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RESEARCH ARTICLE

Robust fault detection filter design for uncertain neutral systems with time delays: A case study of two-stage recycle reactors

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ABSTRACT

This study introduces a robust fault detection filter (RFDF) for uncertain neutral-type systems with time delays in both state variables and their derivatives. Such systems are frequently encountered in manufacturing processes, where delays and parametric uncertainties can significantly impact system reliability and efficiency. The proposed method addresses the simultaneous challenges presented by unknown inputs and parameter uncertainties. A Lyapunov–Krasovskii functional is employed to derive sufficient conditions for ensuring the asymptotic stability of the estimation error dynamics. The design problem is formulated as a convex optimization problem solved via linear matrix inequalities. A new fault detection scheme has been developed comprising three key components: a model-based fault detection filter, a reference residual generator operating under nominal conditions, and a resilient residual generator that is impervious to disturbances yet responsive to faults. An H∞ optimization criterion is applied to maximize fault sensitivity while minimizing the effect of unknown disturbances. The effectiveness of the proposed RFDF is validated through simulation of a two-stage chemical reactor system subject to neutral time delays and recycle streams. The results demonstrate the proposed filter's ability to detect actuator and sensor faults under uncertain conditions, confirming its robustness and effectiveness.

1. Introduction

Robust fault detection filters (RFDFs) for neutral systems with time delays have attracted considerable attention in industrial applications over the past few years.[1–3]

Neutral delay systems are dynamical systems in which the system behavior depends not only on the present state but also on past and delayed states, including their derivatives. Such delays are common in modeling diverse processes across mechanics, physics, chemistry, economics, and related disciplines.[4–8]

In the literature, a substantial body of research has addressed the synthesis of linear uncertain

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neutral systems with time delays, with a particular focus on differentiating the types of delays involved.[9–11]

Delays often introduce undesirable effects, which may manifest as faults in actuators, sensors, or system models, potentially leading to instability. Consequently, stabilization, robust stability, and fault diagnosis remain central objectives in developing reliable control strategies.[12,13]

In the literature, various approaches have been proposed to ensure the stability or stabilization of delayed systems. Among these, Lyapunov–Krasovskii functionals provide a general theoretical framework applicable to a broad class of differential equation systems.

In particular, time-varying delays and switching topologies have been extensively studied in the context of multiagent consensus systems. For instance, the consensus problem for noisy multiagent systems with Markovian-switching topologies and time-varying delays has been investigated using linear matrix inequality (LMI)-based stability analysis.[14] Similarly, continuous-time average consensus under dynamically changing topologies and multiple time-varying delays has been addressed, further demonstrating the versatility and generality of delay-dependent LMI frameworks in networked and distributed control systems.[15]

Furthermore, robust stability conditions are frequently established using the LMI framework, which has been widely adopted for delay-dependent control and estimation design.[16] Unlike robust control, robust fault detection (RFD) aims to distinguish between the effects of faults, parametric uncertainties, and external disturbances. Consequently, the performance of an RFD scheme should be assessed in terms of robustness (disturbance/uncertainty attenuation) and fault sensitivity.

Numerous physical processes exhibit time delays and uncertainty; a suitable representation for such processes is an uncertain neutral system with state delay. Since delays in both the state and its derivative frequently degrade performance and may compromise system stability, there has been growing interest in designing robust fault-detection filters (RFDFs) for linear, uncertain, neutral, state-delayed systems.[17–19] LMIs have been widely employed for RFDF synthesis in neutral uncertain systems.[20–22] However, the performance indices commonly used to define the reference residual model for retarded systems do not directly extend to neutral systems due to simultaneous delays in the state and its derivative. As a result, the RFDF design that simultaneously accounts for time delay, exogenous disturbances, and parametric uncertainty remains relatively underexplored.

This work addresses that gap by developing an H∞ based on the RFDF for a class of linear neutral time-delay systems subject to exogenous disturbances and parameter uncertainties. Our contributions are threefold:

- We synthesized a residual generator that is robust against structured parametric uncertainty and unknown inputs while preserving fault sensitivity;
- We developed open-loop stability conditions using a Lyapunov–Krasovskii functional and derived convex LMI constraints; and,
- We formulated an H∞ optimization that yields residuals selectively sensitive to faults affecting the system under study.

The remainder of this paper is organized as follows: Section 2 formulates the problem and system model. Section 3 presents the RFDF design. Section 4 illustrates the proposed approach through a simulation example. Section 5 draws the conclusions.

2. Framework and problem formulation

2.1. Framework

Time delays arise from sensing, actuation, and transport phenomena and are ubiquitous across engineered and natural systems. In our setting, they primarily reflect measurement/actuation latencies and transport lags. In addition, delays occur, particularly when a phenomenon related to the transport of matter, energy, or information is present.[23,24]

The manipulation of physical processes requires a modeling phase. These mathematical models are viewed as sets of equations that reflect subjective understanding of the mechanisms. Thus, they can be seen as a procedure for identifying the behavior of the system's inputs/outputs, which depends on parameters whose values are often poorly known or vary over time.[25–27] The obtained models did not provide a perfect representation of reality. The parameters characterizing a system cannot always be identified with exactitude. However, during the modeling phase, uncertainties naturally arise from sensor imprecision and model errors.[28–30]

The parameter uncertainties, whose origins are multiple (ignorance of certain system parameters, poorly accounted-for dynamics, linearization of the physical equations, parameter fluctuations, model complexity), can affect system dynamics. However, it is necessary to take these different factors into account when modeling.

In the literature, numerous works address the synthesis of linear uncertain neutral time-delay systems, but the differences lie in the types of delays.

2.2. Problem formulation

Consider a class of uncertain neutral systems with time delays, which is defined by the following state-space **Equation 1**:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (A_h + \Delta A_h)x(t-h) + \\ (A + \Delta A_d)\dot{x}(t-d) \quad + (B + \Delta B)u(t) + \\ \qquad B_d d(t) + B_f f(t) \\ \qquad y(t) = Cx(t) + D_f f(t) + D_d d(t) \\ \qquad x(t) = \theta(t) \quad ; \; t \in [-k, 0] \end{cases} \tag{1}$$

where x(t)$\in R^n$ is the state vector, u(t)$\in R^m$ is the input vector, y(t)$\in R^p$ indicates the output vector. A, A_h, A_d, B, and C are known constant matrices with appropriate dimensions. K = max {h, d} and $\theta(t) \in R^n$ is a continuous initial function. While both h $>$ 0 and d $>$ 0 indicate, respectively, the state and its derivative, constant time-delays. d(t)$\in R^{m_l}$ denotes the disturbance input, and f(t)$\in R^l$ represents the fault vector. In addition, B_f, B_d, D_f, and Dd are well-known matrices.

Thus, the argument Δ of uncertain matrices depends on the time t with ΔA, ΔA_h, ΔA_d, and ΔB. The uncertain parameter terms are given as **Equation 2**:

$$\begin{bmatrix}\Delta A & \Delta A_h & \Delta A_d & \Delta B\end{bmatrix} = [E_1 \sum_1 F_1 \tag{2}$$

$$E_2 \sum\nolimits_2 F_2 \quad E_3 \sum\nolimits_3 F_3 \quad E_4 \sum\nolimits_4 F_4]$$

where E_1, E_2, E_3, E_4, F_1, F_2, F_3, and F_4 are known constant matrices. which specify how the uncertain parameters in $\sum_1$, $\sum_2$, $\sum_3$, and $\sum_4$ enter the nominal matrices A, A_h, A_d, and B.

$\sum_1$, $\sum_2$, $\sum_3$, and $\sum_4$, which are real unknown matrices with Lebesgue measurable elements, may vary over time, and satisfy$\sum_1^T \sum_1 \leq I$, $\sum_2^T \sum_2 \leq I$,$\sum_3^T \sum_3 \leq I$, and $\sum_4^T \sum_4 \leq I$.

We model parametric uncertainty by adding structured perturbations to the nominal state-space matrices. The uncertainty blocks are norm-bounded and enter multiplicatively, yielding an LMI-amenable representation.

As mentioned above, the considered system describes the delay impacts (that is, the values of the past state x(t−h) and its derivative $\dot{x}(t-d)$.

This model accounts for parameter uncertainties. As a result, the structural complexity of this class of systems requires a specific analysis using mathematical tools (for example, LMIs[31]). During this work, we are interested in the stability analysis of **Equation 2** with/without structured parameter uncertainties ΔA, ΔA_h, ΔA_d, and ΔB. In the literature, the analysis of neutral systems with time delays is a recent topic and is little studied.[1]

3. A new architecture for robust fault detection issue

The proposed RFD architecture, dedicated to a class of uncertain neutral systems with time delays in both states and state derivatives simultaneously, consists of three steps, described as follows:

Step 1. Synthesis of a model-based fault detection filter: This filter is designed to generate fault indicators (residual signals) that are sensitive to faults and disturbances, and take into account delayed states and their derivatives.

Step 2. Design of the fault detection reference filter: This is synthesized under nominal conditions, assuming that the structured parametric uncertainties are 0.

Step 3. Design of the RFDF: The scheme is based on an augmented system subject to unknown inputs, and the filter generates a robust residual signal in their presence (**Figure 1**).

Details of the proposed RFD scheme are included in the next section.

