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RESEARCH ARTICLE

A process-incapability–based optimization framework for product quality inspection using variable repetitive group sampling

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ABSTRACT

Conventional variable single sampling (VSS) plans are commonly employed due to their straightforward implementation; however, the large sample sizes required result in substantial inspection costs and reduced quality control efficiency. To address the challenge of ensuring reliable quality assurance while minimizing inspection effort, this study proposes a novel acceptance sampling plan that integrates the process incapability index (C_{pp}) with a variable repetitive group sampling (VRGS) scheme. The objective is to minimize the average sample number (ASN) while satisfying specified producer's and consumer's risk requirements and quality standards. A nonlinear optimization model was developed under four scenarios to determine the optimal sampling parameters, with ASN minimization as the primary objective. The proposed C_{pp}-VRGS plan was evaluated through sensitivity analysis and comparative analysis against an existing C_{pp}-based VSS plan. The results demonstrate that the proposed C_{pp}-VRGS plan consistently achieves lower ASN values while maintaining the required risk protection levels. Different scenarios exhibit superior performance under different quality conditions, providing greater flexibility in selecting an appropriate inspection strategy. Sensitivity and comparative analyses further confirm the robustness and efficiency of the proposed approach. A case study involving monocrystalline silicon wafers validates its practical applicability in a real manufacturing environment. This study contributes to the acceptance sampling literature by integrating process incapability assessment with VRGS within an optimization-based framework. To support practical implementation, a user-friendly graphical user interface was developed to assist quality engineers in determining plan parameters and making inspection decisions efficiently. The proposed methodology offers a flexible, reliable, and cost-effective solution for industrial quality control.

1. Introduction

The growth of industrial production systems has demonstrated that the digital transformation associated with the fourth industrial revolution has driven significant development and new ideas. Technical breakthroughs in this domain have enabled manufacturers to reduce costs, optimize operational efficiency, ensure regulatory compliance, and elevate product quality.[1,2] As a key component of these technical advances, quality management systems are particularly crucial for ensuring that products meet high-quality standards before they are delivered to customers.[3] Statistical quality control (SQC) serves as an essential quality mechanism for monitoring and enforcing these standards across production lines.[4] SQC systems generally comprise two primary components: (i) control charts used to monitor and evaluate production processes, and (ii) acceptance sampling plans for validating the quality of raw, semi-finished, and

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finished goods. The latter serves as a pragmatic alternative to comprehensive inspection, utilizing sample-based assessments to verify compliance with specified quality traits.[5,6]

Acceptance sampling plans are categorized into attribute and variable schemes based on the nature of the inspection data. Variable sampling plans are particularly advantageous when quality characteristics are continuous; by leveraging the full information contained in measurements, they often achieve equivalent decision accuracy with smaller sample sizes than attribute-based counterparts.[7-13] One such plan is the attribute repetitive group sampling (RGS) plan, introduced by Sherman[14] and later extended by Shankar *et al.*[15] via a two-phase inspection framework. A notable advancement in acceptance sampling is the integration of variable data into the RGS framework, resulting in the development of the VRGS plan.[16,17]

Subsequent advanced studies have focused on enhancing VRGS plans using various process capability indices, including process loss functions,[18] $C_{pm,}$[19] $C_{pk,}$[13] and $C_{pmk,}$[20] as well as S_{pk}[21] and one-sided specification schemes.[22] These developments demonstrate the growing interest in improving the efficiency and applicability of VRGS methodologies. Despite this progress, these studies often focus on optimizing the average sample number (ASN) at the acceptable quality level (AQL), the rejected quality level (RQL), or the average of ASN evaluated at RQL and AQL, which provide efficient sample sizes only when lot quality is either very good or very poor. When the actual quality falls between the critical acceptance values, the existing objective functions may not yield efficient ASN, highlighting a need for more nuanced approaches.

Furthermore, existing VRGS sampling plans predominantly assess lot quality based on the proportion of products conforming to specification limits.[17-22] While effective at distinguishing conforming goods, this binary approach neglects subtle performance variations among products that satisfy specifications, thereby overlooking valuable process information. In contrast, process incapability indices (C_{pp}) provide a quantitative measure of the degree to which a process fails to meet requirements. Specifically, the process incapability index (C_{pp}) integrates information on both process variability and deviations from target specifications, thereby offering a more comprehensive assessment of process performance.[23] Consequently, C_{pp} has become an effective tool for evaluating product quality and supporting quality-related decision-making in manufacturing environments.[24]

Despite the advantages of C_{pp} in process performance assessment, its integration into acceptance sampling design remains limited. To our knowledge, only a few studies have explicitly developed acceptance sampling plans based on the process incapability index. Most notably, the pioneering work of Sheu *et al.*[24] introduced a variable single sampling (VSS) plan using C_{pp} as the quality assessment criterion. More recently, Darmawan[23] and Darmawan *et al.*[25] extended this research stream by developing a C_{pp}-based quick-switch and resubmission sampling plan. These studies demonstrated the potential of incorporating process incapability information into lot sentencing decisions and highlighted the benefits of capability-based quality evaluation.

However, these approaches exhibit certain limitations. The C_{pp}-based VSS and quick-switch plan rely on a single-stage inspection procedure, which may require relatively large sample sizes and lacks flexibility in handling lots with extremely high or low quality levels. As a result, inspection effort may not be optimally utilized. Although the C_{pp}-based resubmission sampling plan introduces an additional decision opportunity through lot resubmission, its inspection structure remains relatively rigid and may still require substantial sampling effort before a final decision can be reached. Consequently, opportunities to further reduce the ASN and inspection costs while maintaining the required producer's and consumer's risk protection levels remain largely unexplored.

The VRGS framework presents a promising solution to these challenges. Unlike single-stage and resubmission sampling plans, VRGS incorporates repetitive inspection stages that allow earlier acceptance or rejection decisions once sufficient evidence regarding lot quality is available. This adaptive decision mechanism can substantially reduce ASN, particularly for lots exhibiting either exceptionally high or exceptionally low quality, while preserving the desired risk protection levels. Furthermore, the flexibility of VRGS enables more efficient utilization of inspection resources under varying quality conditions.

Motivated by these considerations, this study develops a novel C_{pp}-based VRGS (C_{pp}-VRGS) plan with four objective function scenarios—AQL, RQL, average ASN at AQL and RQL, and average ASN at critical values—and comprehensively examines the behavior of the plans. By integrating the process incapability index with a VRGS framework and determining plan parameters through a nonlinear optimization model, the proposed approach seeks to enhance inspection efficiency while maintaining quality assurance requirements. In doing so, this study fills an important gap in the acceptance sampling literature by extending the limited body of research on C_{pp}-based sampling plans and providing a more flexible and cost-effective alternative to existing C_{pp}-based single-stage and resubmission sampling schemes.

The remainder of this paper is organized as follows. In Section 2, we discuss the theoretical foundation of the proposed model. We define the process incapability index and examine the statistical properties of its estimator. Section 3 explicitly describes how the proposed system works, including the operational procedure, the ASN, and operating characteristic (OC) functions. Section 4 presents a detailed examination and discussion of plan parameters across four scenarios under different quality conditions, and then compares them to the existing VSS scheme. Section 5 demonstrates the practical applicability of the proposed approach through a case study, and Section 6 concludes with the study's primary findings.

2. Literature review

2.1. Variable sampling plan

Acceptance sampling is a fundamental component of SQC and serves as an important decision-making tool for determining whether a production lot should be accepted or rejected based on information obtained from a sample. Depending on the nature of the quality data collected, acceptance sampling plans are broadly categorized into attribute sampling plans and variable sampling plans. The development of variable sampling methodologies has a long history in quality control literature. Early investigations by Jennett and Welch[8] established the conceptual foundation for incorporating measurement data into acceptance sampling decisions. Subsequently, Hamaker *et al.*[9] introduced one of the earliest variable double sampling procedures based on cumulative sample information, providing a more flexible framework for lot disposition decisions. The practical implementation of variable sampling plans was further strengthened through the development of military and industrial standards, particularly MIL-STD-414, for which comprehensive OC curves and sampling tables were subsequently developed by Lieberman and Resnikoff.[26] Later, Owen[11] advanced the theoretical development of variable sampling plans by formulating procedures under the assumption of normally distributed quality characteristics, which remains one of the most widely adopted assumptions in acceptance sampling theory.

The primary motivation for employing variable sampling plans is their superior efficiency relative to attribute-based approaches. Since variable sampling plans exploit the magnitude and variability of measured quality characteristics, they require substantially fewer observations to achieve the same discriminatory power between acceptable and unacceptable lots.[9] In practical terms, a variable sampling plan can be designed to possess OC properties equivalent to those of an attribute sampling plan while using a considerably smaller sample size. This reduction in inspection effort translates directly into lower inspection costs, reduced testing time, and improved operational efficiency, particularly in industries where inspection is expensive, destructive, or time-consuming.[10-13]

Beyond sample size reduction, variable sampling plans provide several additional advantages. First, measurement data offer richer information about process performance and product quality.[12,19,27] Rather than merely identifying the presence or absence of defects, variable measurements reveal the degree of conformity to specification limits, enabling practitioners to assess process centering, variability, and capability. Consequently, variable sampling plans facilitate not only acceptance decisions but also broader quality improvement initiatives. Second, because variable sampling plans make explicit use of process variability, they are often more sensitive to shifts in process performance and can provide earlier indications of quality deterioration.[13] This characteristic is particularly valuable in modern manufacturing environments where maintaining process stability and minimizing variability are critical competitive factors.

