the problem $\mathbb{P}_N^{ttsmpc}(x_t)$, the constraints Equations (29)-(32) are satisfied by $\boldsymbol{v}^*(x_t)$ and $\boldsymbol{s}^*(x_t)$. The shifted input sequence of $\boldsymbol{v}^*(x_t)$ is denoted as

$\vec{\boldsymbol{v}}^*(x_t) = [v^*_{1|t}, \cdots, v^*_{N-1|t}, Ks^*_{N|t}]$ and the shifted state sequence of $\boldsymbol{s}^*(x_t)$ is denoted as $\vec{\boldsymbol{s}}^*(x_t) = [s^*_{1|t}, \cdots, s^*_{N|t}, A_{cl}s^*_{N|t}]$. We choose the candidate solution $\mathcal{M}_{t+1} = [s^*_{1|t}, v^*_{1|t}, \cdots, v^*_{N-1|t}, Ks^*_{N|t}]$. Hence, the first $N-1$ elements of $\vec{\boldsymbol{v}}^*(x_t)$ satisfy the hard input constraint Equation (29) and the first N elements of $\vec{\boldsymbol{s}}^*(x_t)$ satisfy the time-varying tightened constraints Equation (29) due to the fact that if $s^*_{k+1|t} \in \mathcal{C}_{k+1+t}$, then $s^*_{k+1|t} \in \mathcal{C}_{k+t}$ as $\mathcal{C}_{k+1+t} \subseteq \mathcal{C}_{k+t}$. Because $s^*_{N|t} \in \tilde{\mathcal{X}}_f$, it follows from **A1** that $A_{cl}s^*_{N|t} \in \tilde{\mathcal{X}}_f$ and $Ks^*_{N|t} \in \mathbb{U} \ominus K\mathcal{Z}$. Therefore, the last element $Ks^*_{N|t}$ of $\vec{\boldsymbol{v}}^*(x_t)$ satisfies constraint Equation (29) and the last element $A_{cl}s^*_{N|t}$ of $\vec{\boldsymbol{s}}^*(x_t)$ satisfies the terminal constraint Equation (29). Moreover, from Proposition 1 we have that if $x_t \in s^*_{0|t} \oplus \mathcal{W}_t$, then $x_{t+1} \in s^*_{1|t} \oplus \mathcal{W}_{t+1}$ for all $w \in \mathbb{W}$, i.e., the initial constraint Equation (29) is satisfied. To summarize, $\mathcal{M}_{t+1}$ satisfies the feasibility conditions for the problem $\mathbb{P}^{ttsmpc}_N(x_{t+1})$. This completes the proof for point (i).

The proof of point (ii) is as follows. From the optimal control problem $\mathbb{P}^{ttsmpc}_N(x_t)$ in Equation (27), we have that $e_t = x_t - s^*_{0|t} \in \mathcal{W}_t$, $t \in \mathbb{N}_0$. Using the fact $\mathcal{D}_{t+1} = A_{cl}\mathcal{W}_t \oplus \mathcal{E}_{1-\varepsilon}$, we get that $\mathbf{Pr}(A_{cl}e_t + w_t \in \mathcal{D}_{t+1} | e_t \in \mathcal{W}_t) \geq 1-\varepsilon$ for all $w_t \in \mathbb{W}$ following from Equation (18). Since $s^*_{1|t} \in \mathcal{C}_{t+1}$, $\mathbf{Pr}(s^*_{1|t} + A_{cl}e_t + w_t \in \mathcal{C}_{t+1} \oplus \mathcal{D}_{t+1} | e_t \in \mathcal{W}_t) \geq 1-\varepsilon$ holds. Consequently, as $x_{t+1} = s^*_{1|t} + A_{cl}e_t + w_t$ and $\mathbb{X} = \mathcal{C}_{t+1} \oplus \mathcal{D}_{t+1}$, it follows that $\mathbf{Pr}(x_{t+1} \in \mathbb{X}) \geq 1-\varepsilon$, $t \in \mathbb{N}_0$. To prove the satisfaction of the closed-loop input hard constraint, note that $x_t - s^*_{0|t} \in \mathcal{W}_t$ and $v^*_{0|t} \in \mathcal{V}$, $t \in \mathbb{N}_0$, due to the recursive feasibility of $\mathbb{P}^{ttsmpc}_N(x_t)$. Since $u_t = K(x_t - s^*_{0|t}) + v^*_{0|t}$, $\mathcal{V} = \mathbb{U} \ominus K\mathcal{Z}$ and $\mathcal{W}_t \subseteq \mathcal{Z}$, then $u_t \in K\mathcal{W}_t \oplus (\mathbb{U} \ominus K\mathcal{Z}) \subseteq K\mathcal{Z} \oplus (\mathbb{U} \ominus K\mathcal{Z}) \subseteq \mathbb{U}$, $t \in \mathbb{N}_0$, holds.

