

AN EFFICIENT AUTOMATIC CURVELET-CONTOURLET FAULT DETECTION METHOD USING FUZZY ENTROPY

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ABSTRACT

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Accurate detection of faults in seismic data can affect various disciplines such as subsurface geological studies, petroleum exploration, drilling strategies and production. Still, it is in general a manual and time consuming task. Therefore, bringing automatic methods to this field which can accurately extract useful information from seismic images and locate faults would be valuable.

In this paper we propose a novel method for automatic seismic fault detection using combination of multiresolution multidirectional algorithms and fuzzy entropy theory. The paper employs Curvelet (CV) and Contourlet (CN) transforms for feature extraction from transformed domain and to capture both detail and smooth information content of the data. The proposed framework introduces a novel feature space by extracting features in temporal domain using CN transform to capture smooth contour information and CV transform to capture details along the curve features in order to improve detection performance. It also introduces an automatic feature selection algorithm using differentiation which highlights fault information, to isolate faults from reflectors adaptively. The reduced coefficients are used as feature vectors to locate faults more accurately. Then, a multi-level thresholding based on fuzzy partition of the histogram and entropy theory is applied to classify image pixels into fault and non-fault. According to results and assessments, this method is very efficient in accurately locating faults and eliminates the need for manually interpret fault surfaces.

KEY WORDS: seismic, fault detection, curvelet, contourlet, fuzzy entropy.

INTRODUCTION

Detection of faults is a key topic and a necessity in seismic data interpretation. Manual interpretation and visual investigation of seismic data is the most common method for seismic fault detection which needs interpreters to pick faults manually. Considering large volume of seismic data, manual interpretation is a time consuming process and takes significant amounts of manual work. To accelerate this procedure, in the past decade researches have made great efforts into developing computer-aided tools to extract fault information from seismic images.

In classical seismic interpretation, it is typical to use attribute analysis for fault detection. Many attributes have been introduced to assist interpreters (Bahorich and Farmer, 1995; Lisle, 1994; Marfurt et al., 1998; Van Bemmelen and Pepper, 2000; Pepper and Van Bemmelen, 2011; Deng and Gao, 2014; Roden et al., 2015). Although attributes are regularly used by interpreters and improve the image of fault within seismic data, the fault need to be interpreted manually. Introduction of image processing techniques such as multiresolution and multidirectional filters brought attention to make use of these filters in seismic processing. Different filters in spatial and frequency domains have been developed to pull out the useful features from images.

The conventional Discrete Wavelet Transform (DWT) (Matos and Osorio, 2002) is a main part of many signal and image processing techniques. It has a good performance when dealing with one and two-dimensional signals with point singularity features, but as other methods, the wavelet transform (WT) has its own limitations. When it comes to two-dimensional images, the filter has poor directionality and do not possess the directional information. It can only isolate discontinuities across horizontal or vertical edges and cannot be used in case of representing high dimensional signals with linear or surface singularities and will introduce artifacts during processing of curves.

In order to overcome shortcoming of wavelet transform and process images of high dimension more effectively, Curvelet (CV) and Contourlet (CN) transforms were introduced. Curvelet Transform and Contourlet Transform are capable of capturing the directional information with multiresolution representation. Concept of Curvelet transform was first proposed by Candès and Donoho (2000) in image processing to overcome the problem of edge representation and drawbacks caused by poor directionality of conventional WT and to capture smooth discontinuity curve more effectively. Their procedure was based on windowed Ridgelet. This was called first generation CV transform but it had limited applications. Candès and Demanet (2002) and Candès and Guo (2002) presented computationally simpler CV approach by introduction a new tight frame.

Subsequently, Candès and Donoho (2004) developed the CV based frequency partitioning. This approach found many useful applications in different fields such as image and seismic processing (Starck et al., 2003). Yu and Yan (2011) also used the CV to attenuate multiples. Cao and Zhao (2016) and Zhang et al. (2017), applied CV transform for 3D simultaneous seismic data reconstruction and noise suppression, Lu et al. (2021) used CV for seismic resolution enhancement.

The Contourlet (CN) transform is one of the transforms with the property of representing images containing contours and textures. It can effectively capture the smooth contours in the images. CN transform was first proposed by Do and Vetterli (2001) and is one of the image processing transforms for feature extraction applications. Bamberger and Smith (1992), established the directional filters used in CN transform. Do and Vetterli (2001) combined directional filters with multiresolution filters to form this transform. The CN transform is an effective multiresolution image representation, which considers multiresolution, multiscale, multi-directionality, and anisotropy properties (Liu et al., 2021). This transform is capable of capturing contours and fine details in images. Zhao et al. (2016) and Sang et al. (2019), utilized CN transform for random noise attenuation. Golpardaz et al. (2020) used CN transform for SAR image segmentation.