2.2. Variable repetitive group sampling plan

Sherman[14] originally introduced the concept of RGS for attribute inspection. The underlying principle of RGS is closely related to sequential sampling procedures, whereby additional samples are collected only when the available evidence is insufficient to make a definitive acceptance or rejection decision. Building upon this framework, Shankar *et al.*[15] proposed a two-stage RGS procedure that further improved inspection efficiency. Building on this concept, Balamurali *et al.*[16] and Balamurali and Jun[17] developed the VRGS plan, which achieves smaller sample sizes than attribute-based RGS and single sampling plans while maintaining the same AQL and RQL requirements. Leveraging the richer information content of continuous measurements, VRGS schemes demonstrate superior sampling efficiency compared with traditional attribute-based RGS and single sampling plans. Specifically, they achieve substantial reductions in ASN while maintaining the desired protection against producer's and consumer's risks defined through AQL and RQL requirements.[21] These advantages have established VRGS as a promising and economically attractive inspection strategy for contemporary manufacturing environments.

Over the past decade, research on VRGS plans has expanded considerably, with particular emphasis on integrating process capability indices into acceptance sampling design. For example, Aslam *et al.*[18] developed VRGS schemes incorporating process loss considerations, whereas Wu[19] proposed a VRGS framework utilizing the Taguchi loss index (C_{pm}). Subsequent studies extended this line of research by incorporating alternative capability measures, including C_{pk}, C_{pmk}, and S_{pk}, as well as their modified and mixed formulations. Notable contributions include the multiple-state VRGS model integrated with process loss functions, VRGS plans for unilateral specifications, mixed VRGS schemes, and adjustable VRGS procedures under one-sided capability frameworks. More recently, RGS concepts have also been incorporated into statistical process monitoring applications, such as control chart design under non-normal lifetime distributions.

Despite these significant advancements, the existing VRGS literature has primarily focused on capability-index-based acceptance criteria and ASN optimization. Comparatively less attention has been devoted to the simultaneous incorporation of quality performance and risk considerations into the design framework. Given the increasing complexity of modern manufacturing systems and supply chains, there remains a need for more comprehensive VRGS methodologies that integrate both quality and risk dimensions to support robust and economically efficient acceptance decisions.

2.3. Process incapability index (C_{pp})

Building on Taguchi's quality loss function,[28] the C_{pm} index was introduced to evaluate production floor performance. However, its statistical properties pose analytical challenges.[28,29] In response, Greenwich and Jahr-Schaffrath[30] proposed the process incapability index, C_{pp}, which assesses production process performance through two components: C_{ia} (inaccuracy index) for process accuracy and C_{ip} (imprecision index) for process precision.[31] The C_{pp} index is defined as the sum of these two components: $C_{pp} = C_{ia} + C_{ip}$. Thus, the C_{pp} index can be formulated as **Equation 1**:

$$C_{pp} = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2 \tag{1}$$

From the above formula, the C_{pp} index is formulated based on key process parameters: μ (process mean), σ (process standard deviation), T (target value), and specification limits (upper specification limit [USL] and lower specification limit [LSL]). $D = (d/3)$, where d is one-half of the specification width, calculated as . The C_{pp} index comprises two components: C_{ia}, which measures process accuracy as , where M is the midpoint of the specification interval, , and C_{ip}, which quantifies process precision as . Notably, the C_{pp} index refines the C_{pm} index by distinctly separating process accuracy and precision components.

From a statistical standpoint, the C_{pp} index offers a more tractable alternative to C_{pm}, with derived relationships $C_{pp}=(C_{pm})^{-2}$, $C_{ia}=(3-3C_a)^2$, and $C_{ip}=(C_p)^{-2}$. A key advantage of C_{pp} is its estimator's superior statistical properties, stemming from its avoidance of reciprocal transformations of the mean and variance. While indices like C_{pk} and C_{pm} provide lower-bound estimates of process yield via the formula $yield \geq 2\Phi(3 \times index\ value)-1$, C_{pp} enables a similar estimation through its relationship with C_{pm}. Specifically, C_{pp} facilitates a lower-bound yield assessment as ($yield>2\Phi(3(1/C_{pp})^{1/2})-1$), underscoring its utility in process evaluation.

When population parameters are unknown, as is often the case, sample-based estimation is necessary for calculating the C_{pp} index. A natural estimator, $\hat{C}_{pp}$, is thus employed to assess process capability via the maximum likelihood estimation method, yielding an estimate of C_{pp} denoted as **Equation 2:**

$$\hat{C}_{PP} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{S_n^2}{D^2} = \left(\frac{(\bar{X}-T)^2}{D^2}\right) + \frac{1}{n}\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{D^2} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2} \tag{2}$$

where $\bar{X}=\sum_{i=1}^{n} X_i / n,\ S_n^2=\sum_{i=1}^{n}(X_i-\bar{X})^2/n.$

Given the normal distribution premise and the mathematical framework provided in **Equation 2**, the connection between the C_{pp} index and its estimator $\hat{C}_{pp}$ can be formally expressed as **Equation 3:**

$$\frac{\hat{C}_{PP}}{C_{PP}} = \frac{\sum_{i=1}^{n}(X_i-T)^2}{nD^2} \times \frac{D^2}{\left[\sigma+(\mu-T)^2\right]} = \frac{\dfrac{\sum_{i=1}^{n}(X_i-T)^2}{\sigma^2}}{n\left[1+\left(\dfrac{(\mu-T)^2}{\sigma^2}\right)\right]} = \frac{\chi^2_{n,\delta}}{n+\delta} \tag{3}$$

with δ representing the non-centrality parameter, defined as $\delta = n\xi^2 = n(\mu - T)^2/\sigma^2$, the distribution of the estimator $\hat{C}_{pp}$ can be characterized as $C_{pp}\chi^2_{n,\delta}/n+\delta$. Notably, when the process mean aligns perfectly with the target value T, this parameter (ξ) equals zero. Building on this relationship, the cumulative distribution function of the estimator can be derived as **Equation 4**:

$$F_{C_{pp}}(y) = P(\hat{C}_{pp} \le y) = P\left(\chi^2_{n,n\left(1+\xi^2\right)} \le \frac{n\left(1+\xi^2\right)y}{C_{pp}}\right). \tag{4}$$

Recent studies have explored the evolution of the process incapability index, building on its initial introduction. Researchers have made significant contributions to its development, thereby enhancing its applicability and effectiveness in assessing process performance.[32-35] These foundational studies laid the groundwork for subsequent research, such as the study by Chen *et al.*[36,] Lin,[37] and Ke *et al.*[38] More recent investigations, including those by Kaya[39] and Kaya and Baracli,[40] have further expanded the applicability of the index. Sheu *et al.*[24] first proposed the development of a VSS plan based on the process incapability index. Additionally, contributions from Liao,[41] Abbasi Ganji and Sadeghpour Gildeh,[42] Ganji,[43] Sadeghpour Gildeh and Abbasi Ganji,[44] Shirani Bidabadi *et al.*,[45] Leony and Lin,[46] Pakzad and Basiri,[47] Chen *et al.*,[48] Bera and Anis,[49] and Darmawan *et al.*[25] have continued to refine and enhance the incapability index, demonstrating its evolving nature and the ongoing efforts to improve its utility in assessing process performance.

3. Methodology

3.1. Flowchart of C_{pp}-VRGS sampling plan

This section outlines the development of an RGS strategy based on the process incapability index. The present study considers a quality characteristic that is normally distributed and subject to bilateral specification limits. When the quality characteristic surpasses the established specification limits, the product is deemed nonconforming. The existing quality improvement strategy is based on the premise of minimizing the rate of nonconformance and divergence from the target value. Consequently, in addressing the previously described issue, the formulation of the acceptance sampling plan integrates the process incapability index (C_{pp}), which serves as a quality performance measure. **Figure 1** illustrates the sequential operational procedure of the proposed strategy using a flowchart, which is detailed below.

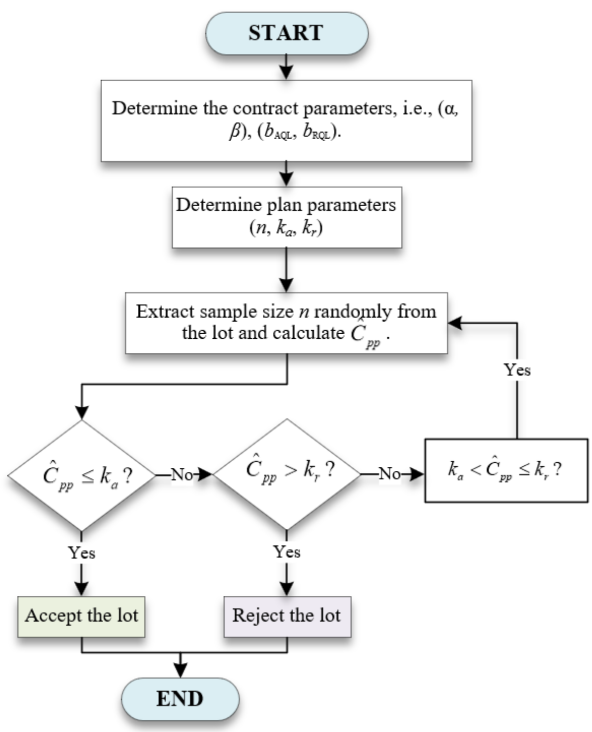

Figure 1. Flowchart of the process incapability index (C_{pp})-based variable repetitive group sampling plan

(i) Step 1 (define contract parameters): Specify the AQL (b_{AQL}) and RQL (b_{RQL}) in terms of the C_{pp} index, along with the producer's risk (α) and consumer's risk (β).
(ii) Step 2 (determine plan parameters): Obtain the plan parameters n, k_a, and k_r by solving the optimization model or referring to the provided table.
(iii) Step 3 (conduct sampling and calculation): Randomly extract a sample of size n from the submitted lot, calculate the sample statistic and $\hat{C}_{pp}$.
(iv) Step 4 (make a disposition decision): Determine the fate of the submitted lot based on the following decision rule:
- Accept the lot if the estimated $\hat{C}_{pp} \le k_a$
- Reject the lot if the estimated $\hat{C}_{pp} > k_r$.
- If the estimated $\hat{C}_{pp}$ falls within the continuation region $k_a < \hat{C}_{pp} \le k_r$, repeat the sampling process (Step 3).