3.1. Fault detection filter synthesis

We adopt a two-filter architecture (reference and residual-generating blocks) **Equation 1** to decouple disturbance effects and amplify fault signatures. The synthesis problem is posed in state space with decision variables collected in the filter gain matrices (**Equation 3**):[2]

$$\begin{cases} \dot{\widehat{x}}(t) = A\widehat{x}(t) + A_h\widehat{x}(t-h) + A_d\dot{\widehat{x}}(t-d) + \\ Bu(t) + H(y(t) - \widehat{y}(t)) \\ \qquad \widehat{y}(t) = C\widehat{x}(t) \\ \qquad r(t) = \; V\left[y(t) - \widehat{y}(t)\right] \end{cases} \tag{3}$$

In this instance,$\widehat{x}(t) \in R^n$, $\widehat{y}(t) \in R^q$ represent, respectively, the estimated state and output vectors. The state error e(t) is determined as though the difference between the actual value of a system x(t) and the desired value $\widehat{x}(t)$. Then, the relation is given as e(t) = x(t) - $\widehat{x}(t)$. Thereafter, the residual signal form r(t) is introduced by **Equation 4**:

$$r(t) = V\left[y(t)) - \widehat{y}(t)\right] \tag{4}$$

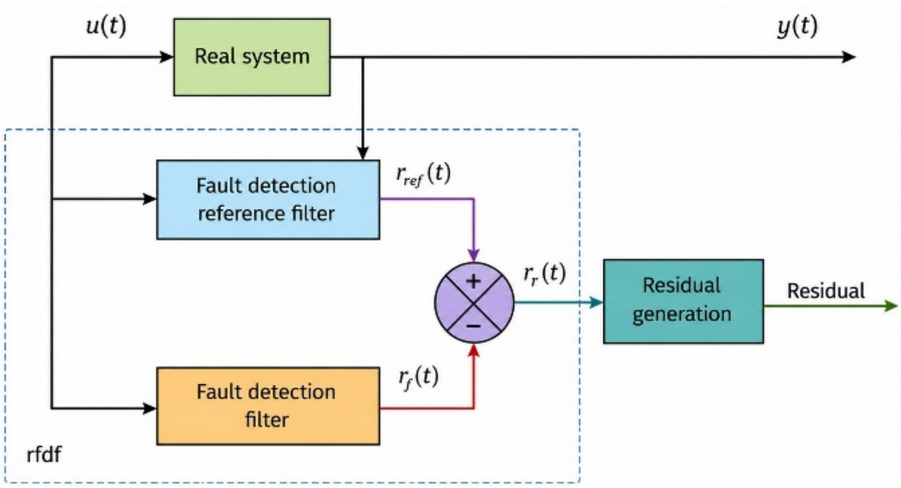

Figure 1. New scheme for robust fault detection
Abbreviation: RFDF: Robust fault detection filter.

where a weighting matrix V is necessary to speed up the convergence of the fault indicators. While the output error y(t)$-\widehat{y}(t)$ herein indicates a fault indicator. Both matrices H and V are the observer gain and the weight related to the residual generator r(t), respectively.

The state reconstruction error dynamic $\dot{e}(t)$ is defined by **Equation 5**:

$$\dot{e}(t) = \dot{x}(t) - \dot{\widehat{x}}(t) \tag{5}$$

This should be asymptotically stable. Using **Equations 1–3**, this dynamic state error can be rewritten as **Equation 6**:

$$\begin{aligned}\dot{e}(t) &= (A - HC)e(t) + A_h e(t-h) + A_d \dot{e}(t-d)\\ &+\Delta A x(t) + \Delta A_h x(t-h) + \Delta A_d x(t-d)\\ &+\Delta B u(t) + (B_d - HD_d)d(t) + (B_f - HD_f)f(t)\end{aligned} \tag{6}$$

The residual signal is obtained in the following manner (**Equation 7**):

$$r(t) = VCe(t) + VD_d d(t) + VD_f f(t) \tag{7}$$

Thus, the equation system is obtained (**Equation 8**):

$$\begin{cases}\dot{e}(t) = (A - HC)e(t) + A_h e(t-h)\\ +A_d \dot{e}(t-d)\\ +\Delta A x(t) + \Delta A_h x(t-h))\\ +\Delta A_d \dot{x}(t-d) + \Delta B u(t)\\ +(B_d - HD_d)d(t) + (B_f - HD_f)f(t)\\ r(t) = VCe(t) + VD_d d(t) + VD_f f(t)\end{cases} \tag{8}$$

According to **Equation 6**, the residual response is a function not only of the fault vector f(t) and the perturbation d(t) but also depends on the state vector x(t), the delayed state x(t-h), and the delayed-state derivative $\dot{x}(t-d)$.

Nevertheless, the typical RFDF designed in previous studies cannot easily lead to finding a filter expression in terms of e(t), e(t−h), and $\dot{e}(t-d)$.[32,33]

To overcome common difficulties with neutral uncertain systems with time-delays, the issue can thus be reformulated, taking into account model-matching problems H∞. In fact, the solution consists of generating an ideal reference residual, denoted $r_{ref}(t)$, and minimizing the error between the residual signal obtained from the fault detection filter, r(t), and the reference residual signal $r_{ref}(t)$.

Noting that, owing to a reference filter devoted to the fault detection target, the design of an RFDF dedicated to neutral delay systems can be readily achieved.

3.2. Design of the reference fault detection filter

In the sequel, an appropriate reference fault detection filter should be developed to synthesize a pertinent RFDF for neutral systems with time delays and parameter uncertainties. The fault detection algorithms must improve the system's monitorability.

In this case, we suppose that the structured parameter uncertainties are equal to 0, ΔA=0, ΔA_h=0, ΔA_d=0, and ΔB=0.

The reference filter associated with **Equation 3** is obtained in this form (**Equation 9**):

$$\begin{cases}\dot{e}_{ref}(t) = (A - H_r C)e_{ref}(t) + A_h e_{ref}(t-h)\\ +A_d \dot{e}_{ref}(t-d) + (B_d - H_r D_d)d(t)+\\ (B_f - H_r D_f)f(t)\\ r_{ref}(t) = V_r C e_{ref}(t) + V_r D_d d(t) + V_r D_f f(t)\\ e_{ref}(t) = 0 \quad (t \le 0)\end{cases} \tag{9}$$

With $e_{ref}(t)$ and $r_{ref}(t)$ denote the reference error vector and the reference residual signal, respectively. H_r and V_r are the reference filter gains. These latter are calculated in order to the term $(A - H_r C)$ guarantees asymptotic stability.

The design targets high sensitivity to the fault vector while attenuating unknown inputs and parametric variations. This is enforced through the weighting in the LMI conditions.

If the reference filter is wrongly chosen, the reference residual signal $r_{ref}(t)$ is unable to describe the desired behavior of the fault indicator r(t). For this reason, false alarms will appear.

The reference filter accounts for the effects of external disturbances on the reference residual signal. For this, it is necessary to minimize the disturbance's influence and maximize the fault's effect.

According to previous studies, the reference filter is defined as follows (**Equation 10**):[34–36]

$$r_{ref}(s) = \omega_f\ f(s) + \omega_d\ d(s) \tag{10}$$

where $r_{ref}(s)$ indicates the Laplace transformation of the reference residual signal, $\omega_f(s) \in \mathbb{R}H_\infty$ and $\omega_d(s) \in \mathbb{R}H_\infty$ represent, respectively, the fault and disturbance vectors. The latter are completely decoupled. Subsequently, this residue can satisfy the desired performances.

The robustness of the reference residual signal with respect to disturbances and the fault sensitivity is determined via **Equation 11**:

$$J_f = ||G_{rf}^{ref}||_\infty - ||G_{rd}^{ref}||_\infty \tag{11}$$

with $||G_{rf}^{ref}||_\infty$ and $||G_{rd}^{ref}||_\infty$ represent, respectively, the transfer functions from the fault $f(t)$ and the disturbance $d(t)$ to the reference residual $r_{ref}(t)$.

Then, to maximize the performance index J_f, **Equation 12** must be verified:

$$\begin{cases} ||G_{rf}^{ref}||_\infty \to \max \\ ||G_{rd}^{ref}||_\infty \to \min \end{cases} \tag{12}$$

Consequently, the fault reference filter in **Equation 9** is designed to account for the performance index in **Equation 11**. For simplicity, we assume that two matrices, L and R, are introduced to select appropriate inputs/outputs for channels or channel combinations, as used in reference.[37]

We consider the following transfer function (**Equation 13**):

$$G \triangleq LG_{r_v}^{ref} R = L \begin{bmatrix} G_{rf}^{ref} & G_{rd}^{ref} \end{bmatrix} R \tag{13}$$

with $L \in R^{q\times q}$ and $R \in R^{2l\times l}$. In addition, the selection of β and α scalars can be determined as $\beta > \alpha > 0$ and verified this condition,

$L = I_{q\times q}$, $R = \begin{bmatrix} I_{l\times l} \\ -I_{l\times l} \end{bmatrix}$, we obtain **Equation 14**:

$$\begin{cases} ||G_{rf}^{ref} - G_{rd}^{ref}||_\infty \geq ||G_{rf}^{ref}||_\infty - ||G_{rd}^{ref}||_\infty \\ ||G||_\infty > \beta - \alpha \Rightarrow ||G_{rf}^{ref}||_\infty > \beta \\ and \quad ||G_{rd}^{ref}||_\infty < \alpha \end{cases} \tag{14}$$

where G is the expression of the reference filter (**Equation 15**):

$$G \begin{cases} \dot{e}_{ref}(t) = (A - H_rC)e_{ref}(t) + A_h e_{ref}(t-h) + \\ A_d \dot{e}_{ref}(t-d) + B_1 R v(t) \\ r_{ref}(t) = LV_rCe_{ref}(t) + LD_1Rv(t) \end{cases} \tag{15}$$

where $B_1 = \begin{bmatrix} B_f - H_rD_f & B_d - H_rD_d \end{bmatrix}$, $D_1 = \begin{bmatrix} V_rD_f & V_rD_d \end{bmatrix}$ and $v = \begin{bmatrix} f^T & d^T \end{bmatrix}$.