Fuzzy thresholding is the other approach that we have used in this paper. We have used thresholding method to isolate object of interest in the image from surrounding pixels. Fuzzy definition can be use to describe image classification, particularly for issuing problem with fuzzy nature. De Luca and Termini (2001) first discussed Application of fuzzy approach for image segmentation. It has attracted great interests especially in medical image processing. Orujov et al. (2020) proposed fuzzy based image edge detection algorithm for blood vessel detection in retinal image. Versaci and Morabito (2021) used fuzzy entropy for image edge detection.

In this study, considering unique characteristics of CV and CN transform, we have used these filters as the basis for our designed automatic seismic fault detection along with Fuzzy entropy thresholding. The proposed method was examined over two real data sets containing fault features, and the performance of the method was evaluated by comparing the result with common variance attribute.

THEORY and CONCEPT

Curvelet Transform

The CV construction is based on combination of Ridgelets and Bandpass Filtering as: employing a multiscale Ridgelets which is a pyramid of windowed Ridgelets, renormalized and then transported to a custom range

of scales and locations, then using bandpass filtering for decomposing an object into a series of disjoint scales (Candès and Donoho, 1999) (Fig. 1). CVs are known as result of applying translation, parabolic dilation, and rotation on a basis function. They are indexed by a scaling parameter a , location b , and an orientation θ and obey the following formula (Candès, 2003):

$$\psi_{a,b,\theta}(x) = a^{-\frac{3}{4}}\psi(D_a R_\theta(x - b)), \quad (1)$$

where D_a is the parabolic scaling matrix, and R_θ is amount of rotation defined by θ radians. One of the important CV properties is that it obeys the harmonic analysis principle, which means that it is possible to analyze and rebuilt an optional function $f(x_1, x_2)$ by superposition of a series of CVs (Candès, 2003). The principal advantage of CV is this potential of representing a curve as a series of superimposed functions of various lengths and widths and the ability to capture detail information in the image. The CV transform is a multiscale transform but, unlike the wavelet transform, has a very high directional sensitivity (Alparone et al., 2006).

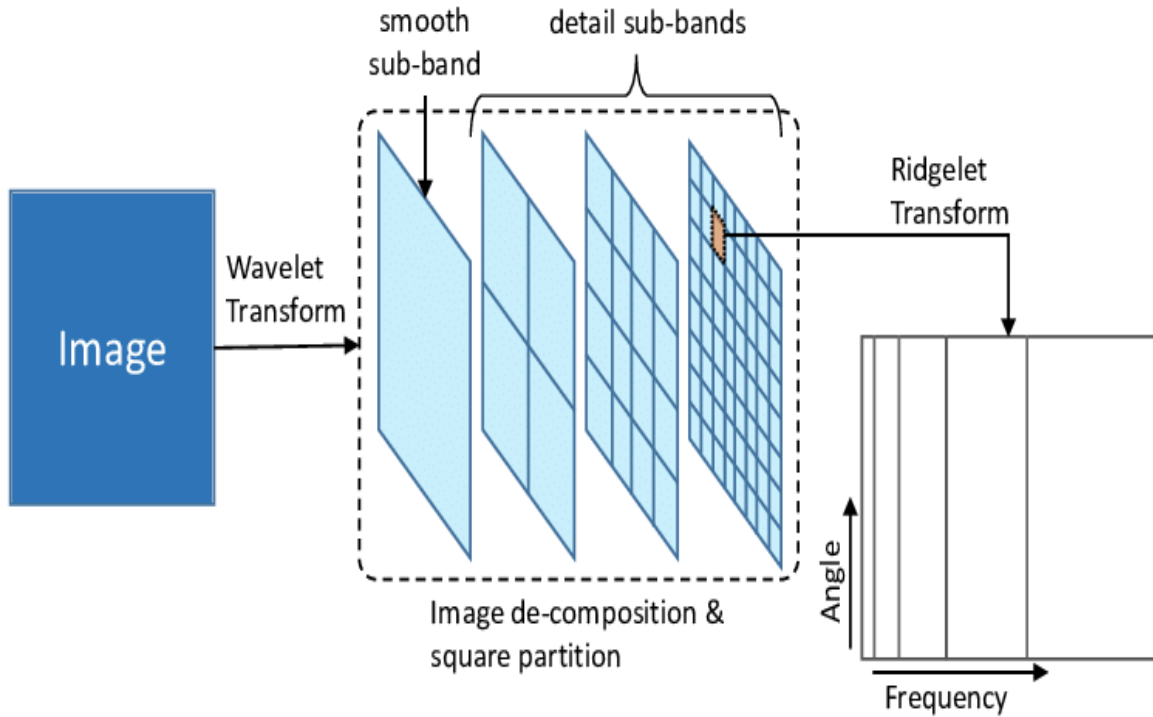


Fig. 1. CV transform flow graph (Mankar et al, 2021).