3.2. Probability of acceptance

Probability functions may be derived from the sample distribution of $\hat{C}_{pp}$, as outlined below. When the lot's quality is specified as $C_{pp} = b$, the probability of lot acceptance can be computed as **Equation 5**:

$$P_a(b) = P(\hat{C}_{pp} \le k_a \mid C_{pp} = b) = 1 - P(C_{pp} > k_a \| C_{pp} = b) = P\left(\chi^2_{n,n(1+\xi^2)} \le \frac{n(1+\xi^2)k_a}{b}\right). \tag{5}$$

Similarly, the probability of lot rejection, denoted as $P_r(b)$, is expressed as **Equation 6**:

$$P_r(b) = P(\hat{C}_{pp} > k_r \mid C_{pp} = b) = 1 - P\left(\chi^2_{n,n(1+\xi^2)} \le \frac{n(1+\xi^2)k_r}{b}\right) \tag{6}$$

The probability of resampling for further sentencing, denoted as $P_R(b)$, conditional on the result of current sampling, is given by the following **Equation 7**:

$$P_R(b) = P(k_a < \hat{C}_{pp} \le k_r) = 1 - P_a(b) - P_r(b) \tag{7}$$

According to the operational methods, the functional properties of the proposed system C_{pp}-VRGS plan, denoted as $\pi_A(b)$, may be articulated as **Equation 8**:

$$\pi_A(b) = P_a(b) + P_R(b)P_a(b) + [P_R(b)]^2 P_a(b) + [P_R(b)]^3 P_a(b) + \ldots = \frac{P_a(b)}{1 - P_R(b)]} = \frac{P_a(b)}{P_a(b) + P_r(b)} \tag{8}$$

It is noteworthy that the C_{pp}-VRGS plan will reduce to a variable single sampling plan contingent upon C_{pp} (i.e., C_{pp}-VSS plan) when the acceptance criteria $k_a = k_r$.[24]

3.3. Nonlinear optimization model

In evaluating the efficacy of the sampling strategy, it is more logical and appropriate to consider the ASN. The ASN denotes the average number of sample units needed to reach a conclusive decision about the lot. The ASN for the proposed C_{pp}-VRGS plan, when $C_{pp} = b$, is formulated as **Equation 9**:

$$\begin{aligned}\text{ASN}(b) &= n[P_a(b) + P_r(b)] \\ &\quad + 2n[1 - P_a(b) + P_r(b)][P_a(b) + P_r(b)] \\ &\quad + 3n[1 - P_a(b) + P_r(b)]^2[P_a(b) + P_r(b)] + \ldots \\ &= \sum_{j=1}^{\infty} jn[1 - P_a(b) + P_r(b)]^{j-1}[P_a(b) + P_r(b)] \\ &= \frac{n[P_a(b) + P_r(b)]}{\{1 - [1 - P_a(b) + P_r(b)]\}^2} = \frac{n}{P_a(b) + P_r(b)}.\end{aligned} \tag{9}$$

As in the prior case, a carefully developed strategy must satisfy the dual criteria on the OC curve: (i) the lot must be ultimately accepted at its quality level with a minimum probability of at least $100(1 - \alpha)\%$; and (ii) the lot must be ultimately accepted at its quality level with a probability surpassing $100\beta\%$. Therefore, the parameters of the RGS plan, which incorporates the process incapability index, must fulfil the following two criteria: **Equations 10** and **11**.

$$\pi_A(b_{\text{AQL}}) = \frac{P_a(b_{\text{AQL}})}{P_a(b_{\text{AQL}}) + P_r(b_{\text{AQL}})} \ge 1 - \alpha , \tag{10}$$

$$\pi_A(b_{\text{RQL}}) = \frac{P_a(b_{\text{RQL}})}{P_a(b_{\text{RQL}}) + P_r(b_{\text{RQL}})} \le \beta , \tag{11}$$

To derive the triple plan parameters (n, k_a, and k_r) for the proposed plan, we formulate the following optimization model, featuring two nonlinear constraints designed to meet the two-point condition on the OC curve. However, the ASN, as derived from **Equation 9**, inherently fluctuates with changes in the actual quality level, given its dependence on it. Determining plan parameters requires knowledge of the actual lot's quality level. Previous research has primarily focused on specific quality levels, such as b_{AQL}, b_{RQL}, the average ASN evaluated at b_{AQL} and b_{RQL}, or the average ASN evaluated at . The study considers four ASN-based scenarios for plan parameter determination, focusing on minimizing ASN at the following points: (i) b_{AQL}, (ii) b_{RQL}, (iii) the average of b_{AQL} and b_{RQL}, and (iv) the midpoint of the critical value range .[21]

In Scenario 1, the objective is to reduce the ASN to its lowest value when evaluated at b_{AQL}. Hence, the determination of plan parameters involves solving the following optimization model as **Equations 12–14**.

$$\min_{n,k_a,k_r} \mathrm{ASN}_{\mathrm{AQL}} = \mathrm{ASN}(b_{\mathrm{AQL}}) = \frac{n}{P_a(b_{\mathrm{AQL}}) + P_r(b_{\mathrm{AQL}})} \tag{12}$$

subject to

$$\pi_A(b_{\mathrm{AQL}}) = \frac{P_a(b_{\mathrm{AQL}})}{P_a(b_{\mathrm{AQL}}) + P_r(b_{\mathrm{AQL}})} \geq 1-\alpha \tag{13}$$

$$\pi_A(b_{\mathrm{RQL}}) = \frac{P_a(b_{\mathrm{RQL}})}{P_a(b_{\mathrm{RQL}}) + P_r(b_{\mathrm{RQL}})} \leq \beta\ , \tag{14}$$

$k_r > k_a > 0$, $n \geq 2$.

For Scenario 2, the ASN function is assessed at the b_{RQL}. Consequently, the objective function of the model can be formulated as in **Equation 15:**

$$\min_{n,k_a,k_r} \mathrm{ASN}_{\mathrm{RQL}} = \mathrm{ASN}(b_{\mathrm{RQL}}) = \frac{n}{P_a(b_{\mathrm{RQL}}) + P_r(b_{\mathrm{RQL}})}\ . \tag{15}$$

In Scenario 3, the minimum ASN is evaluated at the average ASN for both quality conditions, denoted by $\mathrm{ASN}_{\mathrm{AR}}$. The objective function can be expressed as **Equation 16:**

$$\begin{aligned}\min_{n,k_a,k_r} \mathrm{ASN}_{\mathrm{AR}} &= \frac{\mathrm{ASN}(b_{\mathrm{AQL}}) + \mathrm{ASN}(b_{\mathrm{RQL}})}{2} \\ &= \frac{1}{2}\left\{\frac{n}{P_a(b_{\mathrm{AQL}}) + P_r(b_{\mathrm{AQL}})} + \frac{n}{P_a(b_{\mathrm{RQL}}) + P_r(b_{\mathrm{RQL}})}\right\}\end{aligned} \tag{16}$$

In Scenario 4, the minimum ASN is evaluated at the average ASN for both critical values, denoted by ASN_k. The objective function can be expressed as **Equation 17:**

$$\min_{n,k_a,k_r} \mathrm{ASN}_k = \mathrm{ASN}(b_k) = \frac{n}{P_a(b_k) + P_r(b_k)} \tag{17}$$

As previously stated, the (ξ) value in the cumulative distribution function of $\hat{C}_{pp}$—as articulated in **Equation 4**—equals zero, signifying that the process mean corresponds with the process target (T). Consequently, the plan parameters are obtained in this study by solving the previously described nonlinear optimization utilizing the predetermined configurations.

Lastly, to obtain the parameter values n, k_{a}, and k_{r} from **Equations 12–17**, the study utilizes the sequential quadratic programming method implemented through MATLAB's "fmincon" optimization function. Using this approach, simulation and sensitivity analyses were performed to investigate the performance of the proposed plan and compare it with the existing plan. The analyses were carried out across different combinations of quality and risk levels, enabling a comprehensive examination and discussion of their effects.

4. Result, analysis, and discussion

4.1. Average sample number simulation

The process incapability index (C_{pp}) has gained widespread acceptance as a metric for assessing process capability, with its application growing over the past few decades. In practice, the classification outlines the ranges of C_{pp} values corresponding to different levels of process capability. A process is considered "incapable" if its C_{pp} value is greater than 4, while a "super" process has a C_{pp} value below 0.25. The ranges for other conditions are: "capable" (0.5917–1.000), "satisfactory" (0.4444–0.5917), "good" (0.3673–0.4444), and "excellent" (0.2500–0.3673). These ranges indicate that lower C_{pp} values are associated with better process capability, suggesting that C_{pp} measures process variability or dispersion, where smaller values indicating tighter process control and higher quality.