Lastly, we provide a demonstration that the proposed algorithm ensures the stability of the closed-loop system. The pivotal outcome is presented in the subsequent theorem.

Theorem 2. *Stability. Suppose that $\mathcal{Z}$ is the mRPI set for the error dynamics in Equation (8). Considering the optimal control problem $\mathbb{P}^{ttsmpc}_N(x_t)$ in Equation (27) for the system dynamics in (1) under the control law in Equation (33), the actual states converge to $\mathcal{Z}$ asymptotically.*

Proof. It is worth noting that the terminal cost function $\|\cdot\|^2_P$ serves as a control Lyapunov function within the terminal set $\tilde{\mathcal{X}}_f$. Furthermore, the set $\tilde{\mathcal{X}}_f$ complies with the axioms **A1** and **A2**. Consequently, according to Theorem 2.7 and Theorem 2.8 in the study by Kouvaritakis et al.,[28] we can conclude that $\lim_{k\to\infty} s_{k|t} = 0$ and the origin $s = 0$ is exponentially stable. Moreover, since $\lim_{k\to\infty} e_{t+k} \in \mathcal{W}_\infty = \mathcal{Z}$, $\lim_{k\to\infty} x_{t+k} = \lim_{k\to\infty} \left(s_{k|t} + e_{t+k}\right) = \lim_{k\to\infty} e_{t+k} \in \mathcal{Z}$ follows.

3.2. Special case

To simplify the TTSMPC algorithm, we relax all tightened state constraint sets to an infinite-horizon tightening set, thereby formulating an alternative algorithm referred to as the CTSMPC.

It should be noted that $\lim_{t\to\infty} \mathcal{W}_t = \sum_{i=0}^{\infty} \oplus A^i_{cl}\mathbb{W} = \mathcal{Z}$ and the polytope $\mathcal{E}_{1-\varepsilon} \subseteq \mathbb{W}$ is the confidence region of the variable w associated with the confidence coefficient $1-\varepsilon$, as a result, the tightened propagation set of uncertainty at the infinite time step:

$$\mathcal{D}_\infty = A_{cl}\mathcal{Z} \oplus \mathcal{E}_{1-\varepsilon} \tag{34}$$

follows from (23). Therefore, $\mathcal{D}_k \subseteq \mathcal{D}_\infty$, $k \in \mathbb{N}_0$, as $\mathcal{D}_k \subseteq \mathcal{D}_{k+1}$. Thus,

$$\mathbf{Pr}(e_k \in \mathcal{D}_\infty) \geq 1-\varepsilon, k \in \mathbb{N}_0, \tag{35}$$

can be derived from (24).

Construct the constantly tightened state constraint set as:

$$\mathcal{C}_\infty \triangleq \mathbb{X} \ominus \mathcal{D}_\infty, \tag{36}$$

then, the chance constraint in Equation (4) is satisfied. The reason is that, if the nominal state $s_k \in \mathcal{C}_\infty$, then, $\mathbf{Pr}(x_k = s_k + e_k \in \mathbb{X}) \geq 1-\varepsilon$ can be guaranteed by Equation (35). And the satisfaction of hard constraint in Equation (5), $u_k = v_k + Ke_k \in \mathbb{U}$, can be guaranteed by constructing the robust tightened input constraint set $\mathcal{V} \triangleq \mathbb{U} \ominus K\mathcal{Z}$ and letting $v_k \in \mathcal{V}$. Furthermore, in order to guarantee the feasibility of the problem, we formulate the terminal constraint set in the following manner:

$$\bar{\mathcal{X}}_f \triangleq \{s \in \mathbb{R}^n : s_k \in \mathcal{C}_\infty, Ks_k \in \mathcal{V}, k \in \mathbb{N}_0\}, \tag{37}$$

which satisfies the axioms **A1** and **A2**.