Thus, the reference residual model in **Equation 10** can be constructed by defining the following optimization problem (**Equation 16**):

$$\begin{array}{l} \max_{\alpha,\beta,H_r,V_r}(\beta-\alpha) \\ constraints\ (14)\ \text{and}\ (15) \end{array} \tag{16}$$

To determine the reference filter model, it is necessary to find the parameters related to H_r and V_r .

The theorem below enables the determination of the two gains that essentially constitute the reference fault detection filter.

Theorem 1. *Consider the reference fault detection filter in **Equation 9** and the associated augmented error dynamics. Suppose there exist symmetric positive-definite matrices Ξ, T, Y, Z, and scalars $\beta > \alpha > 0$ together with a tuning parameter $k > 0$ such that the following matrix inequality holds:*

$$\begin{bmatrix} \Phi_1 & 0 & A_dY & \Phi_2 & \Xi A^T - FC^T & \Xi & A_hY \\ * & -T & 0 & 0 & TA_h^T & 0 & 0 \\ * & * & -Y & 0 & YA_d^T & 0 & 0 \\ * & * & * & -\Phi_3 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{(1+h)}Y & 0 & 0 \\ * & * & * & * & * & -T & 0 \\ * & * & * & * & * & * & -\frac{1}{h}Y \end{bmatrix} \prec 0 \tag{17}$$

where

$$\Phi_1 = A\Xi + A_h\Xi + \Xi A^T + \Xi A_h^T - C^TZC - FC - C^TF^T - T$$

$$\Phi_2 = (B_f + B_d) + F(D_f + D_d) + C^TZ(D_f + D_d)$$

$$\Phi_3 = (\beta - \alpha)I + (D_f + D_d)^TZ(D_f + D_d)$$

The parameters of the reference fault detection filter H_r and V_r are substituted, respectively, by $\Xi^{-1}Y$ and $Z^{1/2}$.

Proof. The proof of this theorem relies on an adapted Lyapunov–Krasovskii functional. The main idea of this theorem is therefore to determine a positive definite functional $V(e_{ref}(t), t)$ whose derivative along the trajectories is negative definite. □

Consider the following Lyapunov–Krasovskii functional (**Equation 18**):

$$V(e_{ref}, t) = e_{ref}^T(s)Pe_{ref}(s) + \int_{t-h}^{t} e_{ref}^T(s) Qe_{ref}(s)ds$$

where P, Q and S are positive definite, symmetric matrices.

Consider the following index (**Equation 19**):

$$J = \int_0^\infty r_{ref}^T r_{ref} dt - (\beta-\alpha)^2 \int_0^\infty v^T v dt$$
$$= \int_0^\infty \left[r_{ref}^T r_{ref} - (\beta-\alpha)^2 v^T v - \dot{V}_1(e_{ref}, t) \right] dt$$

$V(e_{ref},t)|_{t=0} = 0$under zero initial condition and $V(e_{ref},t)|_{t=\infty} \geq 0$, by substituting the time derivative of $V(e_{ref},t)$ along the solution of **Equation 18** is yielded by **Equation 20**:

$$\dot{V}(e_{ref},t) = \dot{e}_{ref}^T(t) P e_{ref}(t) + {e_{ref}}^T(t) P \dot{e}_{ref}(t)$$
$$+ {e_{ref}}^T(t) Q e_{ref}(t) - {e_{ref}}^T(t-h) Q e_{ref}(t-h)$$

By substituting **Equation 9** and the time derivative expression in **Equation 20**, we obtain **Equation 21**:

$$\begin{gathered}
\dot{V}(e_{ref},t) = \\
e_{ref}^T(t)\left[P(A - H_rC + A_h) + \right. \\
\left. (A - H_rC + A_h)^T P + Q + hPA_hS^{-1}A_h^TP\right] \\
e_{ref}(t) \\
+2{e_{ref}}^T(t)PA_d\dot{e}_{ref}(t-d) + 2{e_{ref}}^T(t)P \\
(B_f - H_rD_f)f(t) \\
2{e_{ref}}^T(t)P(B_d - H_rD_d)d(t) - {e_{ref}}^T(t-h) \\
Qe_{ref}(t-h) + \\
{e_{ref}}^T(t)(A - H_rC)^T\Psi S(A - H_rC)e_{ref}(t) \\
+2{e_{ref}}^T(t)(A - H_rC)^T\Psi SA_h e_{ref}(t-h) \\
+2{e_{ref}}^T(t)(A - H_rC)^T\Psi SA_d\dot{e}_{ref}(t-d) + \\
2{e_{ref}}^T(t)(A - H_rC)^T\Psi S(B_f - H_rD_f)f(t) \\
+2{e_{ref}}^T(t)(A - H_rC)^T\Psi S(B_d - H_rD_d)d(t) \quad (21) \\
+2{e_{ref}}^T(t-h)A_h^T\Psi SA_d\dot{e}_{ref}(t-d) \\
+2{e_{ref}}^T(t-h)A_h^T\Psi S(B_f - H_rD_f)f(t) + \\
2{e_{ref}}^T(t-h)A_h^T\Psi S(B_d - H_rD_d)d(t) \\
+{e_{ref}}^T(t-h)A_h^T\Psi SA_h e_{ref}(t-h) + \\
2\dot{e}_{ref}^T(t-d)A_d^T\Psi S(B_f - H_rD_f)f(t) \\
+2\dot{e}_{ref}^T(t-d)A_d^T\Psi S(B_d - H_rD_d)d(t) \\
+\dot{e}_{ref}^T(t-d)A_d^T\Psi SA_d\dot{e}_{ref}(t-d) \\
+d^T(B_d - H_rD_d)\Psi S(B_f - H_rD_f)f(t) + \\
d^T(t)(B_d - H_rD_d)\Psi S(B_d - H_rD_d)d(t) \\
+f^T(t)(B_f - H_rD_f)^TS(B_f - H_rD_f)f(t) \\
+f^T(t)(B_f - H_rD_f)^TS(B_d - H_rD_d)d(t) \\
-\dot{e}_{ref}^T(t-d)S\dot{e}_{ref}(t-d)
\end{gathered}$$

The reference residual is given by **Equation 22**:

$$r_{ref}(t) = LV_rCe_{ref}(t) + LD_1Rv(t) \quad (22)$$

The term $r_{ref}^T(t)r_{ref}(t)$ is computed as **Equation 23**:

$$\begin{gathered}
r_{ref}^T(t)r_{ref}(t) = \\
[V_rCe_{ref}(t) + V_rD_dd(t) + V_rD_ff(t)]^T \times \\
[V_rCe_{ref}(t) + V_rD_dd(t) + V_rD_ff(t)] \\
= e_{ref}^T(t)C^TZCe_{ref}(t) + e_{ref}^T(t)\left[C^TZD_f\right. \quad (23) \\
\left. -C^TZD_d\right]v(t) + v^T(t)\left[D_f^TZC - D_d^TZC\right]e_{ref}(t) \\
+v^T(t)\left[D_f^TZD_f - D_f^TZD_d - D_d^TZD_f - D_d^TZD_d\right] \\
v(t).
\end{gathered}$$

Finally, the equality in **Equation 19** may be specified in the form of a matrix as presented in **Equation 24**:

$$J = \int_0^\infty \left\{\psi^T \delta \psi\right\} dt$$

Consider $diag(\Xi, T, Y, I, Y)$, Schur complement can be applied (**Equation 25**):

$$\begin{bmatrix}
\Phi_1 & 0 & A_dY & \Phi_2 & \Xi A^T - FC^T & \Xi & A_hY \\
* & -T & 0 & 0 & TA_h^T & 0 & 0 \\
* & * & -Y & 0 & YA_d^T & 0 & 0 \\
* & * & * & -\Phi_3 & 0 & 0 & 0 \\
* & * & * & * & -\frac{1}{(1+h)}Y & 0 & 0 \\
* & * & * & * & * & -T & 0 \\
* & * & * & * & * & * & -\frac{1}{h}Y
\end{bmatrix} \prec 0 \quad (25)$$

where

$\Phi_1 = A\Xi + A_h\Xi + \Xi A^T + \Xi A_h^T - C^TZC - FC - C^TF^T - T_h^T$ $\Phi_2 = (B_f + B_d) + F(D_f + D_d) + C^TZ(D_f + D_d)$

$\Phi_3 = (\beta-\alpha)I + (D_f + D_d)^TZ(D_f + D_d)$

The LMIs of **Equation 25** can be verified when the preceding J criterion is positive definite. Moreover, if we choose the vector $v(t) = 0$ which depends on faults and disturbances, the derivative of the functional $V(e_{ref},t)$ is specified in **Equation 26**:

$$\begin{bmatrix}
G(\Xi, H_r) & 0 & A_d^TY & \Xi A - FC & \Xi & A_h^TY \\
* & -T & 0 & TA_h & 0 & 0 \\
* & * & -Y & YA_d & 0 & 0 \\
* & * & * & -\frac{1}{(1+h)}Y & 0 & 0 \\
* & * & * & * & -T & 0 \\
* & * & * & * & * & -\frac{1}{h}Y
\end{bmatrix} \prec 0 \quad (26)$$

3.3. Robust fault detection filter design

Let the uncertain neutral time-delay system represented in **Equation 1** rewritten in **Equation 27**:

$$\begin{cases}
\dot{x}(t) = (A + \Delta A)x(t) + (A_h + \Delta A_h)x(t-h) \\
+(A_d + \Delta A_d)\dot{x}(t-d) + (B + \Delta B)\omega(t) \quad (27) \\
y(t) = Cx(t) \\
x(t) = \theta(t) \quad ; \ t \in [-k, 0]
\end{cases}$$

With $\omega(t) = \begin{bmatrix} u(t) \\ f(t) \\ d(t) \end{bmatrix}$.