The computations initially focused on evaluating four objective functions, namely $\mathrm{ASN}_{\mathrm{AQL}}$, $\mathrm{ASN}_{\mathrm{RQL}}$, $\mathrm{ASN}_{\mathrm{AR}}$, and ASN_k, to identify the most suitable one for further analysis. This evaluation is also compared to the conventional model to facilitate a comprehensive comparison. These computations covered a range of parameters, including quality levels (b_{AQL}, b_{RQL}) and various risk (α, β) combinations. Additionally, observations were conducted to assess the ASN curve's behavior in relation to C_{pp} values by comparing them across four distinct scenarios. In **Figure 2**, quality settings (b_{AQL}, b_{RQL}) are set at (0.5917, 1.000), four risk combinations (α, β) at (0.01, 0.05), (0.05, 0.01), (0.10, 0.05), (0.10, 0.10). Whereas, in **Figure 3**, quality settings (b_{AQL}, b_{RQL}) are set at (0.2500, 0.3673), four risk combinations (α, β) at (0.01, 0.05), (0.05, 0.01), (0.10, 0.05), and (0.10, 0.10).

The C_{pp}-VSS plan is characterized by a constant ASN (n), as shown in **Figures 2** and **3**, with no variation in response to changes in lot quality. In contrast, the C_{pp}-VRGS plan exhibits distinct behavior, with patterns heavily influenced by the lot's quality. The required ASN for the C_{pp}-VRGS plan for all scenarios is notably lower than that for the C_{pp}-VSS plan when the lot's quality is either very good (below k_a) or very poor (beyond k_r). The C_{pp}-VRGS under Scenario 2 yields the smallest sample size (n); however, it leads to a higher ASN because the broader range between k_a and k_r increases the probability of repeated sampling. Meanwhile, Scenarios 1 and 3 consistently require a moderate ASN value for lot inspection compared to the other scenarios. In anticipation of varying lot quality conditions, Scenarios 1 and 3 provided a balanced outcome between the extremes presented by the other scenarios. Conversely, Scenario 4 necessitates a larger sample size (n) to be inspected from the lot. Moreover, for intermediate quality lots, those between b_{AQL} and b_{RQL}, may demand increased ASN due to conditional resampling requirements ($k_r \leq C_{pp} < k_a$). In accommodating this situation, C_{pp}-VRGS in Scenario4 showcases better performance compared to C_{pp}-VSS. Scenario 4 yields the lowest maximum ASN value compared to the other three scenarios under identical conditions. For detailed information, **Table 1** presents the maximum sampling values for various scenarios with different quality levels and risk tolerance settings.

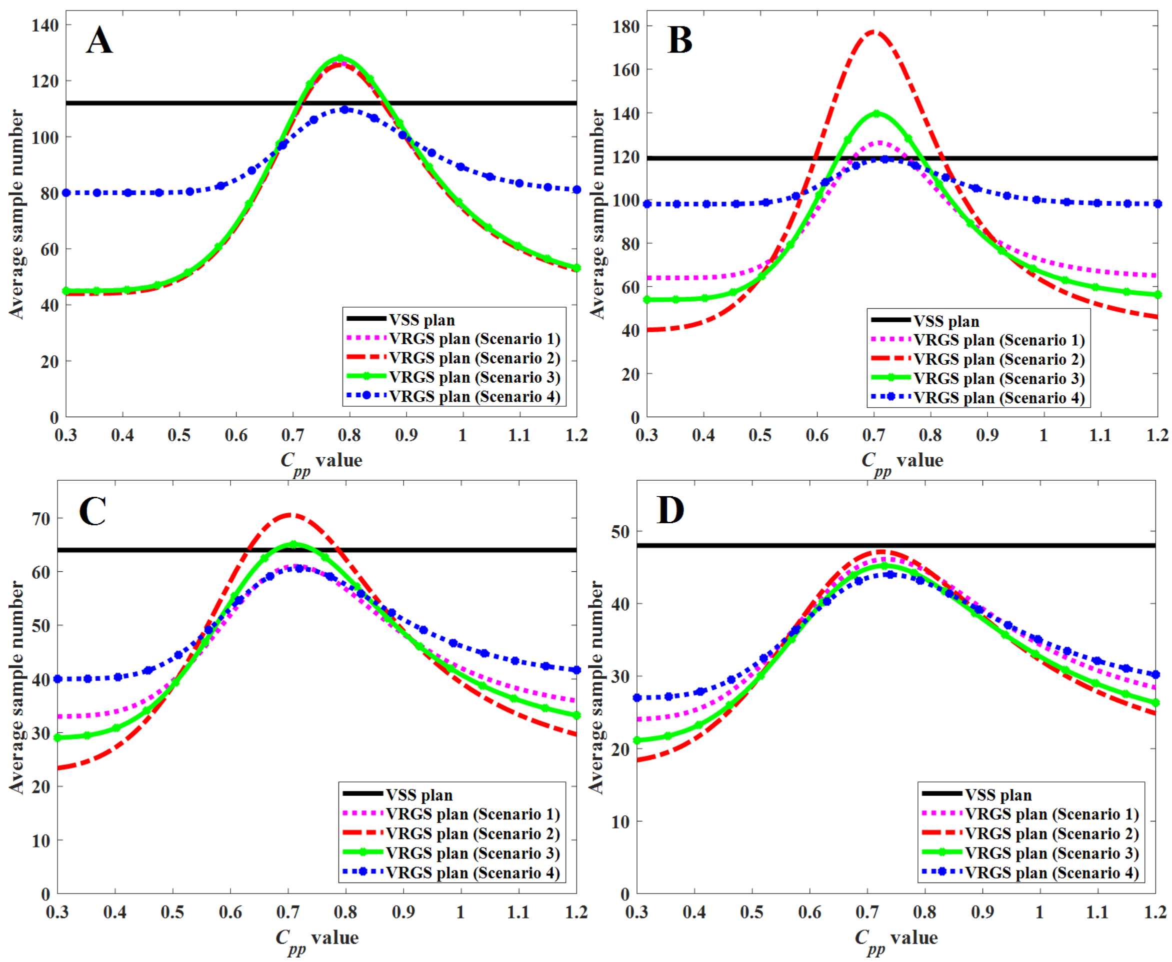

Figure 2. Average sample number curves of the C_{pp}-VSS and C_{pp}-VRGS plans with four scenarios under (b_{AQL}, b_{RQL}) = (0.5917, 1.000) and different risk levels. (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01). (C) (α, β) = (0.10, 0.05). (D) (α, β) = (0.10, 0.10).
Abbreviations: C_{pp}: Process incapability index; VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

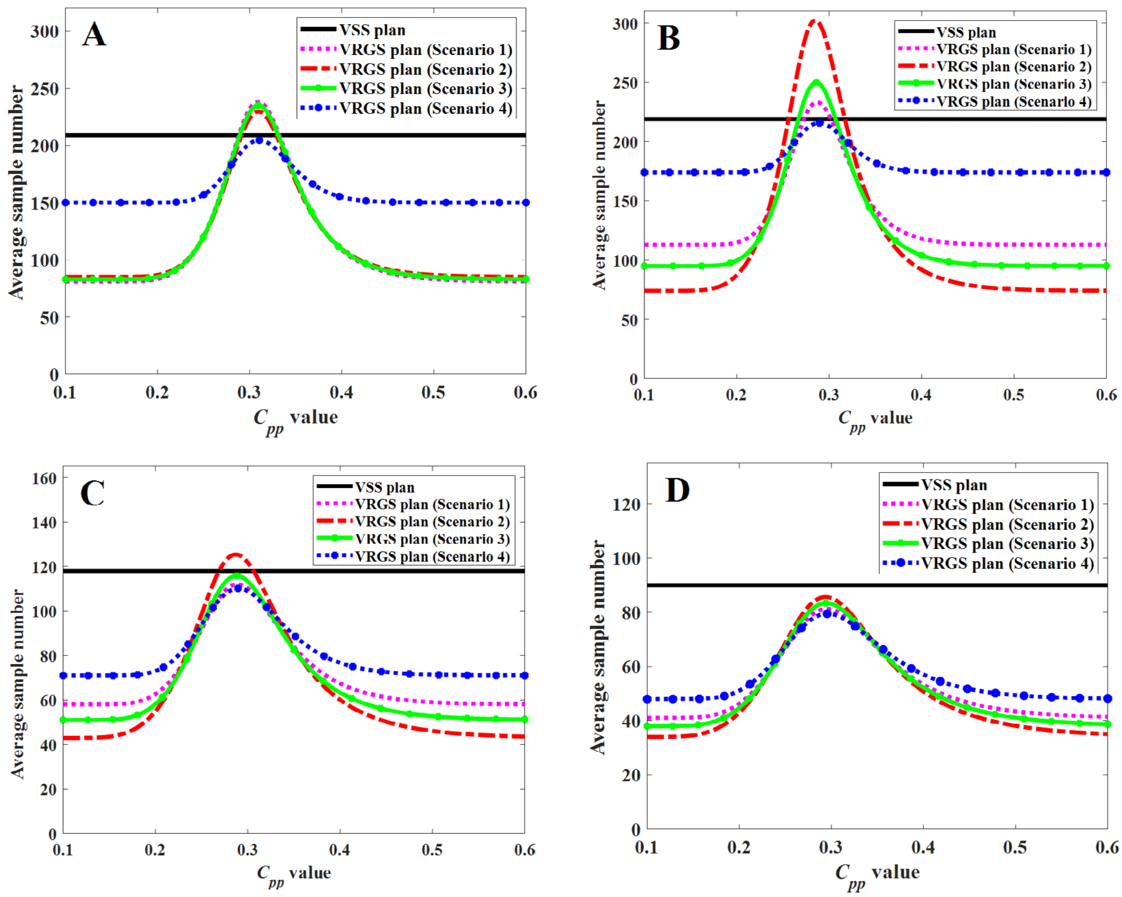

Figure 3. Average sample number curves of the C_{pp}-VSS and C_{pp}-VRGS plans with four scenarios under (b_{AQL}, b_{RQL}) = (0.2500, 0.3673) and different risk levels. (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01). (C) (α, β) = (0.10, 0.05). (D) (α, β) = (0.10, 0.10).
Abbreviations: C_{pp}: Process incapability index; VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

Moreover, it is evident that even in the worst-case situation where the ASN reaches its maximum value, the mandatory ASN for the C_{pp}-VRGS plan (Scenario 4) remains lower compared to that of the C_{pp}-VSS plan. This characteristic can be perceived as a significant advantage of the proposed approach, consistently and substantially reducing inspection costs in comparison to the C_{pp}-VSS plan.