Leveraging the constant constraint tightening strategy, the finite horizon optimal control problem $\mathbb{P}^{ctsmpc}_N(x_t)$, which is required to be solved at every time instant t, can be formulated as follows:

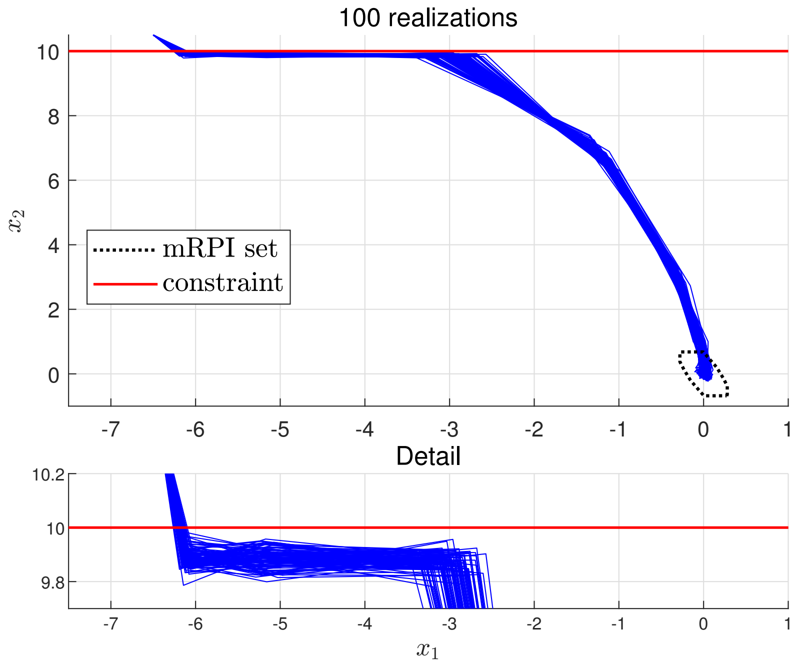

Figure 1. Constraint violation of the robust tube-based model predictive control
Abbreviation: mRPI: Minimal robust positively invariant.

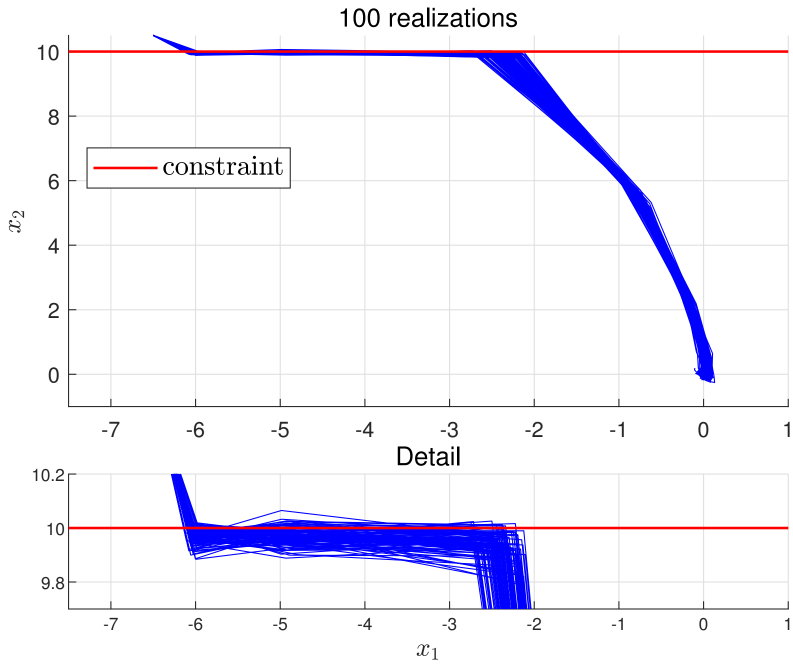

Figure 2. Constraint violation of the variable scaling tube-based stochastic model predictive control

$$
\begin{aligned}
\min_{s_{0|t}, \boldsymbol{v}_t} \quad & \sum_{k=0}^{N-1} (\|s_{k|t}\|_Q^2 + \|v_{k|t}\|_R^2) + \|s_{N|t}\|_P^2 \\
s.t. \quad & s_{k+1|t} = A s_{k|t} + B v_{k|t}, \\
& s_{k|t} \in \mathcal{C}_\infty, k \in \mathbb{N}_1^{N-1}, \\
& v_{k|t} \in \mathcal{V}, k \in \mathbb{N}_0^{N-1}, \\
& x_t - s_{0|t} \in \mathcal{Z}, \\
& s_{N|t} \in \bar{\mathcal{X}}_f.
\end{aligned}
\tag{38}
$$