The vectors x(t), u(t), y(t), d(t), and f(t) are respectively, the state, input vectors, output, disturbances and faults. The theorem below offers a sufficient condition ensuring the asymptotic stability of the system and satisfying the norm H∞.

Theorem 2. *Consider the uncertain linear neutral-type delay system in* ***Equation 27****. Given a prescribed scalar $\gamma > 0$, the system is asymptotically stable, and, under a zero initial condition, the performance $\|y(t)\|_2 < \gamma \|\omega(t)\|_2$ holds if there exist symmetric positive-definite matrices X_1, X_2, X_3, Y, and T, together with positive scalars ε_1, ε_2, ε_3, ε_4, and $h > 0$, such that the following LMI is feasible (****Equation 28****):*

$$[M_{ij}]_{12\times 12} < 0 \tag{28}$$

The nonzero blocks of M_{ij} are specified as:

$M_{11} = AX_1 + A_hX_1 + X_1A^T + X_1A_h^T + \varepsilon_1 F_1^T F_1$
$M_{13} = A_dX_3$, $M_{14} = B$, $M_{15} = X_1A^T$
$M_{16} = X_1C^T$, $M_{17} = X_1$, $M_{18} = A_h^T X_3$
$M_{19} = X_1E_1$, $M_{1,10} = X_1E_2$,
$M_{1,12} = X_1E_4$, $M_{1,11} = X_1E_3$
$M_{22} = -X_2 + \varepsilon_2 F_2^T F_2$, $M_{25} = X_2A_h^T$
$M_{33} = -X_3 + \varepsilon_3 F_3^T F_3$, $M_{35} = X_3A_d^T$
$M_{44} = -\gamma I + \varepsilon_4 F_4^T F_4$, $M_{45} = B^T$
$M_{55} = -\frac{1}{1+h}Y$, $M_{66} = -I$
$M_{77} - T$, $M_{88} = -\frac{1}{h}Y$, $M_{99} = -\varepsilon_1 I$
$M_{10,10} = -\varepsilon_2 I$, $M_{11,11} = -\varepsilon_3 I$
$M_{12,12} = -\varepsilon_4 I$
and $M_{ij} = 0$ otherwise. All matrices are of compatible dimensions and $(A,\ A_h, A_d, B, C, E_l, F_l,\ l = 1, \ldots, 4)$ are known data.

Proof. Consider the Lyapunov–Krasovskii functional defined by **Equation 29**:

$$V(x,t) = x^T(s)P_1x(s) + \int_{t-h}^{t} x^T(s)Q_1x(s)ds + \int_{t-a}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\,ds + \int_{-h}^{0}\left(\int_{t+\alpha}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\,ds\right)d\alpha \tag{29}$$

where P_1, Q_1, and Q_2 positive definite matrices. The index J_1 can be written as **Equation 30**:

$$J_1 = \int_0^\infty y^Ty - (\beta-\alpha)^2\omega^T\omega dt \tag{30}$$

□

Then, this criteria is rephrased. To admit that $V(x,t)_{|t=0} = 0$ and $V(x,t)_{|t=\infty} = 0$, we get **Equation 31**:

$$J_1 \le \int_0^\infty \left[y^Ty - (\beta-\alpha)^2\omega^T\omega - \dot{V}_2(x,t)\right] dt \tag{31}$$

Then the dynamics of $V(x,t)$ is given by **Equations 32** and **33**:

$$\begin{aligned}\dot{V}(x,t) &= 2\dot{x}^T(t)P_1x(t) + x^T(t)Q_1x(t) - x^T(t-h)Q_1x(t-h) + \dot{x}^T(t)Q_2\dot{x}(t) \\ &\quad -\dot{x}^T(t-d)Q_2\dot{x}(t-d) + \int_{-h}^{0}\left[\dot{x}^T(t)Q_2\dot{x}(t) - \dot{x}^T(t+\alpha)Q_2\dot{x}(t+\alpha)\right]d\alpha \\ &= 2\dot{x}^T(t)Px(t) + x^T(t)Q_1x(t) - x^T(t-h)Q_1x^T(t-h) + (1+h)\dot{x}^T(t)Q_2\dot{x}(t) \\ &\quad -\dot{x}^T(t-d)Q_2\dot{x}(t-d) - \int_{-h}^{0}\dot{x}^T(t+\alpha)S\dot{x}(t+\alpha)d\alpha \\ &= x^T(t)(P_1(\bar{A}+\bar{A}_h) + (\bar{A}+\bar{A}_h)^TP_1 + Q_1)x(t) \\ &\quad -2x^T(t)P_1\int_{-h}^{0}\bar{A}_h\dot{x}(t+\alpha)d\alpha + 2x^T(t)P_1\bar{A}_d\dot{x}(t-d) \\ &\quad +2x^T(t)P_1\bar{B}u(t) - x^T(t-h)Q_1x^T(t-h) \\ &\quad +(1+h)\dot{x}^T - \dot{x}^T(t-d)Q_2\dot{x}(t-d) - \\ &\quad \int_{-h}^{0}\dot{x}^T(t+\alpha)Q_2\dot{x}(t+\alpha)d\alpha - 2x^T(t)P_1\int_{-h}^{0}\bar{A}_h\dot{x}(t+\alpha)d\alpha\end{aligned} \tag{32}$$

$$\begin{aligned}&= -\int_{-h}^{0}2x^T(t)P_1\bar{A}_h\dot{x}(t+\alpha)d\alpha \\ &\le \int_{-h}^{0}x^T(t)P_1\bar{A}_hQ_2{}^{-1}\bar{A}_h^TP_1x(t) + \dot{x}^T(t+\alpha)Q_2\dot{x}(t+\alpha)d\alpha \\ &\le hx^T(t)P_1\bar{A}_hQ_2{}^{-1}\bar{A}_h^TP_1x(t) + \int_{-h}^{0}\dot{x}^T(t+\alpha)Q_2\dot{x}(t+\alpha)d\alpha \\ &= x^T(t)\left[P_1(\bar{A}+\bar{A}_h) + (\bar{A}+\bar{A}_h)^TP_1 + Q_1 + hP_1\bar{A}_hQ_2{}^{-1}A_h^TP_1\right]x(t) + 2x^T(t)P_1\bar{A}_d\dot{x}(t-d) \\ &\quad +2x^T(t)P_1\bar{B}u(t) - x^T(t-h)Q_1x(t-h) \\ &\quad +x^T(t)\bar{A}^T(1+h)Q_2\bar{A}x(t) + 2x^T(t)\bar{A}^T(1+h)Q_2\bar{A}_hx(t-h) \\ &\quad +2x^T(t)\bar{A}^T(1+h)Q_2\bar{A}_d\dot{x}(t-d) + 2x^T(t)\bar{A}^T(1+h)Q_2\bar{B}u(t) \\ &\quad +2x^T(t-h)\bar{A}_h^T(1+h)Q_2\bar{A}_d\dot{x}(t-d) \\ &\quad +2x^T(t-h)\bar{A}_h^T(1+h)Q_2\bar{B}u(t) \\ &\quad +x^T(t-h)\bar{A}_h^T(1+h)Q_2\bar{A}_hx(t-h) + 2\dot{x}^T(t-d)\bar{A}_d^T(1+h)Q_2\bar{B}u(t) \\ &\quad +\dot{x}^T(t-d)\bar{A}_d^T(1+h)Q_2\bar{A}_d\dot{x}(t-d) + u^T(t)\bar{B}^T(1+h)Q_2\bar{B}u(t) \\ &\quad -\dot{x}^T(t-d)Q_2\dot{x}(t-d)\end{aligned} \tag{33}$$

where $y^Ty = x^T(t)C^TCx(t)$, then we can calculate the following expression as **Equation 34**:

$$y^Ty - (\beta-\alpha)^2\omega^T\omega - \dot{V}_2(x,t) =$$

$$\begin{bmatrix} M(X_1,\bar{A},\bar{A}_h) & 0 & \bar{A}_dX_3 & \bar{B} \\ * & -X_2 & 0 & 0 \\ * & * & -X_3 & 0 \\ * & * & * & I \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} X_1\tilde{A}^T & X_1C^T & X_1 & \tilde{A}_hX_3 \\ X_2\tilde{A}_h^T & 0 & 0 & 0 \\ X_3\tilde{A}_d^T & 0 & 0 & 0 \\ -\frac{1}{1+h}Y & 0 & 0 & 0 \\ * & -I & 0 & 0 \\ * & * & -X_2 & 0 \\ * & * & * & -\frac{1}{h}X_3 \end{bmatrix} < 0 \tag{34}$$

Lemma 1. (Equation 35).