The sensitivity analysis highlighted that ASN is evaluated in four scenarios within the objective function, and the C_{pp}-VRGS plan shows better performance than the C_{pp}-VSS plan when the quality of the submitted lot falls into either high- or low-quality categories. Thus, this approach can lead to a reduction in inspection costs. Notably, Scenario 4 is particularly recommended for determining plan parameters, especially when lot quality information is insufficient. By adopting Scenario 4, acceptable lot quality decisions can be ensured even in ambiguous conditions (i.e., when the quality lies between k_a and k_r). Moreover, this scenario guarantees a minimum ASN compared to the other scenarios under the same conditions, while also outperforming the C_{pp}-VSS plan overall.

4.2. Plan parameters determination

To facilitate practical application for quality engineers, we calculated plan parameters for various quality levels (b_{AQL}, b_{RQL}) and risk levels (α, β). The results, presented in **Tables 2–5**, cover four scenarios and illustrate the relationship between risk levels and sample sizes. For instance, under Scenario 4 with $b_{\mathrm{AQL}} = 0.5917$, $b_{\mathrm{RQL}} = 1.000$, $\alpha = 0.05$, and $\beta = 0.05$, the plan parameters are $n = 50$, $k_{\mathrm{a}} = 0.6825$, and $k_{\mathrm{r}} = 0.8136$ (as shown in **Table 5**). These parameters inform the sampling

Table 1. Comparison of the maximum ASN values for the C_{pp}-VRGS plan (under four scenarios) and the C_{pp}-VSS plan under different quality levels and risk settings

(l_{AQL}, l_{RQL})	(α, β)	The highest point on the ASN curve for each scenario				
		1	2	3	4	VSS
(0.5917, 1.000)	(0.01, 0.05)	126.18	125.61	128.01	109.65	112
	(0.05, 0.01)	126.18	177.15	139.56	118.56	119
	(0.05, 0.05)	78.32	88.10	79.45	75.59	79
	(0.05, 0.10)	57.75	62.68	60.61	56.80	62
(0.4444, 0.5917)	(0.01, 0.05)	434.82	410.75	422.74	369.95	380
	(0.05, 0.01)	414.15	513.18	446.51	385.92	393
	(0.05, 0.05)	258.92	272.59	264.16	246.98	265
	(0.05, 0.10)	196.03	198.45	196.83	188.21	207
(0.3673, 0.4444)	(0.01, 0.05)	993.17	923.10	956.14	837.36	860
	(0.05, 0.01)	929.88	1114.29	989.50	861.89	880
	(0.05, 0.05)	585.63	603.70	593.17	553.94	597
	(0.05, 0.10)	444.11	441.49	443.24	423.28	469
(0.2500, 0.3673)	(0.01, 0.05)	238.16	229.60	235.02	204.58	209
	(0.05, 0.01)	233.12	302.17	249.80	215.76	219
	(0.05, 0.05)	142.95	154.02	146.84	137.88	147
	(0.05, 0.10)	106.95	111.13	109.92	104.55	115

Abbreviations: ASN: Average sample number; VSS: Variable single sampling.

process, including the determination of sample size and the execution of statistical evaluations. With these plan parameters, quality engineers can proceed to randomly select 50 samples from the lot, perform the inspection and measurement procedures, and compute the sample statistics $(\bar{X}, S_n^2)$ and $\hat{C}_{pp}$. In addition, the tables reveal an inverse relationship between risk levels and sample size, where reduced risks for producers and/or customers necessitate larger samples to minimize misclassification of good products as bad products and vice versa. Furthermore, critical values (k_a and k_r) are directly proportional to β-risk and inversely proportional to α-risk, ceteris paribus. The interval width between b_{AQL} and b_{RQL} significantly impacts sample size (n) and acceptance values. Wider intervals—such as (0.5917, 1.0000)—allow for smaller samples but require stricter acceptance criteria (k_a and k_r), whereas narrower intervals—such as (0.2500, 0.3673)—necessitate larger samples and more lenient criteria. Moreover, for a fixed sampling duration, increasing the difference between b_{AQL} and b_{RQL} reduces the required sample size under given α-risk, β-risk, and b_{RQL} constraints. The decision process is more straightforward with larger differences between b_{AQL} and b_{RQL}.

Furthermore, variations in α and β reflect the trade-off between producer and consumer protection, influencing sample size and critical values. Decreasing α (enhanced producer protection) results in larger sample sizes and stricter critical values, whereas increasing β (greater consumer risk tolerance) leads to smaller sample sizes and more lenient criteria. By judiciously selecting α, β, and the (b_{AQL}, b_{RQL}) pair, practitioners can tailor the C_{pp}-VRGS plan to balance inspection costs with quality assurance objectives in a rigorous and statistically controlled manner.

4.3. Operating characteristics curve

The OC curve is a key performance metric for evaluating the discriminatory power of sampling schemes, alongside the ASN. The OC curve must

Table 2. Plan parameters of the C_{pp}-VRGS plan under Scenario 1

α	β	$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.5917, 1.000)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.4444, 0.5917)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.3673, 0.4444)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.2500, 0.3673)**		
		n	k_a	k_r	n	k_a	k_r	N	k_a	k_r	n	k_a	k_r
	0.01	70	0.6380	0.8657	220	0.4628	0.5559	483	0.3774	0.4283	125	0.2641	0.3346
0.01	0.05	45	0.6406	0.9466	143	0.4645	0.5854	316	0.3784	0.4436	81	0.2652	0.3581
	0.1	35	0.6436	1.0000	113	0.4662	0.6042	253	0.3794	0.4532	64	0.2663	0.3734
	0.01	64	0.6289	0.7957	195	0.4583	0.5305	424	0.3747	0.4151	113	0.2609	0.3143
0.05	0.05	37	0.6229	0.8715	114	0.4565	0.5603	250	0.3739	0.4311	65	0.2593	0.3374
	0.1	27	0.6209	0.9247	85	0.4563	0.5805	187	0.3739	0.4417	48	0.2589	0.3532
	0.01	62	0.6241	0.7556	184	0.4557	0.5155	397	0.3731	0.4072	107	0.2592	0.3025
0.1	0.05	33	0.6109	0.8268	100	0.4508	0.5450	217	0.3706	0.4233	58	0.2553	0.3248
	0.1	24	0.6041	0.8783	71	0.4487	0.5657	155	0.3696	0.4345	41	0.2534	0.3407

Table 3. Plan parameters of the C_{pp}-VRGS plan under Scenario 2

α	β	$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.5917, 1.000)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.4444, 0.5917)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.3673, 0.4444)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.2500, 0.3673)**		
		n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
	0.01	55	0.5868	0.9230	190	0.4516	0.5673	440	0.3735	0.4323	104	0.2528	0.3468
0.01	0.05	44	0.6371	0.9507	157	0.4730	0.5757	368	0.3852	0.4363	85	0.2688	0.3541
	0.1	39	0.6690	0.9707	141	0.4864	0.5814	334	0.3926	0.4390	76	0.2789	0.3591
	0.01	40	0.5261	0.9074	132	0.4262	0.5629	302	0.3596	0.4303	74	0.2337	0.3429
0.05	0.05	29	0.5612	0.9406	98	0.4428	0.5743	227	0.3692	0.4360	54	0.2456	0.3523
	0.1	24	0.5825	0.9683	82	0.4531	0.5835	191	0.3751	0.4405	45	0.2529	0.3599
	0.01	34	0.4932	0.8962	110	0.4117	0.5595	249	0.3516	0.4287	62	0.2230	0.3401
0.1	0.05	23	0.5181	0.9298	77	0.4247	0.5719	175	0.3593	0.4350	43	0.2319	0.3499
	0.1	18	0.5319	0.9595	61	0.4323	0.5825	141	0.3639	0.4405	34	0.2371	0.3584

Table 4. Plan parameters of the C_{pp}-VRGS plan under Scenario 3

α	β	$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.5917, 1.000)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.4444, 0.5917)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.3673, 0.4444)**			$(b_{\text{AQL}}, b_{\text{RQL}})$ **(0.2500, 0.3673)**		
		n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
	0.01	64	0.6188	0.8864	207	0.4582	0.5607	462	0.3756	0.4301	116	0.2595	0.3395
0.01	0.05	45	0.6391	0.9484	150	0.4688	0.5807	341	0.3819	0.4399	83	0.2668	0.3563
	0.1	37	0.6550	0.9868	127	0.4766	0.5926	291	0.3864	0.4456	69	0.2724	0.3664
	0.01	54	0.5940	0.8298	167	0.4463	0.5423	366	0.3687	0.4210	95	0.2511	0.3240
0.05	0.05	33	0.5991	0.8970	107	0.4508	0.5663	239	0.3718	0.4333	60	0.2537	0.3434
	0.1	26	0.6054	0.9419	84	0.4550	0.5819	189	0.3745	0.4412	47	0.2563	0.3561
	0.01	49	0.5805	0.7962	149	0.4397	0.5307	325	0.3648	0.4151	86	0.2464	0.3145
0.1	0.05	29	0.5762	0.8625	89	0.4402	0.5559	197	0.3657	0.4283	51	0.2460	0.3343
	0.1	21	0.5759	0.9085	67	0.4417	0.5729	148	0.3670	0.4372	38	0.2466	0.3479