Contourlet transform

CN transform proposed by Do and Vetterli (2001) is an image processing transforms. The CN construction is based on combination directional filters and multiresolution filters. The directional filters were introduced by Bamberger and Smith (1992). Then, Do and Vetterli (2001) combined directional filters with multiresolution filters where maximum information is grouped into a small number of samples. CN is a powerful method in visual information processing and capturing smooth contours in the images. The transform is capable of performing multiresolution, multiscale, multidirectionality, and anisotropy properties (Liu et al., 2021). It can efficiently capture the intrinsic geometrical structures by achieving the best approximation rate for piecewise smooth functions (Dong and Ma, 2012).

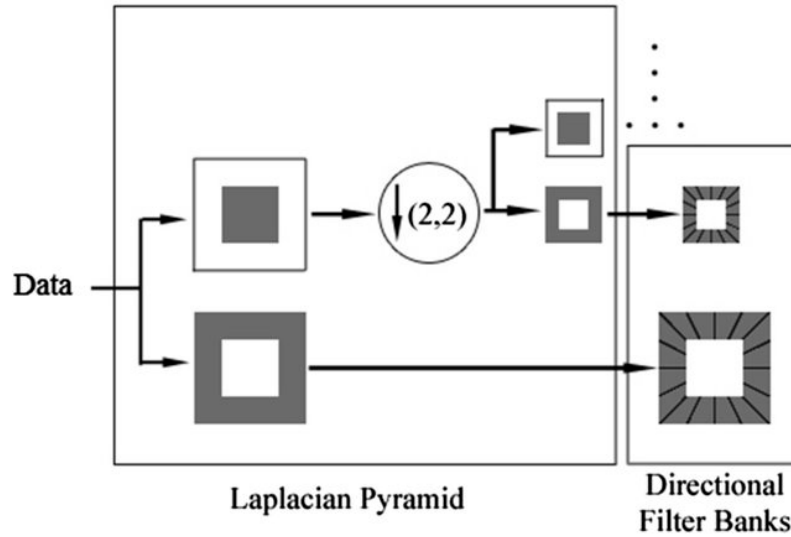


Fig. 2. CN transform flow graph, Laplacian pyramid followed by directional filter bank (Hameed, 2021).

In this transform the original image is first decomposed into frequency subsets (bandpass subsets and a lowpass subset). Then, bandpass subsets are directed to various orientation using directional filters, to be decomposed into bandpass subsets with particular directional features. The procedure can be iterated on the lowpass subband, resulting in multiple subbands with various scales and directions (Liu et al., 2021) (Fig. 2). Let us suppose that an image x is fed onto CN transform and is decomposed into J bandpass images b_j and a lowpass image a_j (Do and Vetterli, 2001). By applying directional filters, it decomposes each band-pass image b_j into the directional coefficients d_j with $\|b_j\|^2 = \|d_j\|^2$ where $d_j: x \rightarrow (d_1, d_2, \dots, d_j, a_j)$ by following equation (Do and Vetterli, 2001):

$$\|x\|^2 = \sum_{j=1}^J \|d_j\|^2 + \|a_j\|^2 . \quad (2)$$

Selecting higher number of direction and scaled decomposition, will lead to a more detailed decomposition. Decomposed images in CN domain contain a sequence of CN transform coefficients. The dominant features in the original image are expressed by high-magnitude coefficients and images mostly containing noise is represented by smaller coefficients (Liu et al., 2021).