$$\Omega = \Omega_1 + \Omega_1 * diag\{\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4\} * diag\{F_1,F_2,F_3,F_4\} * diag\{F_1,F_2,F_3,F_4\} + \mathrm{diag}\{F_1^T,F_2^T,F_3^T,F_4^T\} * \mathrm{diag}\{\Sigma_1^T,\Sigma_2^T,\Sigma_3^T,\Sigma_4^T\}\Omega\frac{T}{2} \tag{35}$$

where **Equation 36**:

$$\Omega = \begin{bmatrix} M_{11} & 0 & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\ * & M_{22} & 0 & 0 & M_{25} & 0 & 0 & 0 \\ * & * & M_{33} & 0 & M_{35} & 0 & 0 & 0 \\ * & * & * & M_{44} & M_{45} & 0 & 0 & 0 \\ * & * & * & * & M_{55} & 0 & 0 & 0 \\ * & * & * & * & * & M_{66} & 0 & 0 \\ * & * & * & * & * & * & M_{77} & 0 \\ * & * & * & * & * & * & * & M_{88} \end{bmatrix} \tag{36}$$

Lemma 2.
$E\Sigma F + F^T\Sigma^T E^T \leq E\Lambda E^T + F^T\Lambda F$
$E = \Omega_2$, $F = diag\{F_1,F_2,F_3,F_4\}$
$\Gamma = diag\{\varepsilon_1 I,\varepsilon_2 I,\varepsilon_3 I,\varepsilon_4 I\}$, $\Sigma = diag\{\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4\}$

This inequality warrant that $\dot{V}(x,t) < 0$. If $\omega(t) = 0$. LMI in **Equation 17** that satisfy the feasibility of the LMI in **Equation 34** this implicit that the system in **Equation 19** is asymptotically stable (**Equation 37**):

$$||y(t)||_2 < ||\omega(t)||_2 \tag{37}$$

Therefore, the augmented system corresponds to the linear uncertain system with neutral time-delays and it is described by **Equation 38**:

$$\begin{cases} \dot{X}_F = (A_F + \Delta A_F)X_F + (A_{h_F} + \Delta A_{h_F}) \\ X_F(t-h) + \\ (A_{d_F} + \Delta A_{d_F})\dot{X}_F(t-d) + (B_F + \Delta B_F)\omega(t) \\ r_F = C_F X_F + D_F\omega(t) \end{cases} \tag{38}$$

where $\dot{X}_F = \begin{bmatrix} \dot{e} \\ \dot{e}_{ref} \\ \dot{x} \end{bmatrix}$, $\omega = \begin{bmatrix} u \\ f \\ d \end{bmatrix}$, $A_F = \begin{bmatrix} A-HC & 0 & 0 \\ 0 & A-H_r & 0 \\ 0 & 0 & \overline{A} \end{bmatrix}$, $A_{h_F} = \begin{bmatrix} A_h & 0 & 0 \\ 0 & A_h & 0 \\ 0 & 0 & \overline{A}_h \end{bmatrix}$, $A_{d_F} = \begin{bmatrix} A_d & 0 & 0 \\ 0 & A_d & 0 \\ 0 & 0 & \overline{A}_d \end{bmatrix}$, $B_F = \begin{bmatrix} 0 & B_f - HD_f & B_d - HD_d \\ 0 & B_f - H_rD_f & B_d - H_rD_d \\ \overline{B} & B_f & B_d \end{bmatrix}$, $C_F = \begin{bmatrix} VC & V_rC & 0 \end{bmatrix}$, $D_F = \begin{bmatrix} 0 & VD_f - V_rD_f & VD_d - V_rD_d \end{bmatrix}$, $A_F = E_{1F}\sum_1 F_{1F}$, $A_{h_F} = E_{2F}\sum_2 F_{2F}$, $A_{d_F} = E_{3F}\sum_3 F_{3F}$, $B_F = E_{4F}\sum_4 F_{4F}$, $E_{1F} = \begin{bmatrix} E_1 \\ 0 \\ E_1 \end{bmatrix}$, $E_{2F} = \begin{bmatrix} E_2 \\ 0 \\ E_2 \end{bmatrix}$, $E_{3F} = \begin{bmatrix} E_3 \\ 0 \\ E_3 \end{bmatrix}$, $E_{4F} = \begin{bmatrix} E_4 \\ 0 \\ E_4 \end{bmatrix}$, $F_{1F} = \begin{bmatrix} 0 & 0 & F_1 \end{bmatrix}$, $F_{2F} = \begin{bmatrix} 0 & 0 & F_2 \end{bmatrix}$, $F_{3F} = \begin{bmatrix} 0 & 0 & F_3 \end{bmatrix}$, $F_{4F} = \begin{bmatrix} 0 & 0 & F_4 \end{bmatrix}$

To calculate the gain H and the weighting matrix V associated with the robust filter for fault detection given by **Equation 8**, it is necessary to ensure the stability of the augmented system in **Equation 38**.

Theorem 3. *Consider the linear uncertain neutral time-delay system in **Equation 38**. Given a constant scalar $\gamma > 0$, the system is asymptotically stable and the $H\infty$ performance $\|r_F(t)\|_2 < \gamma\|\omega(t)\|_2$ holds if there exist symmetric positive definite matrices P, Q_1, Q_2, and $\Gamma = diag\{\varepsilon_1 I,\varepsilon_2 I,\varepsilon_3 I,\varepsilon_4 I\}$ and free matrices Y and V, such that the following LMI is satisfied (**Equation 39**):*

$$[N_{ij}]_{14\times 14} < 0 \tag{39}$$

The symmetric block entries are defined by:
$N_{11} = P_1A + A^TP_1 + P_1A_h + A_h^TP_1 - Y_1C - C^TY_1^T$
$N_{15} = P_1B_f - Y_1D_f$, $N_{16} = P_1B_d - Y_1D_d$
$N_{17} = C^TV^T$, $N_{1,10} = P_1A_d$, $N_{1,11} = P_1E_1$
$N_{1,12} = P_1E_2$, $N_{1,13} = P_1E_3$, $N_{1,14} = P_1E_4$
$N_{22} = P_2A + A^TP_2 + Q_2 - P_2H_rC - C^TH_r^TP_2$
$N_{25} = P_2B_f - P_2H_rD_f$, $N_{26} = P_2B_d - P_2H_rD_d$
$N_{27} = C^TV_r^T$, $N_{29} = P_2A_h$, $N_{2,10} = P_2A_d$
$N_{33} = P_3A + A^TP_3 + Q_3 + \varepsilon_1F_1^TF_1$, $N_{34} = P_3B$
$N_{35} = P_3B_f$, $N_{36} = P_3B_d$, $N_{39} = P_3A_h$
$N_{3,10} = P_3A_d$, $N_{3,11} = P_3E_1$, $N_{3,12} = P_3E_2$
$N_{3,13} = P_3E_3$, $N_{3,14} = P_3E_4$, $N_{44} = -\gamma^2I + \varepsilon_4F_4^TF_4$, $N_{55} = -\gamma^2I$
$N_{57} = D_f^TV^T - D_f^TV_r^T$, $N_{66} = -\gamma^2I$
$N_{67} = D_d^TV^T - D_d^TV_r^T$, $N_{77} = I$
$N_{88} = Q_2$, $N_{99} = -Q_1 + \varepsilon_2F_2^TF_2$
$N_{1010} = Q_2 + \varepsilon_3F_3^TF_3$, $N_{1111} = -\varepsilon_1I$
$N_{1212} = -\varepsilon_2I$, $N_{1313} = -\varepsilon_3I$
$N_{1414} = -\varepsilon_4I$,
$N_{ij} = 0$ otherwise.

Moreover, the observer gain and weighting are recovered by $H = P^{-1}Y$ and $V = Z^{1/2}$, respectively.

The fault detection phase is completed by the development of a reference filter, allowing the generation of the fault indicators r_{ref} sensitive only to the fault and robust with respect to the parametric uncertainties.

4. Modeling and analysis of two-stage recycle reactors

4.1. System description

The effectiveness and capabilities of the proposed fault detection algorithms will be demonstrated through analysis of a two-stage chemical reactor with delayed recycle streams. Both reactors are assumed to be continuous stirred tank reactors that are isothermal.[38–42] In industry, the recycling reactor is most frequently utilized. In addition to increasing conversion overall, it lowers reaction costs.

A reactor is a container or enclosed space for conducting and optimizing a chemical reaction and for converting materials in general. In addition, the recycling reactor is used when the reaction is self-catalyzed, when the reactor must operate under nearly isothermal conditions, or when selectivity is required. This type of reactor can also be applied to biochemical operations.

The schematic diagram of the two-stage reactor with recirculation is depicted in **Figure 2**.

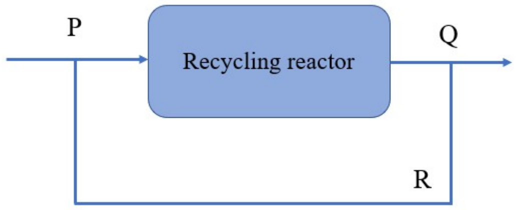

Figure 2. Principle of operation of a recycling reactor

Referring to the schematic diagram in **Figure 2**, the reused jet is removed at point Q and merged with the fresh feed at point P. The recycling parameter R is defined as the moles recycled per mole of product removed at point Q.