Table 5. Plan parameters of the C_{pp}-VRGS plan under Scenario 4

α	β	(b_{AQL}, b_{RQL}) (0.5917, 1.000)			(b_{AQL}, b_{RQL}) (0.4444, 0.5917)			(b_{AQL}, b_{RQL}) (0.3673, 0.4444)			(b_{AQL}, b_{RQL}) (0.2500, 0.3673)		
		n	k_a	k_r	n	k_a	k_r	n	k_a	k_r	n	k_a	k_r
	0.01	147	0.7481	0.7646	483	0.5066	0.5138	1083	0.4011	0.4051	269	0.2976	0.3030
0.01	0.05	80	0.7474	0.8339	273	0.5085	0.5385	623	0.4026	0.4178	150	0.2983	0.3228
	0.1	59	0.7547	0.8783	208	0.5129	0.5531	481	0.4053	0.4251	113	0.3013	0.3350
	0.01	98	0.6961	0.7411	306	0.4857	0.5067	673	0.3898	0.4018	174	0.2816	0.2968
0.05	0.05	50	0.6825	0.8136	160	0.4825	0.5344	357	0.3886	0.4164	90	0.2784	0.3185
	0.1	35	0.6795	0.8646	115	0.4832	0.5527	259	0.3894	0.4258	64	0.2784	0.3332
	0.01	82	0.6712	0.7195	251	0.4752	0.4993	545	0.3839	0.3980	144	0.2738	0.2907
0.1	0.05	40	0.6493	0.7909	124	0.4682	0.5281	273	0.3806	0.4136	71	0.2678	0.3128
	0.1	27	0.6394	0.8433	85	0.4661	0.5481	190	0.3800	0.4240	48	0.2656	0.3284

meet the specific two-point requirements, passing through $(b_{AQL}, 1-\alpha)$ and (b_{RQL}, β). **Figures 4** and **5** depict OC curves for C_{pp}-V-SSP and C_{pp}-VRGS (four scenarios) under specified conditions, including $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.01)$, and $(b_{AQL}, b_{RQL}) = (0.5917, 1.000)$. The plan parameters under risk tolerance conditions $(\alpha, \beta) = (0.01, 0.05)$ are $(n, k) = (112, 0.7910)$ for C_{pp}-VSS; $(n, k_a, k_r) = (45, 0.6406, 0.9466)$ for Scenario 1 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (44, 0.6371, 0.9507)$ for Scenario 2 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (45, 0.6391, 0.9484)$ for Scenario 3 of the C_{pp}-VRGS plan; and $(n, k_a, k_r) = (80, 0.7474, 0.8339)$ for Scenario 4 of the C_{pp}-VRGS plan.

The plan parameters under risk tolerance conditions $(\alpha, \beta) = (0.05, 0.01)$ are $(n, k_a, k_r) = (64, 0.6289, 0.7957)$ for Scenario 1 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (40, 0.5261, 0.9074)$ for Scenario 2 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (54, 0.5940, 0.8298)$ for Scenario 3 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (98, 0.6961, 0.7411)$ for Scenario 4 of the C_{pp}-VRGS plan; and $(n, k) = (119, 0.7233)$ for the C_{pp}-VSS plan.

The C_{pp}-VSS and the C_{pp}-VRGS plan (four scenarios) OC curves are presented in **Figures 6** and **7**, corresponding to specific parameter settings: $(b_{AQL}, b_{RQL}) = (0.2500, 0.3673)$ and $(\alpha, \beta) = (0.01, 0.05)$ and $(0.05, 0.01)$. The plan parameters under risk tolerance conditions $(\alpha, \beta) = (0.01, 0.05)$ are $(n, k) = (209, 0.3104)$ for the C_{pp}-VSS plan; $(n, k_a, k_r) = (81, 0.2652, 0.3581)$ for Scenario 1 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (85, 0.2688, 0.3541)$ for Scenario 2 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (83, 0.2668, 0.3563)$ for Scenario 3 of the C_{pp}-VRGS plan; and $(n, k_a, k_r) = (150, 0.2983, 0.3228)$ for Scenario 4 of the C_{pp}-VRGS plan. The plan parameters under risk tolerance conditions $(\alpha, \beta) = (0.05, 0.01)$ are $(n, k_a, k_r) = (113, 0.2609, 0.3143)$ for Scenario 1 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (74, 0.2337, 0.3429)$ for Scenario 2 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (95, 0.2511, 0.3240)$ for Scenario 3 of the C_{pp}-VRGS plan; $(n, k_a, k_r) = (174, 0.2816, 0.2968)$ for Scenario 4 of the C_{pp}-VRGS plan; and $(n, k) = (219, 0.2906)$ for the C_{pp}-VSS plan—illustrating each plan's discriminatory power in distinguishing between acceptable and unacceptable quality levels.

With respect to discriminatory power, **Figures 4–7** reveal that the OC curves for the C_{pp}-VRGS plan exhibit a similar shape, meeting the two-point requirements and passing through $(b_{AQL}, 1-\alpha)$ and (b_{RQL}, β). This suggests the C_{pp}-VRGS plan provides adequate protection for both producers and consumers while requiring smaller sample sizes. Notably, the C_{pp}-VRGS plan demonstrates superior discriminatory power compared to the conventional plan, regardless of sample size.

The C_{pp}-VRGS plan outperforms the C_{pp}-VSS plan in terms of efficiency, requiring a lower ASN value. C_{pp}-VRGS (Scenario 2) is the most efficient and provides balanced protection to both producers and consumers. Meanwhile, C_{pp}-VRGS (Scenario 4) is ideal for uncertain or worst-case situations, providing balanced protection for

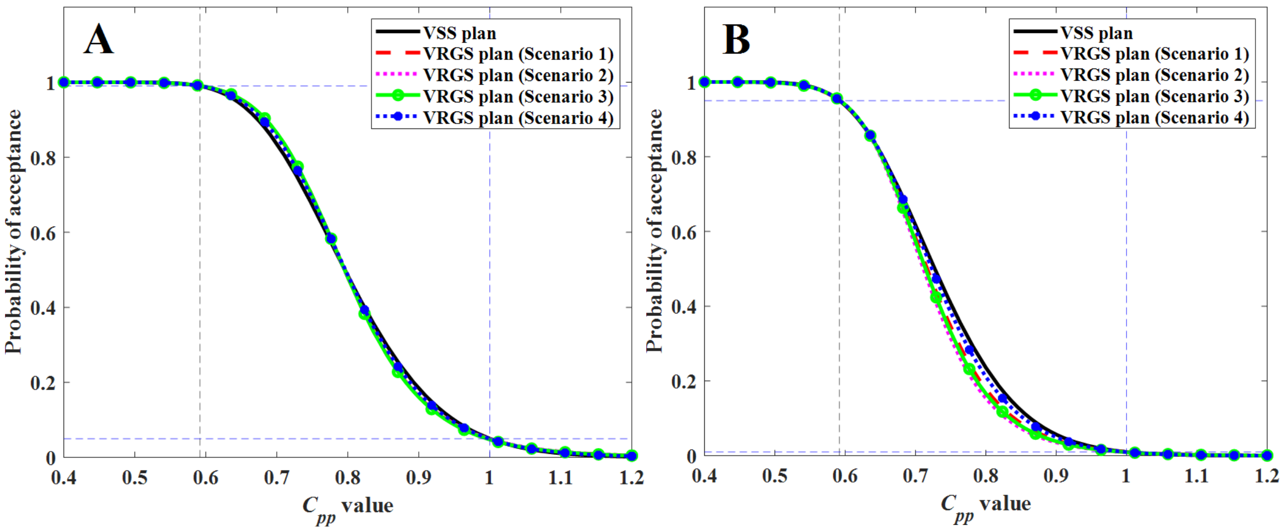

Figure 4. Comparison of the C_{pp}-VSS and C_{pp}-VRGS plans (four scenarios) operating characteristic curves under (b_{AQL}, b_{RQL}) = (0.5917, 1.000). (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01).
Abbreviations: VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

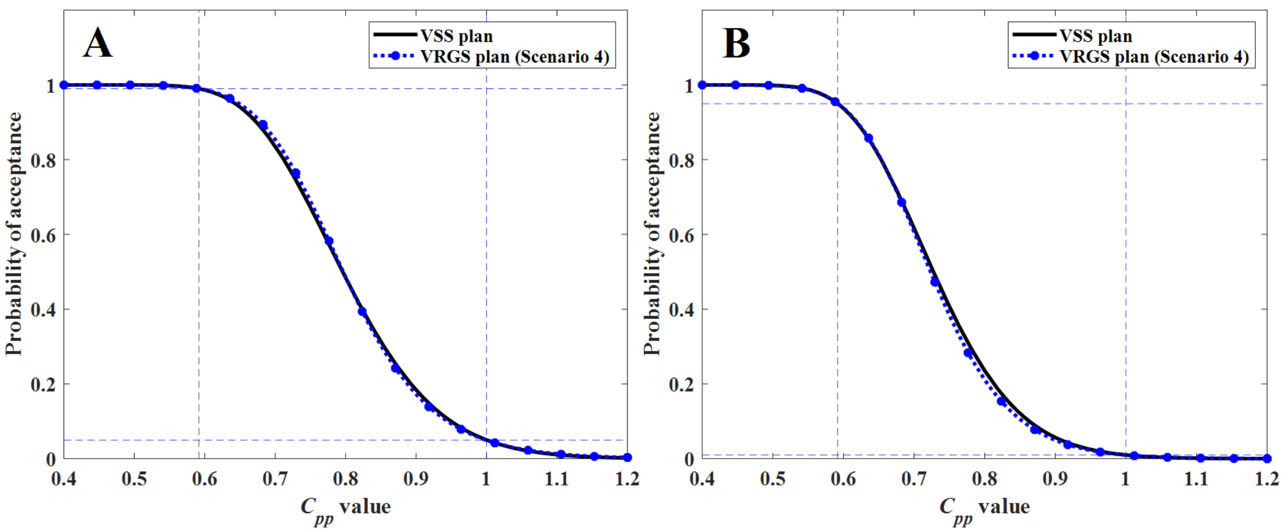

Figure 5. Comparison of the C_{pp}-VSS and C_{pp}-VRGS plans (Scenario 4) operating characteristic curves under (b_{AQL}, b_{RQL}) = (0.5917, 1.000). (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01).
Abbreviations: VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

both producers and consumers in such scenarios. This approach allows for re-evaluation, providing operational flexibility in decision-making. It ensures equivalent protection while optimizing sample size. Overall, the C_{pp}-VRGS plan is a more efficient and effective sampling plan, particularly in situations where information is limited or uncertainty is high. Its flexibility and reliability make it a valuable tool for quality control applications, as it can be tailored to specific needs and scenarios.