The optimal solution to the problem $\mathbb{P}_N^{ctsmpc}(x_t)$ comprises two key components: the nominal state $s^*_{0|t}$ and input sequence $\boldsymbol{v}^*(x_t) = [v^*_{0|t}, v^*_{1|t}, \cdots, v^*_{N-1|t}]$. Correspondingly, the associated optimal state sequence for the nominal system is denoted as $\boldsymbol{s}^*(x_t) = [s^*_{0|t}, s^*_{1|t}, \cdots, s^*_{N|t}]$. At each time instant t, the problem $\mathbb{P}_N^{ctsmpc}(x_t)$ is solved to generate the optimal input sequence $\boldsymbol{v}^*(x_t)$ and the initial nominal state $s^*_{0|t}$. Based

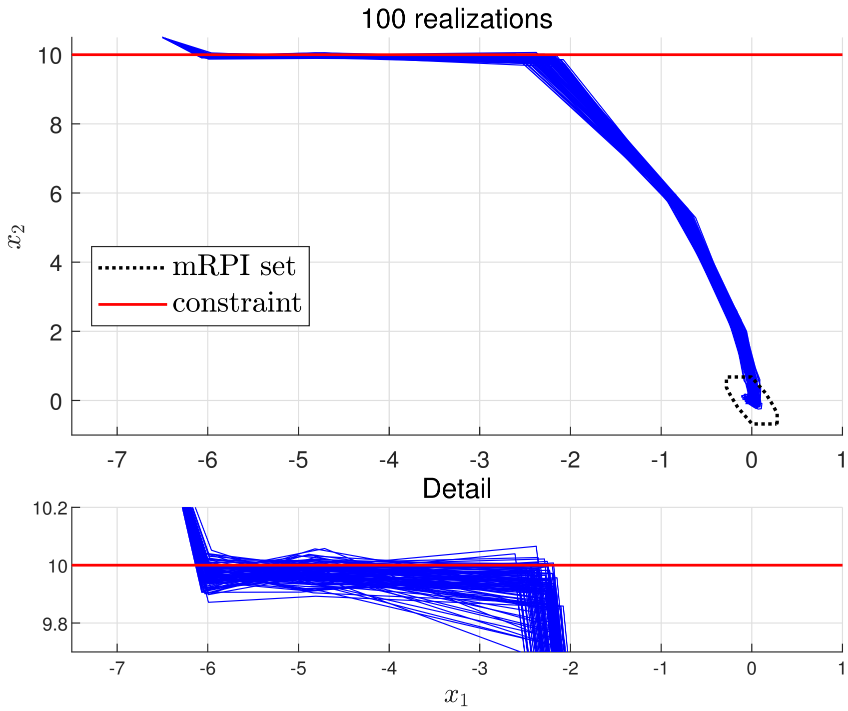

Figure 3. Constraint violation of the time-varying tube-based stochastic model predictive control
Abbreviation: mRPI: Minimal robust positively invariant.

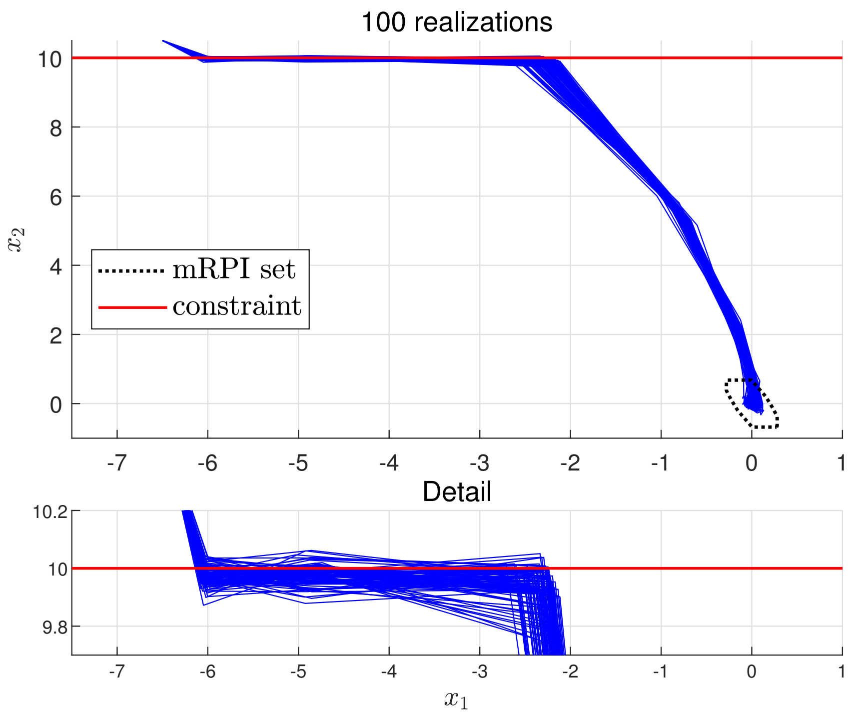

Figure 4. Constraint violation of the constant tube-based stochastic model predictive control
Abbreviation: mRPI: Minimal robust positively invariant.