Fuzzy Entropy

Fuzzy Entropy thresholding approach is an entropy based thresholding method used to partition an image to sets of pixels with similar characteristics (entropy). It is an effective and adaptive tool for isolating target and background regions of an image. The principle of thresholding is searching for an adequate threshold to separate object from background. The multilevel thresholding is an extension of thresholding technique and is used in the case of segmenting several objects from background (Rajini, 2019). However, it takes great deal of time and mathematical calculation when searching multilevel thresholds. Differential Evolution as a global optimization technique found to be very helpful for this optimization issue. The method quickly and efficiently suggests an adequate solution. It gives close optimal solution, acceptable for real time applications (Rajini, 2019). The definition of Fuzzy Entropy for image segmentation was proposed by De Luca and Termini (2001) based on Shannon's function. The Shannon's function is given in the following equation (Al-Sharhan et al., 2001):

$$H(X) = - \sum_{j=1}^n P_j \log P_j , \quad (3)$$

where P_j is the set of probabilities of a set of random variables X (Al-Sharhan et al., 2001). The thresholding method based on entropy values is done by considering histogram of entropy values of all image pixels. For threshold selection from image histogram, a novel technique depending on Differential Evolution had been projected. The selection of threshold values depends always over the image gray level histogram. Through appropriate function optimization, the best threshold is computed (Rajini, 2019). For categorizing the normalized histogram into n classes, n_t thresholds (t) is needed, where the entropy for each class can be computed as (Sarkar et al., 2014):

$$H_n(X) = - \sum_{j=t_{n-1}+1}^{I^n} \frac{P_j}{\sum_{i=t_{n-1}+1}^{t_n} P_i} \ln \frac{P_j}{n} \quad (4)$$

Trapezoidal membership function was used for this method to calculate membership of n segmented areas, therefore by adding estimated memberships ($\mu_i, i = 1, 2, \dots, n$) we can rewrite above equation as following to find maximum fuzzy entropy for each segment (Sarkar et al., 2014):

$$H_n = - \sum_{j=t_{n-1}+1}^{t_n} \frac{P_j * \mu_n(j)}{\sum_{i=0}^{t_n} P_i * \mu_n(j)} \ln \frac{P_j * \mu_n(j)}{\sum_{i=0}^{t_n} P_i * \mu_n(j)}, \quad (5)$$

where by maximizing total entropy [eq. (14)], we can have optimum value of parameters (Femy and Victor, 2014),

$$\varphi(t_1, t_2, \dots, t_n) = \text{Argmax}(H_1(X) + H_2(X) + \dots + H_n(X)). \quad (6)$$

The optimum value was achieved using Differential Evolution as a global optimization technique, which was recommended by Sarkar et al. (2014) based on the result of comparing different optimization methods for multi-level image thresholding problems. According to the definition of fuzzy entropy, the higher the value of fuzzy entropy in a certain region, the larger the ambiguity fluctuation near the pixel which belongs to the detail region and the lower fuzzy entropy, the smaller fluctuations in the region which belongs to the flat region (Li et al., 2020).

METHODOLOGY

In this paper we have designed a machine fault detection algorithm. The main objective of our research is to help seismic interpreters and reducing the burden of workload. With our method seismic images are automatically scanned to locate faults within the data without the need to manually interpret seismic fault surfaces. The proposed framework consists of three main phases: Feature Extraction, Feature Selection, and Classification of the output image.

Feature Extraction

In the proposed fault detection method, we have used variance attribute as the input. Our designed framework introduces a novel feature space by extracting features in temporal domain using CN and CV transforms, as two most powerful multiresolutional multidirectional filters, in order to improve detection performance, benefits from both transforms unique characteristics and obtain detail and smooth information content of the data. CV transform has the potential to represent curves more effectively. It has better ability to capture curve like features and singularities along the curves and more detailed content of the image. On the other hand, CN transform can obtain flat contour information along different directions in a more effective way. This transform is designed to efficiently represent images made of smooth region and smooth contours by a directional filter bank, DFB, design directionality and anisotropy, which are the important properties of the contourlet. Due to directional filter bank, contourlet is able to develop smooth object boundaries more effectively.

Fig. 3 is a representation of CV ability to decompose images into different scales and directions. Fig. 4(a) shows a sample variance attribute section containing fault feature as input to CV and CN transform. Part (b) is a sample subscale in CV domain containing fault information and (c) is the same subscale of part (b) in time domain after zeroing all other subscales and reconstruction. In the same way, part (d) is the same subscale indices in CN domain, and (e) is the same subscale of part (d) in time domain in time domain. As illustrated, CV domain provide much more coefficient to be able to capture curve like features and contain more detail and the same subscale in CT domain is capable of obtaining smooth and continuous feature more effectively. Hence, output image of our designed feature space would have detail and smooth features.

Feature Selection

The proposed method provides a mechanism for automatic feature selection procedure in transform domains. We have applied CV and CN transforms on variance section to decompose it to various resolutions and directions of subscales and to extract transforms coefficients for each subscale. By using faults and reflectors characteristics and dip difference between these events, CV and CN can discriminate between them. There are some subscales containing fault features mostly and some subscales containing reflector features mostly. Coefficients of CN and CV transform subsets are used as extracted features.