4.2. System modeling

Before applying the methods of fault detection, reactor modeling is crucial. For reactor modeling, consider an element of the reaction phases volume ΔV believed to be uniform in temperature and composition: a general form of a molar balance for any continuous species A_i continuous in a volume V, is presented as **Equation 40**:

$$\frac{dn_i}{dt} = F_{ie} - F_{is} + \delta_i \Delta V \quad \{i = 1......k\} \tag{40}$$

$\frac{dn_i}{dt}$ is the rate of accumulation of A_i in volume ΔV. F_{ie} and F_{is} are, respectively, the molar flow rates entering and leaving by convection and/or diffusion. V_i is the rate of production of the chemical species and is expressed in units of volume. The reactor parameter values are summarized in **Table 1**.

The mass balance equations for the reactor shown in **Figure 3** are deduced in **Equation 41**:

$$\begin{cases} V_1\dot{x}_1(t) = (1-R_2)x_2(t) + \frac{R_1}{R_2}\dot{x}_1(t-d) \\ -\frac{R_1}{R_2}\dot{x}_2(t-d) \\ -F_{p1}x_1(t) - V_1\delta_1 x_1(t) \\ V_2\dot{x}_2(t) = F_2 x_{2f}(t) + R_2 x_2(t-h) \\ +R_1 x_1(t-h) \\ -(R_1+R_2)\dot{x}_1(t-d) + R_2\dot{x}_2(t-d) - V_2\delta_2 x_2(t) \\ -F_2 x_2(t) \end{cases} \tag{41}$$

Equation 41 can be rewritten as **Equation 42** after simplification:

$$\begin{cases} \dot{x}_1(t) = \left(\frac{1}{\theta_1} + \delta_1\right) x_1(t) + \frac{(1-R_2)}{V_1} x_2(t) + \\ \frac{R_1}{R_2 \times V_1}\dot{x}_1(t-d) - \frac{R_1}{R_2 \times V_1}\dot{x}_2(t-d) \\ \dot{x}_2(t) = \left(\frac{1}{\theta_2} + \delta_2\right) x_2(t) + \frac{R_2}{V_2} x_2(t-h) + \\ \frac{R_1}{V_2} x_1(t-h) + \frac{F_2}{V_2} u(t) - \frac{(R_1+R_2)}{V_2}\dot{x}_1(t-d) \\ +\frac{R_2}{V_2}\dot{x}_2(t-h) \end{cases} \tag{42}$$

The state equation is defined as **Equation 43**:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) + Bu(t) \\ y(t) = Cx(t) \\ x_{t_0} = \Phi(\theta) \qquad with \quad [t_0 - h, t_0] \end{cases} \tag{43}$$

with x(t), x(t−h), $\dot{x}(t-d)$, and u(t) are the vector of state, delayed state, neutral state, and the control of second-stage reactor, respectively. y(t) is the output corresponding to the reactor outlet flow rate of the first stage. A, A_h, A_d, B, and C are matrices of appropriate dimensions given as follows:

$$A = \begin{bmatrix} -\left(\frac{1}{\theta_1} + \delta_1\right) & \frac{(1-R_2)}{V_1} \\ 0 & -\left(\frac{1}{\theta_2} + \delta_2\right) \end{bmatrix}, \quad A_h = \begin{bmatrix} 0 & 0 \\ \frac{R_1}{V_2} & \frac{R_2}{V_2} \end{bmatrix}, \quad A_d = \begin{bmatrix} \frac{R_1}{R_2 \times V_1} & -\frac{R_1}{R_2 \times V_1} \\ \frac{-(R_1+R_2)}{V_2} & \frac{R_2}{V_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{F_2}{V_2} \end{bmatrix}$$

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, D=0

5. Simulation results

Figure 4 shows the dynamic behavior of the reactor states, specifically the flow rates, under time delays of h = 3 and d = 1, which are held constant. The figure shows how states change over time, demonstrating the system's evolution under the given delays.

Table 1. Parameters of the two-stage recycling reactor

Symbol	Description	Value
R_1	Recycling flow rate 1	0.5
R_2	Recycling flow rate 2	0.5
θ_1	Residence time of stage 1	1
θ_2	Residence time of stage 2	1
δ_1	Reaction constant of stage 1	1
δ_2	Reaction constant of stage 2	1
F_i	Feed rate	1
V_1	Volume of reactor 1	1
V_2	Volume of reactor 2	1

Figure 5 illustrates how the system output changes with a nonzero time delay. Adding a delay d to the state derivative causes transient oscillations over (2.5–15) seconds before it reaches steady state, indicating the system's ability to suppress disturbances caused by delays.

Reactors are susceptible to faults and disturbances that can affect their performance. How to develop fault detection filters that are robust, as described in the study, to reduce the impact of the faults and maintain the reactor's reliable operation.

While the uncertain parameter characteristics are defined by:

$$\sum_1 = \sum_2 = \sum_3 = \sum_4 = 0.2$$

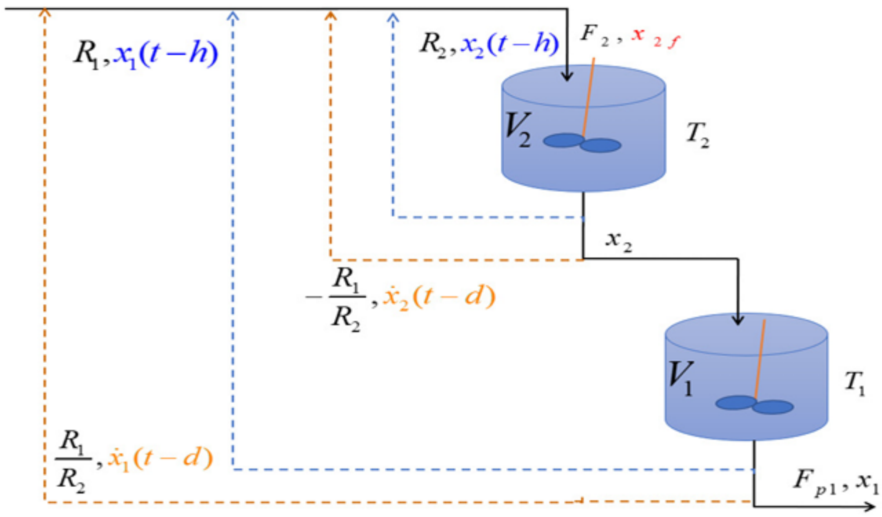

Figure 3. Two-stage chemical reactor with neutral time delay

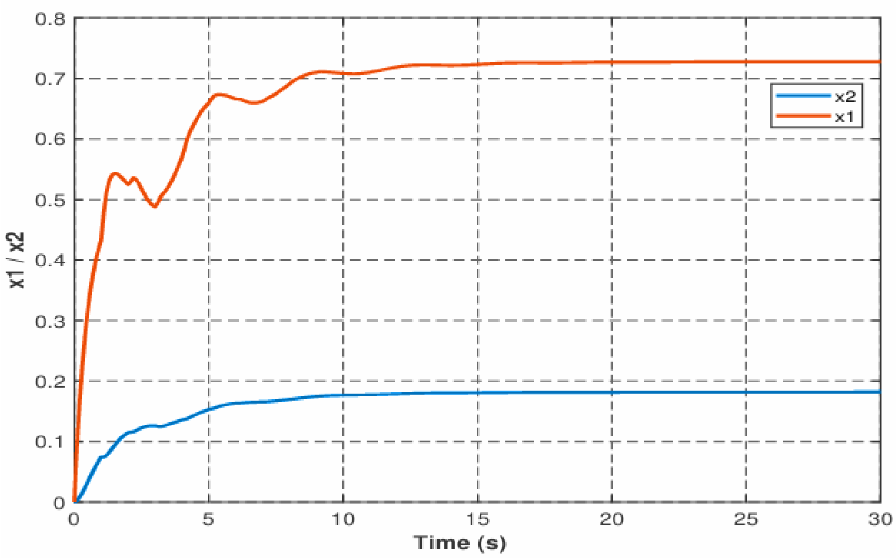

Figure 4. Evolution of the system states in normal operation

The fault occurs at t = 5 s and t = 7 s. Using **Theorem 1**, the reference residual model, given in **Equation 2**, has the observer gain H_r and the weighting matrix V_r as follows:

$$H_r = \begin{bmatrix} 1.5614 & 0.6391 \\ -0.5620 & 0.9493 \end{bmatrix}$$

$$V_r = \begin{bmatrix} 1.9691 & -1.0011 \\ -0.3588 & 1.6496 \end{bmatrix}$$

The value of the constant γ, determined by **Theorem 2**, is equivalent to 3.5195. Based on the LMI's resolution, we can confirm that the disparity $\|z\|_2 < \gamma \|\omega\|_2$ is satisfied. According to the third theorem, the H∞ It is possible to solve the RFDF problem, and the equation yields the desired filter in **Equation 3**, with H and V established as:

$$H = \begin{bmatrix} -0.1641 & 0.5898 \\ 1.8032 & -1.8476 \end{bmatrix}$$

$$V = \begin{bmatrix} 2.0180 & -0.2819 \\ -0.2359 & 0.925 \end{bmatrix}$$

Faults can take many forms. In what follows, we will focus on the fault. The system is affected by two franc-type sensors $f_c(t)$ and actuator $f_a(t)$ faults given as follows (**Equations 44** and **45**):

$$f_c(t) = \begin{cases} 0.2 & 20 < t < 21 \\ 0 & else \end{cases} \tag{44}$$

$$f_a(t) = \begin{cases} 0.1 & 5 < t < 6 \\ 0 & else \end{cases} \tag{45}$$

Figures 6 and **7** illustrate the fault signals injected into the system to evaluate the performance of the proposed fault detection and isolation scheme.