5. Example demonstration

An application was performed on an example of monocrystalline silicon wafer samples integrated into solar cell modules adopted from Wu *et al.*[50] to demonstrate the implementation and efficiency of the proposed approach. The wafers' thickness influenced the manufacturing costs and efficiency of this panel device. Exceeding the specification limits will lead to increased production costs. If the wafer thickness is too low, production costs might decrease, but the wafers will be more

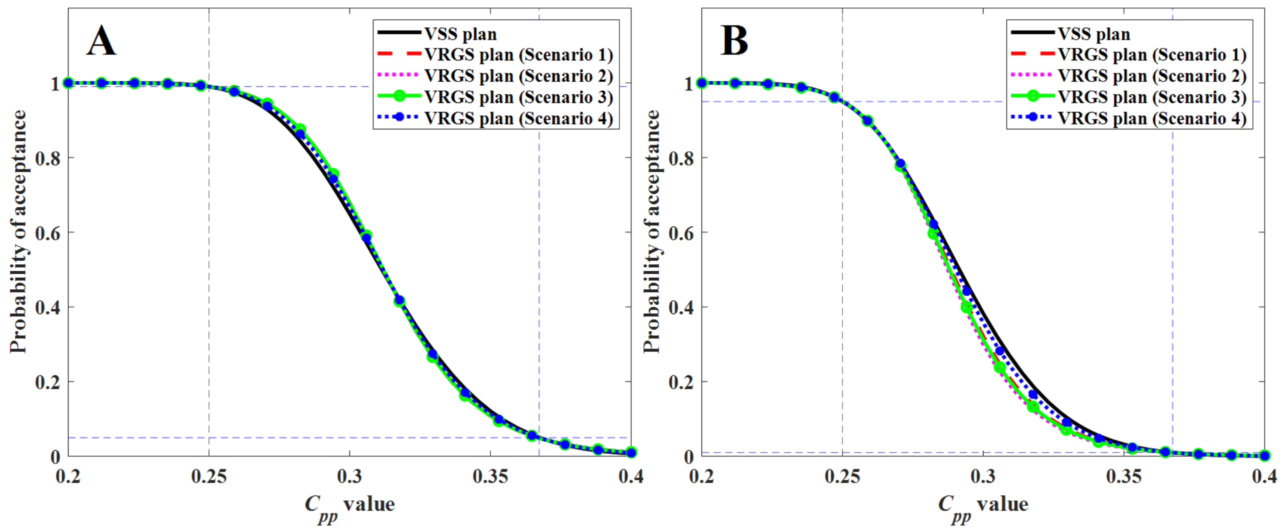

Figure 6. Comparison of the C_{pp}-VSS and C_{pp}-VRGS plans (four scenarios) operating characteristic curves under (b_{AQL}, b_{RQL}) = (0.2500, 0.3673). (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01).
Abbreviations: VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

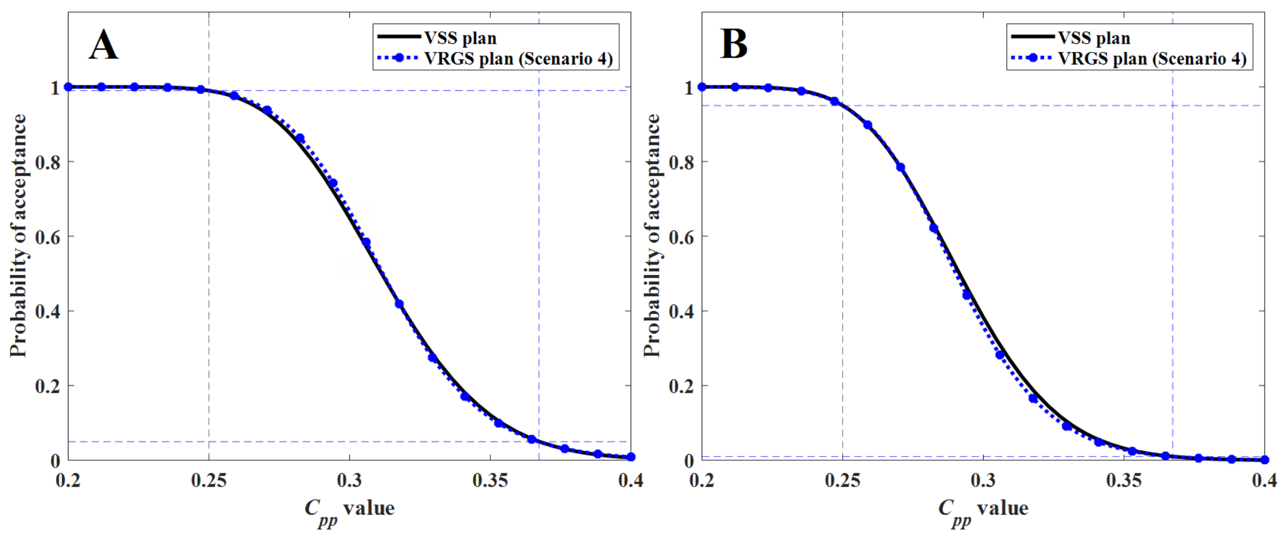

Figure 7. Comparison of the C_{pp}-VSS and C_{pp}-VRGS plans (Scenario 4) operating characteristic curves under (b_{AQL}, b_{RQL}) = (0.2500, 0.3673). (A) (α, β) = (0.01, 0.05). (B) (α, β) = (0.05, 0.01).
Abbreviations: VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

prone to breakage, causing a higher defect rate. Therefore, it is essential to continuously monitor and control the wafer thickness to maintain quality.

The wafer's specified thickness is 200 µm, with a tolerance range of ±20 µm, resulting in an *LSL* of 180 µm and a *USL* of 220 µm. Suppose that, in the contract, the requirements are specified as Scenario 4, quality requirements (b_{AQL};b_{RQL}) = (0.5917, 1.0000), and risk levels $(\alpha, \beta) = (0.01, 0.05)$. Then the plan parameters can be obtained by checking **Table 5**, as $(n, k_a, k_r) = (80,\ 0.7474,\ 0.8339)$.

A random sample of 80 units was drawn from the submitted lot, and their measurements were summarized in **Table 6**. The use of this dataset (**Table 6**) from Wu *et al.*[50] allows for a fair and consistent comparison with existing methodologies and ensures continuity with prior research in process capability-based sampling plans. The sample yielded a mean of 200.0375 and a standard deviation of 5.3140. The calculated $\hat{C}_{pp}$ value was then compared to the critical values. The lot is accepted if $\hat{C}_{pp} \leq 0.7474$ and rejected if $\hat{C}_{pp} \geq 0.8339$. For values between these thresholds, resampling is required, and lot disposition is determined according to the procedure outlined in Section 3. Corresponding to the calculation of $\hat{C}_{pp}$ in this sample, 0.6354 was obtained. Hence, the lot should be accepted since $\hat{C}_{pp}$ less than or equal to $k_a = 0.7474$. Compared with the C_{pp}-VSS plan, a sample of size $n = 112$ (the critical value is 0.7906) is needed for inspection, which exceeds the requirement of the C_{pp}-VRGS plan.

Table 6. The measurement result of the 80 data collected

199	201	202	200	202	200	200	195
194	201	203	207	207	205	202	200
195	200	193	209	205	198	184	202
213	211	199	201	206	201	196	203
200	208	200	204	194	208	197	198
202	198	205	201	196	195	196	192
201	205	199	210	196	194	190	204
207	203	188	203	190	199	201	204
201	198	206	198	192	202	202	198
199	191	196	202	202	201	196	197

Under the same conditions, the proposed C_{pp}-VRGS plan requires a sample size of only 80 for lot disposition. Additionally, **Figure 8** illustrates the ASN curves for both the C_{pp}-VSS plan and the proposed C_{pp}-VRGS plan. In the long term, it is evident that the C_{pp}-VRGS plan offers a cost-saving advantage in terms of inspection expenses, amounting to approximately 20.54% in ASN (89.099), compared to the C_{pp}-VSS plan ($\text{ASN}_{\text{VSS}} = n = 112$), given that the quality of the lot is at C_{pp}=0.6354. Moreover, the suggested approach will exhibit enhanced efficiency, especially in cases when the quality of the lot is either satisfactory or substandard. Even under the most unfavorable circumstances, when the suggested C_{pp}-VRGS plan reaches its maximum ASN value on the ASN curve, the value of $\text{ASN}_{\max} = 109.65$ is still lower than that of C_{pp}-VSS plan. Therefore, the suggested sampling methodology may be regarded as an effective method for determining lot acceptance decisions and ensuring equal protection for the benefit of both parties (the producer and the consumer) in the long term.