The automatic feature selection process as the second phase of our research in both CV and CN transforms domain is carried out in two steps: at first, applying horizontal differentiation on each subscale to highlight the

effect of fault (as a more vertical event). Then, calculating 2D correlation coefficient between input image and each subscale to find subscales containing main features of the input image. Since, higher correlation coefficient corresponds to principal subscales with key features, a threshold for correlation coefficient is set experimentally, to keep subscales with correlation coefficient higher than the threshold and to zero the rest.

Afterward, reconstruction on the processed subscales is applied to transfer them to time domain. Outputs of both transforms are summed up to form detailed and smooth fault information. In this way, we can automatically select subscales containing fault features by the use of differentiation and 2D correlation.

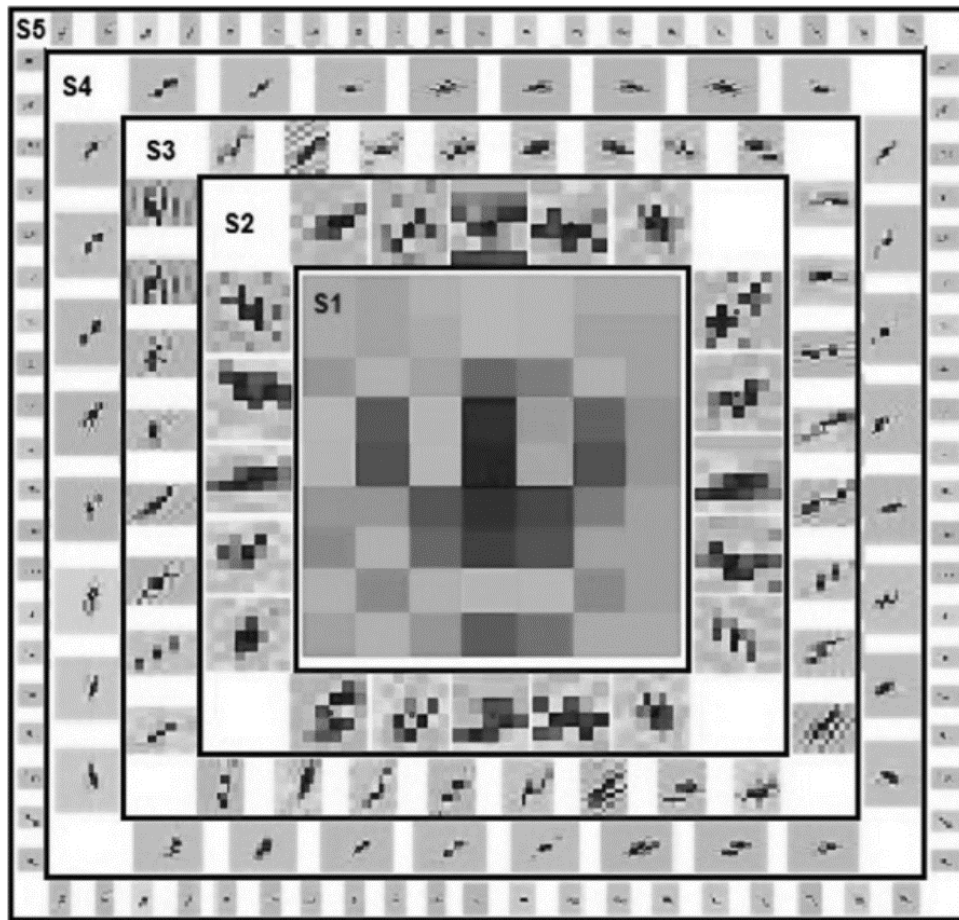


Fig. 3. Schematic presentation of multiscale and multidirectional decomposition in CV domain up to five level (Amirpour Asl and Shad Manaman, 2020).