on these results, the optimal control law is designed as in Equation (33). The complete procedure of the CTSMPC is executed in an iterative fashion for all $t \geq 0$. Such an iterative execution results in a receding horizon control strategy.

The proofs of recursive feasibility and stability for this special case can be derived following a similar approach to that used for the TTSMPC.

Remark 3. *It is noteworthy that the tightened deterministic state constraint set $\mathcal{C}_\infty$ for the special case remains constant across the prediction*

Table 1. Constraint violation of the model predictive control algorithms

Algorithm	Step 1	Step 2	Step 3	Average
RTMPC	0%	0%	0%	0%
VTSMPC	16.32%	17.11%	8.96%	14.13%
TTSMPC	19.89%	20.16%	9.82%	16.62%
CTSMPC	19.90%	20.15%	9.35%	16.47%

Abbreviations: CTSMPC: Constant tube-based stochastic model predictive control; RTMPC: Robust tube-based model predictive control; TTSMPC: Time-varying tube-based stochastic model predictive control; VTSMPC: Variable scaling tube-based stochastic model predictive control.

Table 2. Time consumption of the model predictive control algorithms

Algorithm	RTMPC	VTSMPC	TTSMPC	CTSMPC
Time(s)	0.027	0.030	0.032	0.029

Abbreviations: CTSMPC: Constant tube-based stochastic model predictive control; RTMPC: Robust tube-based model predictive control; TTSMPC: Time-varying tube -based stochastic model predictive control;
VTSMPC: Variable scaling tube-based stochastic model predictive control.

horizon and can be computed offline. Consequently, the resulting CTSMPC algorithm exhibits the same computational complexity as the RTMPC algorithm.

Remark 4. *Given that $\lim_{t\to\infty} \mathcal{W}_t = \mathcal{Z}$, the optimal control problem $\mathbb{P}_N^{ttsmpc}(x_t)$ becomes equivalent to $\mathbb{P}_N^{ctsmpc}(x_t)$ as $t \to \infty$. It should be noted that the CTSMPC algorithm is essentially identical to the work presented by Rosolia et al.,* [29] *albeit with a distinct derivation process.*

4. Numerical Simulation

In this section, we verify the chance constraint violation behaviors of the proposed algorithms. The discrete-time LTI system subject to truncated normal distributed additive uncertainties is considered: [17]

$$x_{k+1} = \begin{bmatrix} 1.6 & 1.1 \\ -0.7 & 1.2 \end{bmatrix} x_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k + w_k, \qquad (39)$$

where $\mathbb{X} \triangleq \left\{ x \in \mathbb{R}^2 : \begin{bmatrix} -10 \\ -2 \end{bmatrix} \le x \le \begin{bmatrix} 2 \\ 10 \end{bmatrix} \right\}$, $\mathbb{U} \triangleq \{u \in \mathbb{R} : |u| \le 10\}$, and $\mathbb{W} \triangleq \{w \in \mathbb{R}^2 : \|w\|_\infty \le 0.1, w \sim \mathcal{N}(0, 0.03^2 I)\}$.

The state and input constraints are specified as follows: $\mathbf{Pr}(x_k \in \mathbb{X}) \ge 0.8$ and $u_k \in \mathbb{U}$, $\forall w_k \in \mathbb{W}$, $k \in \mathbb{N}_0$. The weights of the cost functions are $Q = I$ and $R = 1$, respectively. K is chosen as the linear quadratic regulator feedback gain for the unconstrained optimal problem (A, B, Q, R). The prediction horizon N is set sufficiently large to ensure algorithm feasibility and maintain acceptable control performance. However, to limit computational overhead, N should be minimized as much as possible. In this particular simulation scenario, the prediction horizon is set to $N = 6$, the number of simulation steps is $N_{\text{sim}} = 11$ and the initial state of the system is given by $x_0 = [-6.5, 10.5]^T$.