Figure 6 shows the sensor fault signal $f_c(t)$. This is defined as a rectangular pulse with an amplitude of 0.2 and is active only during the time interval 20 < t < 21s. Outside this interval, the signal is 0 (**Equation 44**). This fault is intentionally brief, lasting only one second, and occurs relatively late in the simulation, making it challenging to detect.

Figure 7 represents the actuator fault signal $f_a(t)$ defined as a rectangular pulse of amplitude 0.1, occurring over the interval 5 < t < 6 seconds, and 0 elsewhere (**Equation 45**).

Figure 8 illustrates the dynamic evolution of a neutral-type delay system under the influence of switching faults and disturbances. The peak in time (5 < t < 6) is significant, because it detects the presence of an actuator fault, and the same for the peak between (20 < t < 21 seconds) for the sensor fault. The oscillatory behavior also reveals, to some extent, system stability problems. To reliably detect faults, the filter must be sufficiently sensitive to faults but robust enough to withstand the typical dynamics and disturbances of normal operation.

Figure 9 demonstrates the time-like evolution of the residual signal, which is often employed in fault detection and isolation strategies. The residual compares the measured system output with the anticipated (observer-assessed) output.

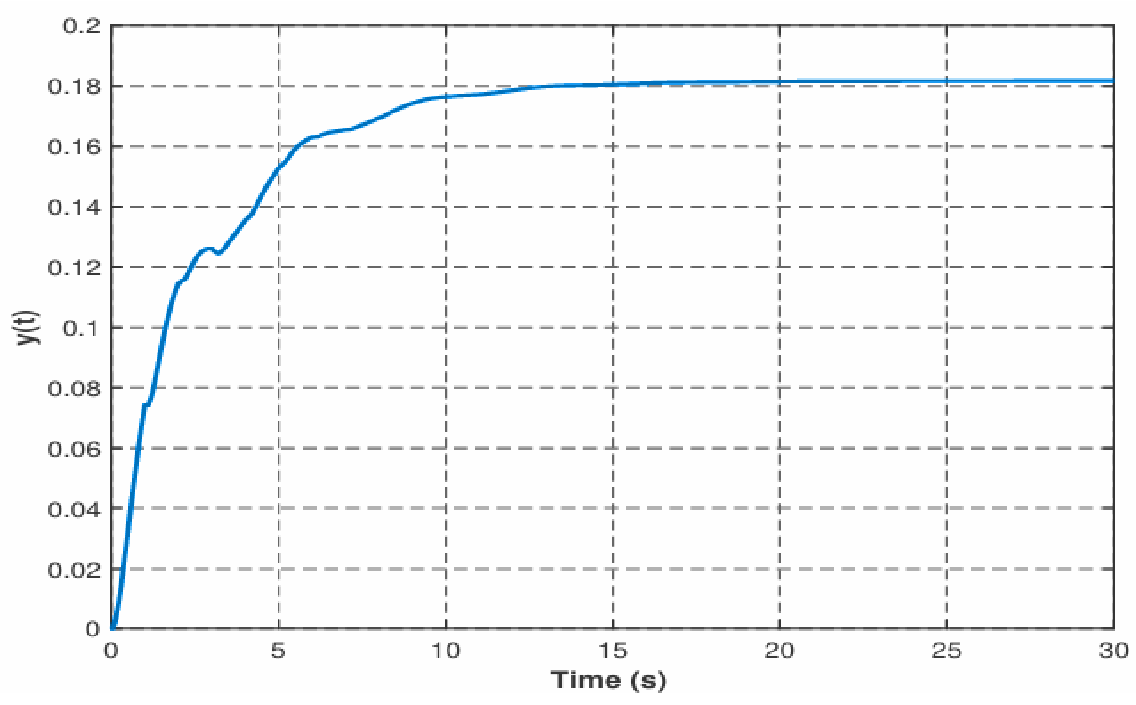

Figure 5. Output trajectory analysis of the neutral-type time-delay system

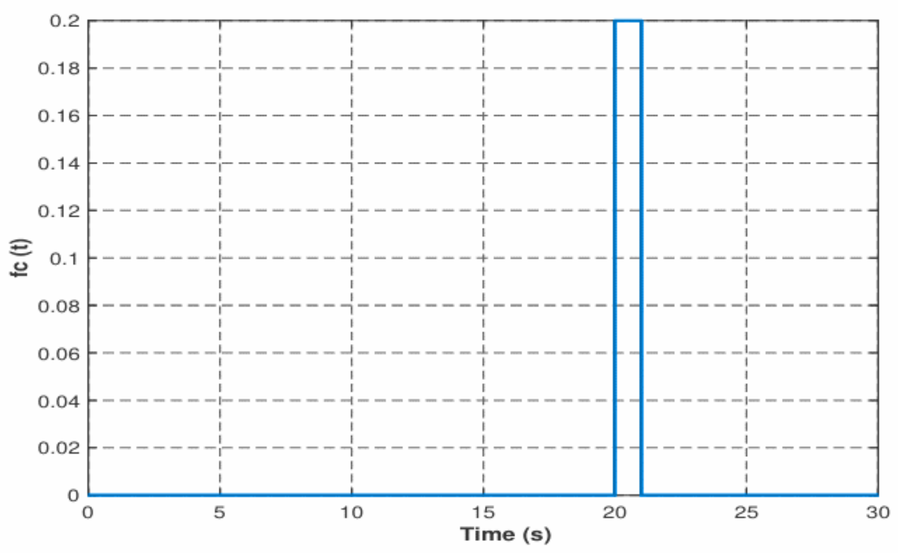

Figure 6. The sensor fault

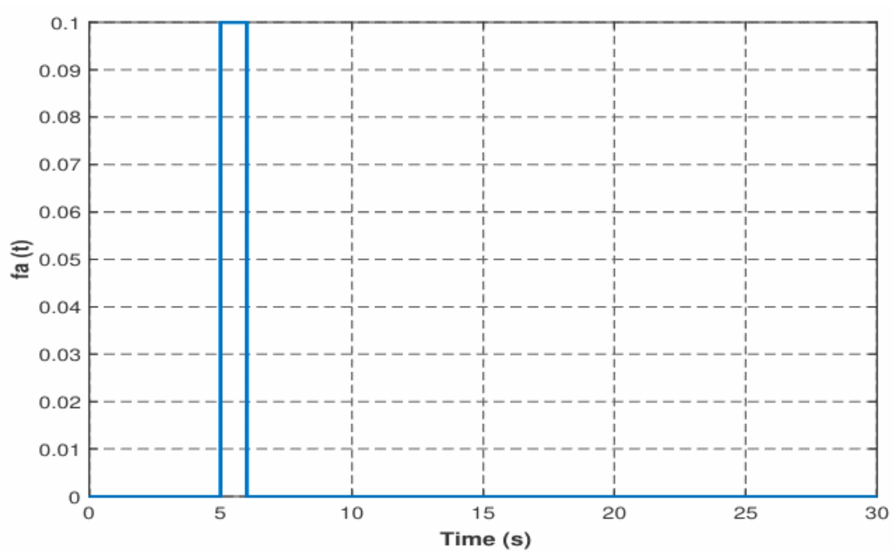

Figure 7. The actuator fault

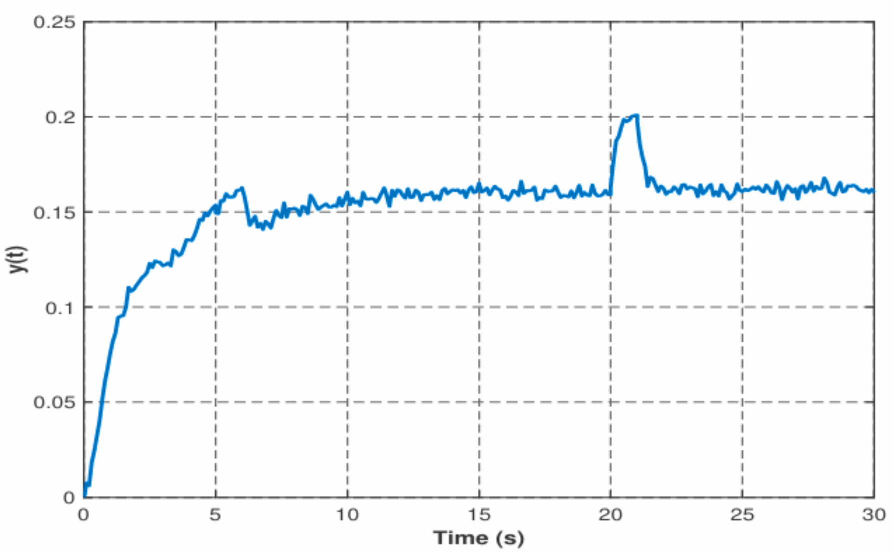

Figure 8. Output feedback of a neutral-type delay system under faults and disturbances