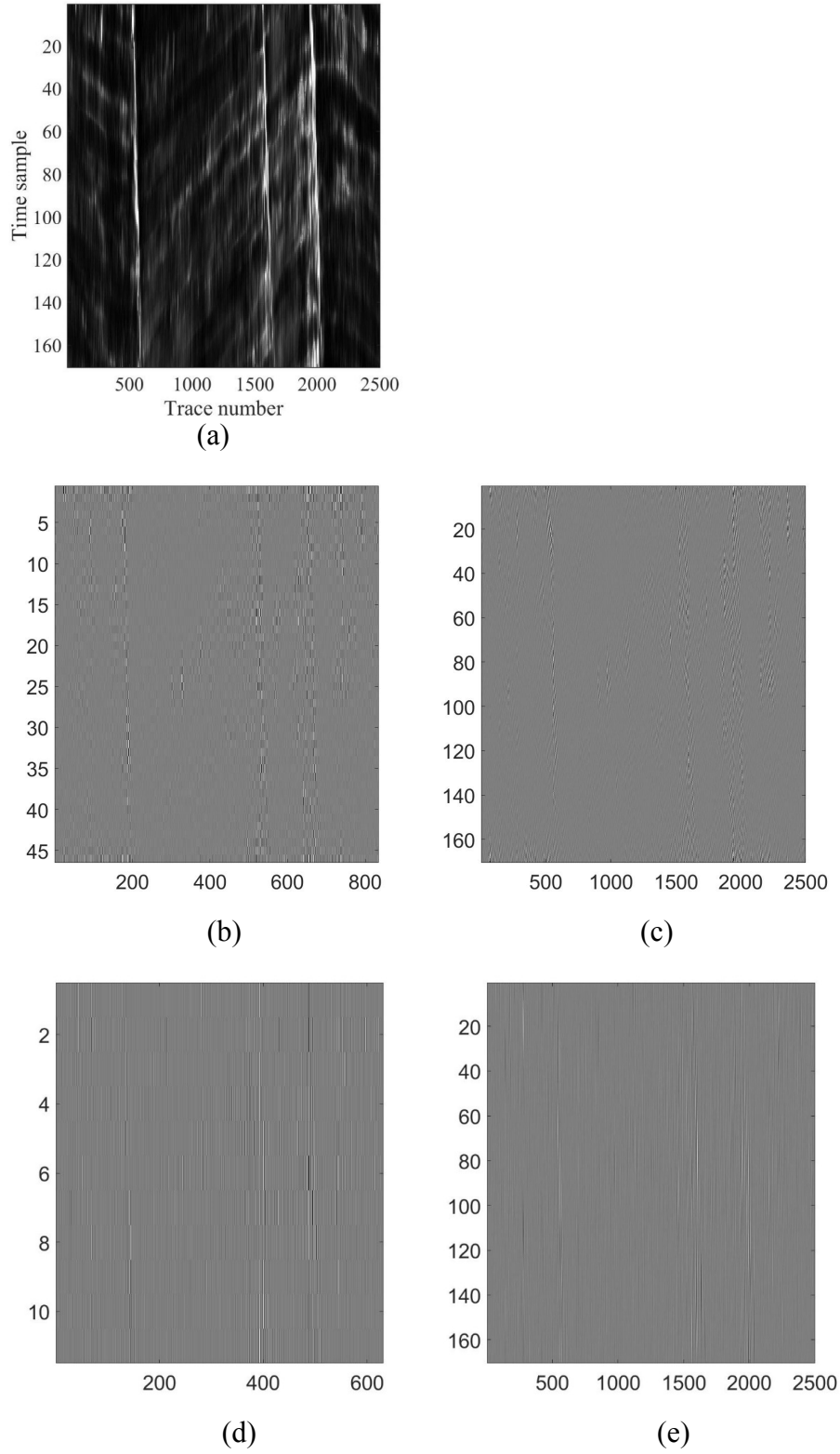


Fig. 4. (a) Variance attribute section containing fault feature as input to CV and CN transform, (b) a sample subscale in CV domain containing fault information, (c) the same subscale of part (b) in time domain, (d) a sample subscale in CN domain containing fault information, (e) the same subscale of part (d) in time domain.

Classification

The final phase of our proposed workflow is classification. We have employed fuzzy entropy thresholding technique developed by Sarkar et al. (2014) as classifier to distinguish fault and non-fault areas of processed output of both transforms. To partition the processed image to target (fault) and background pixels, considering histogram of the image, initial threshold values is assigned to fuzzify the processed output image of both transforms using trapezoidal membership function. Then, it searches for an adequate threshold to separate faults from the background. Differential Evolution technique is used to maximize fuzzy entropy values and find optimized threshold values for histogram partitioning. Fig. 5 is the flowchart of our designed technique.