4.1. Chance Constraint Violation

In this subsection, the chance constraint satisfaction of the proposed algorithms are verified. Figures 1–4 report the simulation results with 100 realizations of the RTMPC, [24] variable scaling tube-based SMPC (VTSMPC), [16] TTSMPC and CTSMPC algorithms for system in Equation (39). These figures illustrate the closed-loop trajectories along with detailed views of the first three steps. To precisely characterize the violation of chance constraints, 10,000 simulation realizations were conducted. The constraint violations within the first three steps are summarized in Table 1. Consistent with the observations from Figures 1–4, the RTMPC exhibited an average constraint violation of 0%, whereas the VTSMPC, TTSMPC, and CTSMPC yielded average violations of 16.47%, 16.62%, and 16.47%, respectively. All the three SMPC satisfied the chance constraints $\mathbf{Pr}(x_k \in \mathbb{X}) \ge 0.8$. Furthermore, a higher violation ratio indicates greater optimization potential within the expanded feasible region. Consequently, the proposed SMPC approaches exhibited less conservative with the violation ratios were closer to the predefined threshold of 20%.

4.2. Computational Time

The average computational times for one step of the four strategies are shown in Table2. The proposed TTSMPC algorithm and the CTSMPC algorithm require the comparable computational load with the RTMPC algorithm and the VTSMPC algorithm. The reason is that the calculations of the tubes were performed offline.

5. Conclusion

In this paper, an SMPC approach is developed within the RTMPC framework for linear constrained systems subject to bounded additive uncertainties. The proposed SMPC scheme mitigates conservatism by tightening constraints based on uncertainty propagation while accommodating chance constraints. Moreover, we have established the recursive feasibility and closed-loop stability of the proposed method. In addition, the tightened constraints within the SMPC scheme can be precomputed in an offline manner, rendering the proposed approach comparable in computational requirements to the RTMPC. Numerical simulations validate that the proposed SMPC approach reduces conservatism in terms of constraint violations.

Acknowledgments

None.

Funding

This research was funded in part by Henan Provincial Funds for Key Research and Development Special Project (Grant no. 251111211800); in part by Henan Provincial Funds for Major Science and Technology Special Project (Grant no. 251100210200); in part by the Natural Science Foundation of Henan Province (Grant no. 252300420380, 252300420253); in part by Henan Provincial Funds for Science and Technology Project (Grant no. 242102311244, 252102211094); in part by Henan Provincial Funds or Higher Education Institutions Key Research Project Plan (Grant no. 24A413005); and in part by Postgraduate Education Reform and Quality Improvement Project of Henan Province (Grant no. YJS2025AL140).

Conflict of interest

The authors declare they have no competing interests.

Author contributions

Conceptualization: Chao Wu, Shuzhen Wang
Formal analysis: Pengcheng Han, Yan Wang
Investigation: Fei Li, Chuanfeng Li, Chao Wu, Shuzhen Wang
Methodology: Fei Li, Chuanfeng Li, Chao Wu, Shuzhen Wang
Writing–original draft: Fei Li, Pengcheng Han, and Yan Wang
Writing–review & editing: Fei Li, Chuanfeng Li

Availability of data

Not applicable.

AI tools statement

All authors confirm that no AI tools were used in the preparation of this manuscript.

References

1. Rawlings JB, Mayne DQ, Diehl M. *Model predictive control: Theory, Computation, and Design (Vol. 2)*. Madison, WI: Nob Hill Publishing; 2020.
2. Goto H. Model predictive control-based scheduler for repetitive discrete event systems with capacity constraints. *Int J Optim Control Theor Appl.* 2013;3(2):73-83. https://doi.org/10.11121/ijocta.01.2013.00140
3. Geurts M, Baltussen T, Katriniok A, Heemels M. A contingency model predictive control framework for safe learning. *IEEE Control Syst Lett.* 2025;9:1075-1080. https://doi.org/10.1109/LCSYS.2025.3575191
4. Mark C, Liu S. A stochastic MPC scheme for distributed systems with multiplicative uncertainty. *Automatica.* 2022;140:110208. https://doi.org/10.1016/j.automatica.2022.110208
5. Hu K, Liu, T. Robust data-driven predictive control for linear time-varying systems. *IEEE Control Syst Lett.* 2024;8:910-915. https://doi.org/10.1109/LCSYS.2024.3405823
6. Mayne, D. Robust and stochastic model predictive control: Are we going in the right direction?. *Annu Rev Control.* 2016;41:184-192. https://doi.org/10.1016/j.arcontrol.2016.04.006
7. Tsai YK, Malak Jr, RJ. Robust control co-design using tube-based model predictive control. *American Control Conference (ACC).* San Diego, USA, 2023;769-775. https://doi.org/10.23919/ACC55779.2023.10156303
8. Qiao X, Zheng L, Li Y, Zhang Z, Zeng J, Zheng H. A novel stochastic model predictive control considering predictable disturbance with application to personalized adaptive cruise control. *Int J Control Autom Syst.* 2024;22(2):446-459. https://doi.org/10.1007/s12555-022-0389-9
9. Balim H, Carron A, Zeilinger MN, Köhler J. Stochastic data-driven predictive control: Chance-constraint satisfaction with identified multi-step

