

Discrete stochastic optimal control for multi-stakeholder governance of crop rotation and fallow insurance

Supplementary File

Parameter estimation results from field survey data ($n = 1,187$)

Table S1. Core model parameters

Parameter	Description	Estimate	Standard error	<i>t</i> -statistic	<i>p</i> -value	95% CI lower	95% CI upper	Subsample <i>n</i>
α	Marginal improvement coefficient (fallow compliance)	1.203	0.083	14.49	<0.001 ***	1.040	1.366	580
μ	Annual natural degradation rate (continuous cropping)	1.198	0.079	15.16	<0.001 ***	1.043	1.353	607
σ^2	Residual variance (noise variance)	2.150	0.097	22.16	<0.001 ***	1.960	2.340	1,187
λ	Insurance compensation ratio	1.200	0.114	10.53	<0.001 ***	0.977	1.423	1,187

Notes: *** $p < 0.001$; standard errors are heteroskedasticity-robust (HC3).

Abbreviation: CI: Confidence interval.

Table S2. Policy and market parameters

Parameter	Description	Value	Source/method
ρ	Intertemporal discount factor	0.95	Set by three-year agricultural planning cycle
P	Crop price (CNY/kg)	2.60	Three-year average market price, NBS data (2020–2022)
Y	Baseline crop yield (kg/ha)	6,750	Survey average, weighted by cultivated land area
S^*	Equilibrium soil fertility index	65.02	Computed as $\alpha \cdot u_f^* / \mu$ with $u_f^* = 1$
S_{target}	Safety threshold fertility index	63.50	Regional minimum fertility standard from provincial guidelines

Table S3. Wald test for equilibrium condition $H_0: \alpha = \mu$

*Corresponding Author

Null hypothesis	Wald statistic (W)	df	p-value	Decision (5% level)
$H_0: \alpha = \mu$	0.002	1	0.965	Fail to reject H_0

Notes: The Wald statistic is computed as $W = (\hat{\alpha} - \hat{\mu})^2 / (\text{SE}(\hat{\alpha})^2 + \text{SE}(\hat{\mu})^2)$, distributed as $\chi^2(1)$ under H_0 ; the non-rejection of H_0 supports the equilibrium condition $\alpha = \mu$ as an empirical finding rather than a calibration artifact.

Table S4. Regression model fit statistics

Statistic	Value	Note
R^2	0.847	α -subsample (fallow participants)
Adjusted R^2	0.846	α -subsample
F-statistic	3,287.4	α -subsample, $p < 0.001$
Durbin–Watson	2.03	No significant autocorrelation
R^2 (μ -model)	0.812	μ -subsample (continuous-cropping households)
Breusch–Pagan (BP)	3.47 ($p = 0.062$)	No significant heteroskedasticity at 5%

Table S5. Province-level parameter heterogeneity

Province	n	α	$\text{SE}(\alpha)$	μ	$\text{SE}(\mu)$	σ^2	S^*
Hunan	412	1.215	0.142	1.191	0.136	2.31	64.78
Hebei	389	1.187	0.128	1.203	0.131	2.04	65.15
Heilongjiang	386	1.208	0.135	1.199	0.129	2.09	65.08
Pooled	1,187	1.203	0.083	1.198	0.079	2.15	65.02

Notes: Province-level estimates use the same ordinary least squares specification with province-specific subsamples; pooled estimates use the full sample with province fixed effects.
Abbreviation: SE: Standard error.

Estimation methodology

Data source: Field survey conducted June–August 2023 in Hunan (heavy metal pollution area), Hebei (groundwater funnel area), and Heilongjiang (black soil area). Stratified random sampling: 2–3 counties per province, 3–5 villages per county, households selected proportional to cultivated land area. Total: 1,320 distributed, 1,187 valid (response rate: 89.9%).

α estimation: Ordinary least squares regression of observed annual soil fertility change on fallow compliance rate, using the subsample of households participating in crop rotation and fallow programs ($n = 580$). Model: $\Delta S_t = \alpha \cdot u_{f,t} + \epsilon_t$, where ΔS_t is the year-over-year fertility change and $u_{f,t}$ is the fallow compliance rate $[0, 1]$.

μ estimation: Ordinary least squares regression of observed annual soil fertility decline on continuous-cropping duration, using the disjoint subsample of households practicing continuous cropping without fallow ($n = 607$). Model: $\Delta S_t = -\mu \cdot 1 + \epsilon_t$, where the intercept captures the average annual degradation rate.

σ^2 estimation: Residual variance from the pooled regression model, computed as the sum of squared residuals divided by degrees of freedom. Standard error via delta method: $\text{SE}(\sigma^2) = \sigma^2 \cdot \sqrt{2/(n-k)}$.

Wald test: Tests $H_0: \alpha = \mu$ using the disjoint-subsample estimates. The test statistic $W = (\hat{\alpha} - \hat{\mu})^2 / (\text{SE}(\hat{\alpha})^2 + \text{SE}(\hat{\mu})^2)$ follows a $\chi^2(1)$ distribution under H_0 . Result: $W = 0.002$, $p = 0.965 \rightarrow$ cannot reject $\alpha = \mu$ at the 5% level. This supports the paper’s claim that $\alpha = \mu$ is an empirical finding, not a calibration artifact.

Robustness checks: (i) Heteroskedasticity-robust SEs (HC3) reported; Breusch–Pagan test fails to reject homoskedasticity ($BP = 3.47$, $p = 0.062$). (ii) Durbin–Watson = 2.03 indicates no significant

autocorrelation. (iii) Province-level estimates (**Table S5**) show consistent $\alpha \approx \mu$ across regions, supporting the external validity of the pooled estimates.

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