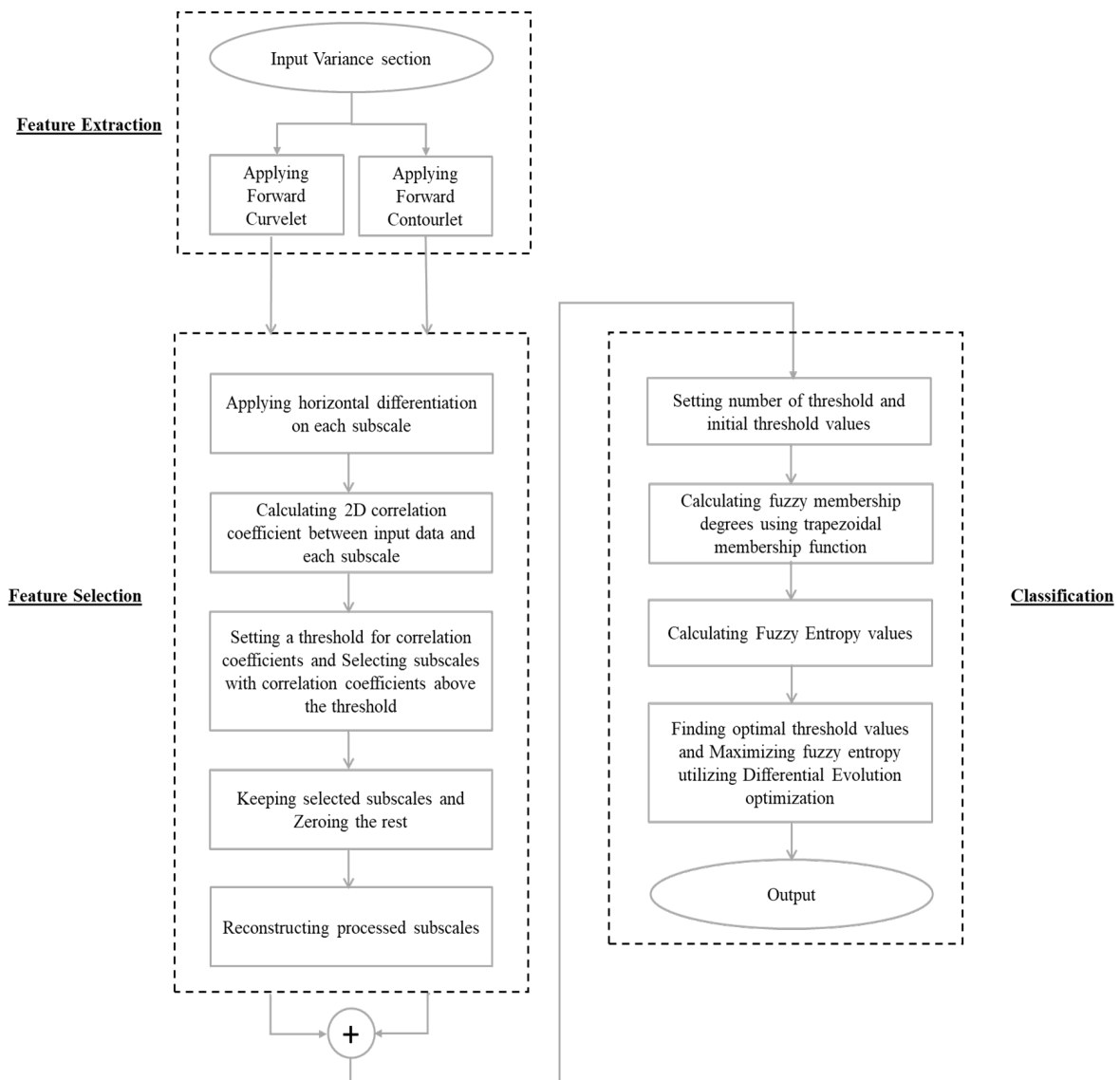


Fig. 5. Flowchart of automatic fault extraction using our designed method.

Experimental result

This paper examines the result of applying the designed technique on two real seismic datasets from two different fields. The first case is shown in Fig. 6. Fig. 6(a) is a real seismic section containing fault event. As the first step of this research, variance section of part (a), shown in part (b), is fed into CV and CN transforms. This paper uses four-level contourlet decomposition and the number of each direction sub-band are 1-8-16-16 for both transforms decomposition. As a result of dip difference between reflectors and faults, and with considering CV and CN ability to discriminate between events with various dip and frequency content in various subscales, there are some subscales with fault data dominantly and some subscales with reflectors dominantly.