predictors. *IEEE Control Syst Lett.* 2024;8:3249-3254.
https://doi.org/10.1109/LCSYS.2024.3523238

10. Mesbah A. Stochastic model predictive control: An overview and perspectives for future research. *IEEE Control Syst Mag.* 2016; 36(6):30-44. https://doi.org/10.1109/MCS.2016.2602087
11. Li F, Li H, He Y. Stochastic model predictive control for linear systems with unbounded additive uncertainties. *J Franklin Inst.* 2022;359(7):3024-3045 https://doi.org/10.1016/j.jfranklin.2022.02.004
12. Farina M, Giulioni L, Scattolini R. Stochastic linear model predictive control with chance constraints - A review. *J Process Control.* 2016;44:53-67. https://doi.org/10.1016/j.jprocont.2016.03.005
13. Yan S, Goulart PJ, Cannon M. Stochastic MPC with dynamic feedback gain selection and discounted probabilistic constraints. *IEEE Trans Autom Control.* 2022;67(11):5885-5899. https://doi.org/10.1109/TAC.2021.3128466
14. Fink M, Wollherr D, Leibold M. Stochastic model predictive control with minimal constraint violation probability for time-variant chance constraints. *IEEE Control Syst Lett.* 2024;8:1385-1390. https://doi.org/10.1109/LCSYS.2024.3408608
15. Li F, Li H, He Y. Stochastic model predictive control for linear systems with bounded additive uncertainties. *Chinese Automation Congress (CAC).* Shanghai, China, 2020;6499-6504. https://doi.org/10.1109/CAC51589.2020.9327755
16. Cannon M, Kouvaritakis B, Raković SV, Cheng Q. Stochastic tubes in model predictive control with probabilistic constraints. *IEEE Trans Autom Control.* 2011; 56(1):194-200. https://doi.org/10.1109/TAC.2010.2086553
17. Kouvaritakis B, Cannon M, Raković SV, Cheng Q. Explicit use of probabilistic distributions in linear predictive control. *Automatica.* 2010;46(10):1719-1724. https://doi.org/10.1016/j.automatica.2010.06.034
18. Hewing L, Zeilinger MN. Scenario-Based Probabilistic Reachable Sets for Recursively Feasible Stochastic Model Predictive Control. *IEEE Control Syst Lett.* 2019;4(2):450-455. https://doi.org/10.1109/LCSYS.2019.2949194
19. Lorenzen M, Dabbene F, Tempo R, Allgöwer F. Stochastic MPC with offline uncertainty sampling. *Automatica.* 2017;81:76-183. https://doi.org/10.1016/j.automatica.2017.03.031
20. Lorenzen M, Dabbene F, Tempo R, Allgöwer F. Constraint tightening and stability in stochastic model predictive control. *IEEE Trans Autom Control.* 2017;62(7):3165-3177. https://doi.org/10.1109/TAC.2016.2625048
21. Tsai YK, Malak Jr, RJ. Control co-design with performance-robustness trade-off using tube-based stochastic model predictive control. *ASME Lett Dyn Syst Control.* 2025;5(3):030903. https://doi.org/10.1115/1.4067768
22. Farina M, Giulioni L, Magni L, Scattolini R. An approach to output-feedback MPC of stochastic linear discrete-time systems. *Automatica.* 2015;55:140-149. https://doi.org/10.1016/j.automatica.2015.02.039
23. Mayne DQ. Competing methods for robust and stochastic MPC. *IFAC PapersOnLine.* 2018;51(20):169-174. https://doi.org/10.1016/j.ifacol.2018.11.010
24. Mayne DQ, Seron MM, Raković SV. Robust model predictive control of constrained linear systems with bounded disturbances. *Automatica.* 2005;41(2):219-224. https://doi.org/10.1016/j.automatica.2004.08.019
25. Hewing L, Carron A, Wabersich KP, Zeilinger MN. On a correspondence between probabilistic and robust invariant sets for linear systems. *European Control Conference (ECC).* Limassol, Cyprus, 2018;1642-1647. https://doi.org/10.23919/ECC.2018.8550160
26. Kolmanovsky I, Gilbert EG. Maximal output admissible sets for discrete-time systems with disturbance inputs. *Am Control Conf.* Seattle, USA, 1995;3:1995-1999. https://doi.org/10.1109/ACC.1995.531239
27. Mayne DQ, Rawlings JB, Rao CV, Scokaert POM. Constrained model predictive control: Stability and optimality. *Automatica.* 2000;36(6):789-814. https://doi.org/10.1016/S0005-1098(99)00214-9
28. Kouvaritakis B, Cannon M.*Model Predictive Control: Classical, Robust and Stochastic.* Cham, Switzerland: Springer;2016.
29. Rosolia U, Zhang X, Borrelli F. A stochastic MPC approach with application to iterative learning. *IEEE Conference on Decision and Control (CDC).* Miami Beach, USA, 2018;5152-5157. https://doi.org/10.1109/CDC.2018.8619268