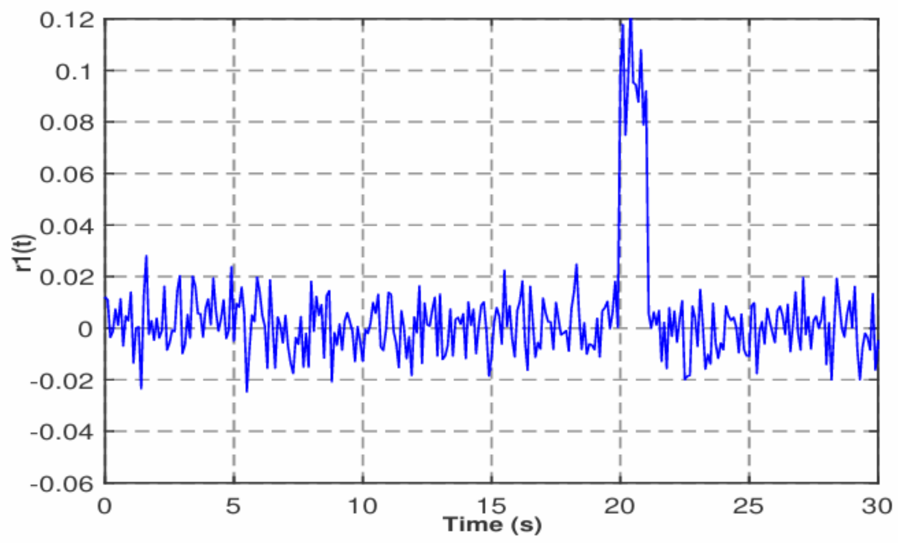

Figure 9. Residual evolution

Fault detection involves evaluating residuals to distinguish between healthy and faulty cases. This is a rather straightforward process: residuals are simply compared to a predetermined threshold; any residual above it confirms the presence of a fault (**Equation 46**).

$$J_{\text{th}} = \sup_{d \in L_2, u \in L_2, f=0} \|r(t)\|_{2,T_e} = \left(\int_{\tau_0}^{t_0+T_e} r_F^T(t) r_F(t)\, dt \right) \tag{46}$$

The evaluation function is based on the robust residual expression. This function enables the detection of faults. The residual evaluation function is defined as **Equation 47**:

$$J(r) = \|r(t)\|_{2,T_e} = \left(\int_{t_0}^{t_0+T_e} r^T(t) r(t)\, \mathrm{d}t \right)^{\frac{1}{2}} \tag{47}$$

The evaluation interval is defined as $t \in [t_0, t_0 + T_e]$, where t = 0 denotes the initial evaluation time and represents the duration of the evaluation window. The value of T_e is limited, since computing the residual over the entire time horizon is impractical. Based on the considerations above, fault occurrence can be detected using the logical relations in **Equation 34**. The decision-making process involves comparing the norm of the residual, calculated over the interval T_e, with an adaptive threshold J_{th} according to the following detection logic (**Equation 48**):

$$\begin{cases} \&r(t)_{2,T_e} > J_{th} \quad \Rightarrow \quad \textit{fault alarm} \\ \&r(t)_{2,T_e} \le J_{th} \quad \Rightarrow \quad \textit{without fault} \end{cases} \tag{48}$$

The adaptive threshold indicates the deviation of the residual from zero. When this threshold is exceeded, the residual indicates a fault has been detected. This threshold is used to suppress false alarms.[38–42] The detection threshold J_{th} must also be based on the model's tolerance to external disturbances and uncertainty. The detection threshold is determined under normal operating conditions (fault-free system) as follows (**Equations 49** and **50**):

$$\|r_d(t) + r_u(t)\|_{2,T_e} \le \|r_d(t)\|_{2,T_e} + \|r_u(t)\|_{2,T_e} \le J_d + J_u \tag{49}$$

- $r_d(t) = r(t)|_{u=0,f=0}$: It represents the portion of the residual that is robust to faults and system inputs, while remaining sensitive to disturbances.
- $r_u(t) = r(t)|_{d=0,f=0}$: It represents the portion of the residual that is robust to faults while being sensitive to the system inputs.

$$J_d = \sup_{d \in L_2} \|r_d(t)\|_{2,T_e} \qquad J_u = \sup \|r_u(t)\|_{2,T_e} \tag{50}$$

Thus, the adaptive threshold J_{th} is chosen as follows (**Equation 51**):

$$J_{\text{th}} = J_d + J_u \tag{51}$$

Assuming $d \in L_2$, we have (**Equation 52**):

$$J_d = \sup_{d \in L_2} \|r_d(t)\|_2 = M, (M \succ 0) \tag{52}$$

We obtain $J_u = \gamma_u \|u\|_{2,T_e}$ and the constant γ_u is determined by (**Equation 53**):

$$\gamma_u = \sup \frac{\|r_u(t)\|_{2,T_e}}{\|u\|_{2,T_e}} \tag{53}$$

As a conclusion, the adaptive threshold can be rewritten as the association of a constant term M and variable ones that depend on the evolution of$\|u\|_{2,T_e}$.

Thus, the adaptive threshold is given below (**Equation 54**):

$$J_{\text{th}} = M+\gamma_u\|u\|_{2,T_e} \tag{54}$$

The generated residual and evaluation functions are shown in **Figures 9** and **10**. With the evaluation time set to t = 0 s, the detection threshold can be obtained directly from the evaluation function. J_{th}= 0.1690 for the 30 s period. We can see that $\|r_F(t)\|_{2,T_e} \approx 0.169 > 0.09$ for $T_e = 5.5$ s. Hence, the system can identify faults once they have occurred.

To further highlight the contribution of this study, **Table 2** provides a comparative overview of recent works on robust filtering and fault detection for time-delay systems. The comparison is structured around three key features: the consideration of neutral-type time delays, the treatment of system uncertainties, and the design of a robust filtering scheme. Several existing approaches have addressed these challenges partially. For instance, some works have focused on robust H∞ filtering for uncertain linear systems with time-varying delays but without accounting for the neutral delay structure, while others have tackled fault detection filter design specifically for neutral systems, yet without incorporating parametric uncertainties. A third line of research has proposed an unknown-input observer-based fault detection approach for uncertain time-delay systems, but without addressing neutral dynamics. In contrast, the present work simultaneously handles all three aspects: neutral time delays, system uncertainties, and RFDF design. This combination, applied to a concrete industrial case study of two-stage recycle reactors, represents a more comprehensive and practically relevant framework than those previously reported in the literature. As summarized in **Table 2**, to the best of our knowledge, our approach is the only one that jointly satisfies all three criteria, and thus closes a significant void in the existing literature.

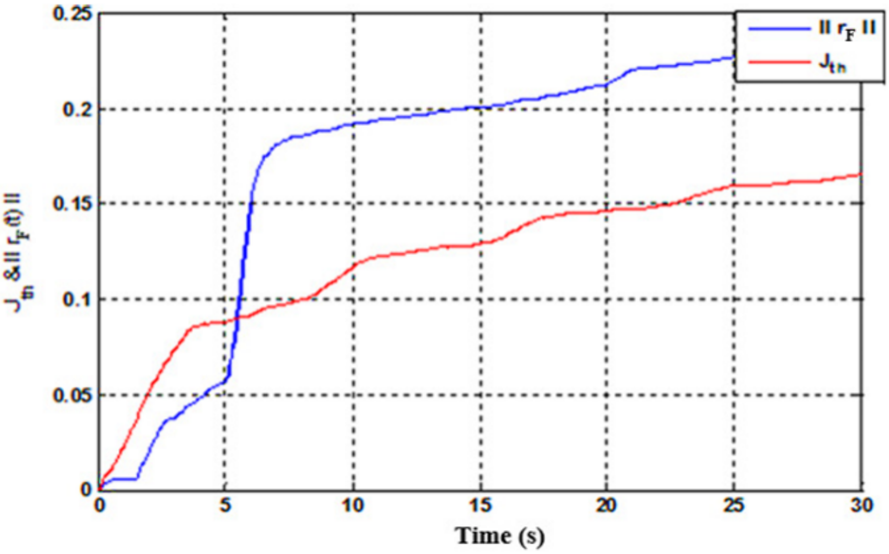

Figure 10. Residual-to-threshold comparison for fault detection purposes

Table 2. An overview of research on robust neutral time delay

Study	**Neutral time delay**	**Uncertain system**	**Robust H_∞ filtering**
Robust H∞ filtering for a class of uncertain linear systems with time-varying delay	No	Yes	Yes
Linear matrix inequality-based fault detection filter design of a neutral system with delay in states	Yes	No	Yes
Robust fault detection of linear uncertain time delay systems using unknown input observers	No	No	Yes
Our study: Robust fault detection filter design for uncertain neutral systems with time delays: A case study of two-stage recycle reactors	Yes	Yes	Yes

6. Conclusion

Our study presented a novel RFDF designed for uncertain neutral-type systems with time delays affecting both the dynamics and its derivative. Using Lyapunov–Krasovskii functionals and LMI methods, stability theorems were formulated to guarantee resilient residual synthesis in the presence of systematic uncertainty and unknown inputs.

The proposed architecture incorporates a reference residual filter and an H∞ optimization-based detection approach to enhance fault sensitivity while reducing the effects of external perturbations.

The theoretical advancements were corroborated via a simulation analysis of a two-stage chemical reactor with delayed recycle streams representative of real industrial processes. The simulation results confirmed the effectiveness of the developed fault detection filter. Sensor and actuator faults could be successfully isolated under uncertain conditions, demonstrating the filter's robustness and practical applicability. Future work could extend this paradigm to encompass nonlinear or large-scale distributed systems, examine advancements in data-driven or adaptive estimation methods, and evaluate experimental validations in hardware models.

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Conflict of interest

The authors have no relevant financial or non-financial interests to disclose.

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Availability of data

The datasets and code generated or analyzed during the current study are available from the corresponding author on reasonable request.

AI tools statement

All authors confirm that no AI tools were used in the preparation of this manuscript.

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