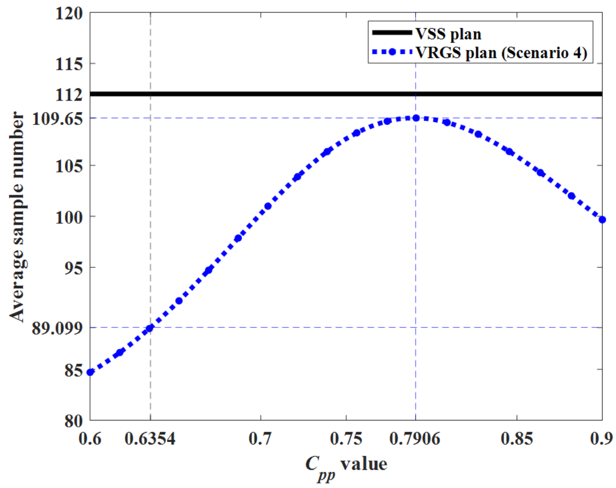

Figure 8. The average sample number curve of the C_{pp}-VSS and C_{pp}-VRGS plans
Abbreviations: VRGS: Variable repetitive group sampling; VSS: Variable single sampling.

Furthermore, a graphical user interface (GUI) application has been developed to facilitate the practical implementation of the proposed plan, making it more accessible and user-friendly to practitioners. The GUI is divided into two components. In the first phase (**Figure 9**), users input the agreed-upon quality levels (b_{AQL}, b_{RQL}), tolerable risk levels (α, β) and select an objective function (with ASN as the default option), and click "Calculate Parameters." Consequently, the plan parameters are computed and displayed to the user. This interface streamlines the process, enabling practitioners to determine plan parameters tailored to their specific needs efficiently.

By simplifying the calculation process, the GUI application enhances the plan's usability in real-world scenarios, providing a practical tool for quality control applications.

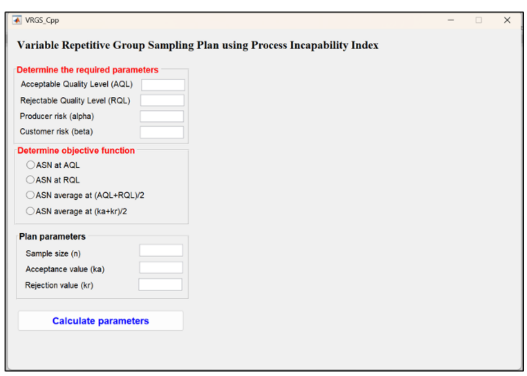

Figure 9. Graphical user interface system for inputting customer requirements and importing or entering input data (first part)

Upon clicking the “Calculate Parameters” button, the GUI will display the next interface panel (**Figure 10**), comprising the following four key components: (i) “Decision Result,” (ii) “Data Input/Import,” (iii) “Data Statistics Output,” and (iv) “Histogram and Normal Probability Plot Generation.” Users must enter specification limits and a target value, and then provide sample data or import an existing dataset. After clicking the “Decision Result” button, the GUI will display comprehensive statistical metrics, including sample mean, standard deviation, and estimated C_{pp}, providing valuable insights into the data.

The GUI program also generates a histogram and a normal probability plot to visually depict the data’s distribution, providing a clear representation of the data’s characteristics (**Figure 11**). Additionally, it furnishes a lot disposition, offering a complete decision output. By streamlining the process of inputting sample data, performing calculations, conducting normality tests, and providing decision support outputs, the program enhances efficiency and accuracy. This integrated approach enables users to make informed decisions with confidence, leveraging the program’s analytical capabilities to drive quality control and assurance.

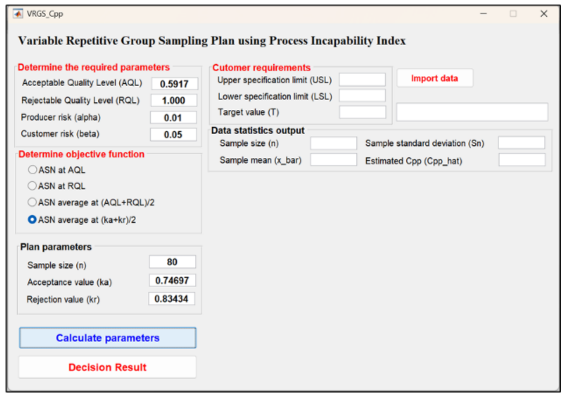

Figure 10. Graphical user interface system for inputing customer requirements and importing or entering data (second part)

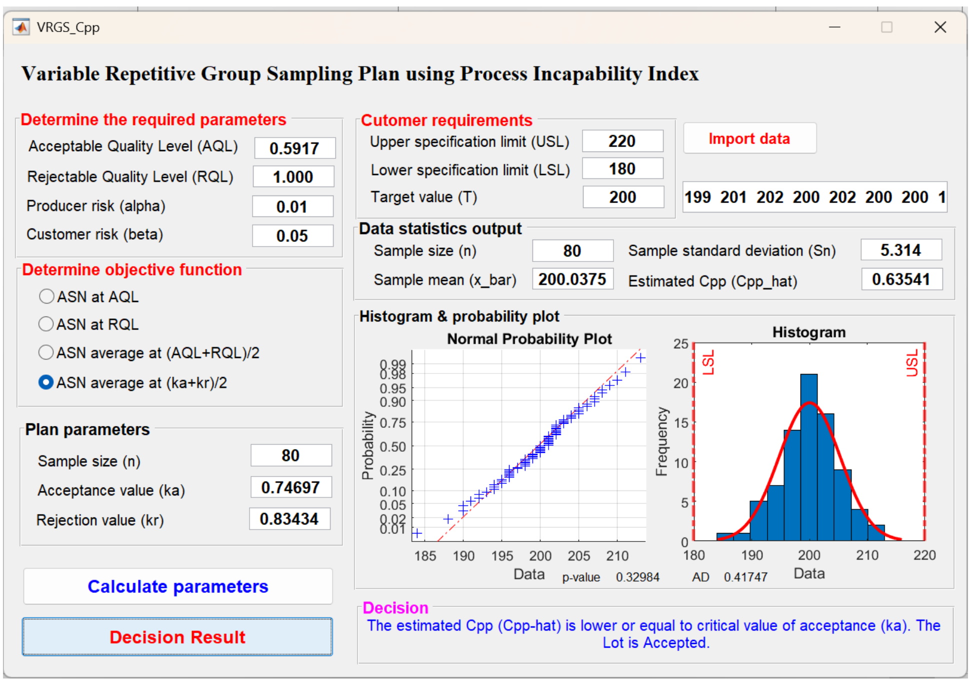

Figure 11. Graphical user interface system for statistical output and decision-making

6. Conclusion

This study proposed a novel acceptance sampling design that integrates the process incapability index (C_{pp}) with a VRGS framework, referred to as the C_{pp}-VRGS plan. To achieve an efficient and flexible inspection strategy, a nonlinear optimization model was developed under four different scenarios to minimize the ASN while satisfying predetermined producer’s and consumer’s risk requirements and quality levels. The proposed model was used to determine the optimal sampling parameters (n, k_a, k_r), and its performance was evaluated through comparative analyses with the existing C_{pp}-VSS plan.

The results demonstrate that the proposed C_{pp}-VRGS plan provides adequate protection for both producers and consumers while reducing inspection effort and associated costs through lower sample size requirements during long-run operations. The comparative analysis confirmed the superior performance of the proposed plan under identical operating conditions. Among the four scenarios examined, Scenario 2 was found to be particularly effective when product quality is either very high or very low, whereas Scenario 4 offered a more economically efficient option when product quality is uncertain. These findings indicate that different C_{pp}-VRGS scenarios can be selected according to specific quality and risk requirements, providing greater flexibility for implementation.

From a theoretical perspective, this study contributes to the acceptance sampling literature by integrating the process incapability index with the VRGS framework within a nonlinear optimization setting. This integration extends existing sampling methodologies by incorporating process performance information while simultaneously improving sampling efficiency. The study also enriches the body of knowledge on risk-based acceptance sampling by demonstrating how ASN can be minimized without compromising producer and consumer protection levels.

From a practical perspective, the proposed C_{pp}-VRGS plan offers quality practitioners an effective and cost-efficient inspection tool that can be adapted to different operational environments. The development of a user-friendly GUI further enhances the applicability of the proposed methodology by simplifying parameter determination, data input, and decision-making. The case study involving monocrystalline silicon wafers confirms the feasibility and effectiveness of the proposed approach in real industrial applications.

Despite these contributions, several limitations should be acknowledged. First, the proposed methodology assumes that the quality characteristic follows a normal distribution or approximately satisfies the normality assumption. Consequently, its applicability may be limited when the underlying process exhibits substantial departures from normality. Second, the current framework focuses on a single quality characteristic and does not explicitly address situations involving multiple quality characteristics that may jointly influence product quality.

Future research may extend the proposed methodology to handle non-normal quality characteristics and investigate robust sampling procedures under non-normal distributional assumptions. In addition, future studies could develop multivariate C_{pp}-VRGS plans capable of simultaneously evaluating multiple quality characteristics. Further exploration of alternative process capability/incapability indices and advanced optimization techniques may also provide opportunities to enhance the efficiency and applicability of acceptance sampling frameworks in increasingly complex manufacturing environments.

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Author contributions

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