Fei Li *received the B.Sc. degree in Automation and M.Sc. degree in Control Theory and Control Engineering from Northeast Petroleum University, Daqing, China, in 2008 and 2011, respectively, and the Ph.D. degree in Weapon Science and Technology from Northwestern Polytechnical University, Xiâ€™an, China, in 2022. He is currently a lecturer with the School of Power Electrical Engineering, Luoyang Institute of Science and Technology, Luoyang, China. His research interests include model predictive control and navigation and control of autonomous systems.*
https://orcid.org/0000-0001-8854-830X

Chuanfeng Li *received the Ph.D. degree in Control Science and Engineering, Huazhong University of Science and Technology, China, in 2011. He is currently a full professor with the School of Computer, Luoyang Institute of Science and Technology, Luoyang, China. His current research interests include computer control and intelligent control.*
https://orcid.org/0000-0002-0286-6444

Chao Wu *received the B.Sc. degree in Information Engineering from Zhejiang University, Hangzhou, China, in 2003 and the M.Sc. degree in Electrical Engineering from Politechnico di Torino, Torino, Italy, in 2009 and the Ph.D. degree in Instrument Science and Technology from Harbin Institute of Technology, China, in 2019. He is currently a full professor with the School of Power Electrical Engineering, Luoyang Institute of Science and Technology, Luoyang, China. His current research interests include control of autonomous systems, battery management systems and fault diagnosis for power batteries.*
https://orcid.org/0009-0000-1062-3779

Shuzhen Wang *received the B.Sc. degree in Electronic Precision Machinery from Tianjin University, Tianjin, China, in 1994 and the M.Sc. degree in Mechatronic engineering from Xidian University, Xi'an, China, in 2005 and the Ph.D. degree in Measurement Technology and Instrumentation from Huazhong University of Science and Technology, Wuhan, China, in 2010. She serves as a full professor at the School of Intelligent Manufacturing, Luoyang Institute of Science and Technology, Luoyang, China. Her research interests include electromechanical control and testing technology.*
https://orcid.org/0009-0005-6290-6099

Pengcheng Han *received the B.Sc. degree in Electrical Engineering and Automation and Ph.D. degree in Electrical Engineering, from Southwest Jiaotong University, Chengdu, China, in 2015 and 2022, respectively. He is currently an associate professor with the School of Power Electrical Engineering, Luoyang Institute of Science and Technology, Luoyang, China. His research interests include multilevel converters, electrified railways, control applications to power electronic converters, and SiC devices and control.*
https://orcid.org/0000-0002-1555-8803

Yan Wang *received the M.Sc. degree in Detection and Automation Device from Zhengzhou University of Light Industry, Zhengzhou, China, in 2007, and a Ph.D. degree in Mechatronics from the Zhejiang University of Technology, Hangzhou, China, in 2010. She is currently an associate professor with the School of Power Electrical Engineering, Luoyang Institute of Science and Technology, Luoyang, China. Her research interests include robotic control technology, sliding mode control, and intelligent algorithms.*
https://orcid.org/0009-0005-4799-4545

An International Journal of Optimization and Control: Theories & Applications
(https://accscience.com/journal/ijocta)