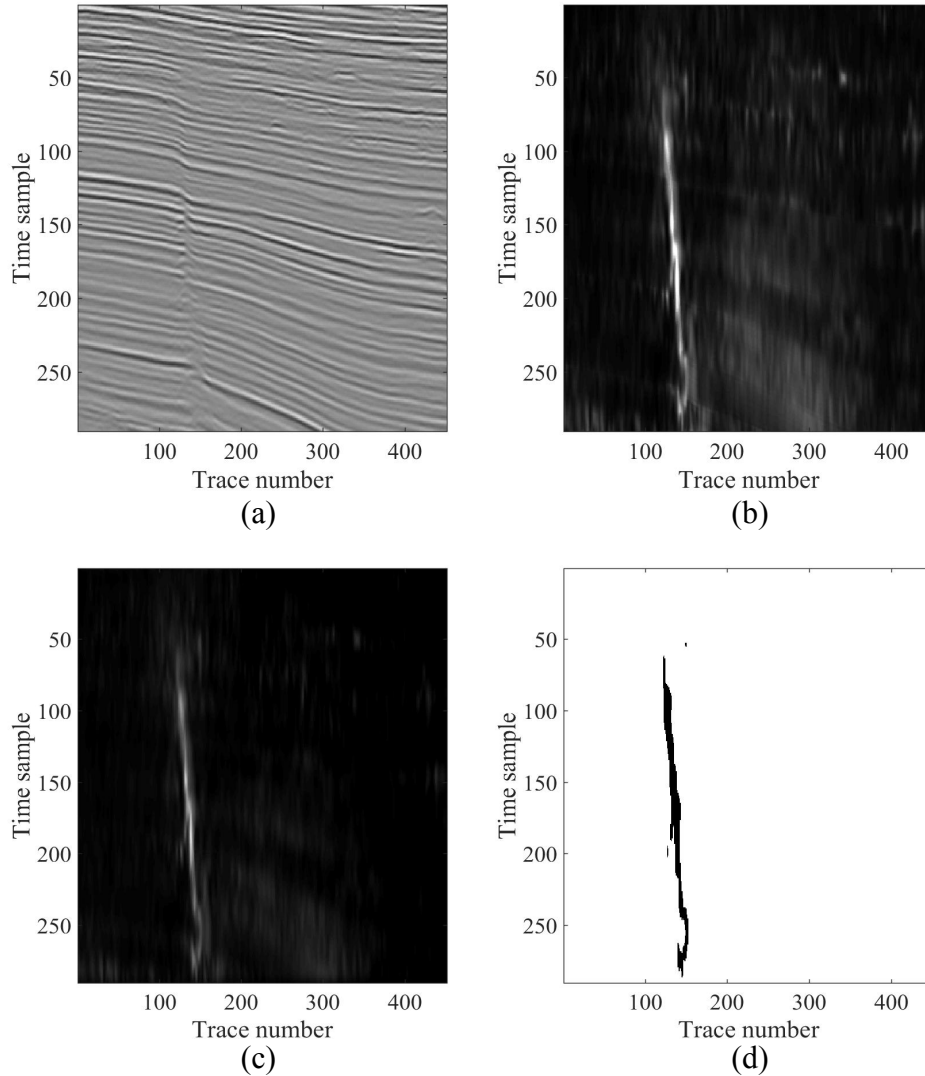


Fig. 6. (a) Real seismic section containing fault feature, (b) variance section of part (a), (c) output of CV-CN domain feature selection before employing fuzzy entropy classification, (d) final output of our designed method (after fuzzy entropy classification).

After applying CV and CN for feature selection, first 2D correlation coefficient between input image and each subscale image was calculated. In this way subscales which are more similar to input image and containing key features of input image were highlighted. Then correlation coefficients were plotted versus subscales indices [Fig. 7(a)]. As illustrated there exists two general peaks in the plot illustrated by black arrows, corresponding to subscales holding two key features of input image, reflectors and faults, other subscales embody useless features such as random noise which can be attenuated by using this method. According to subscale numbers, the first peak corresponds to subscales containing fault features and the second one corresponds to subscales containing reflectors.

To be able to select fault features automatically and discriminate between faults and reflectors, we have applied a horizontal differentiation prior to correlation coefficient calculation, to highlight the effect of fault (as a more vertical phenomena) and decrease the effect of reflectors. Fig. 7(b) is the result of applying horizontal differentiation ahead of correlation coefficient calculation. Higher correlation coefficient corresponds to subscales containing fault information. By setting a threshold, keeping subscales with higher correlation coefficient than threshold and zeroing other subscales, and applying reconstruction on the processed subscales, automatic feature selection is carried out. Afterwards reconstruction is applied on processed subscales of both domains to provide an output fault image. The output of both transforms are added up to have detail and smooth contents of image. Fig. 6(c) is representation of output automatic feature selection in CV-CT domain prior to classification. It is notable that this plan has separated fault feature from other features in the data efficiently.

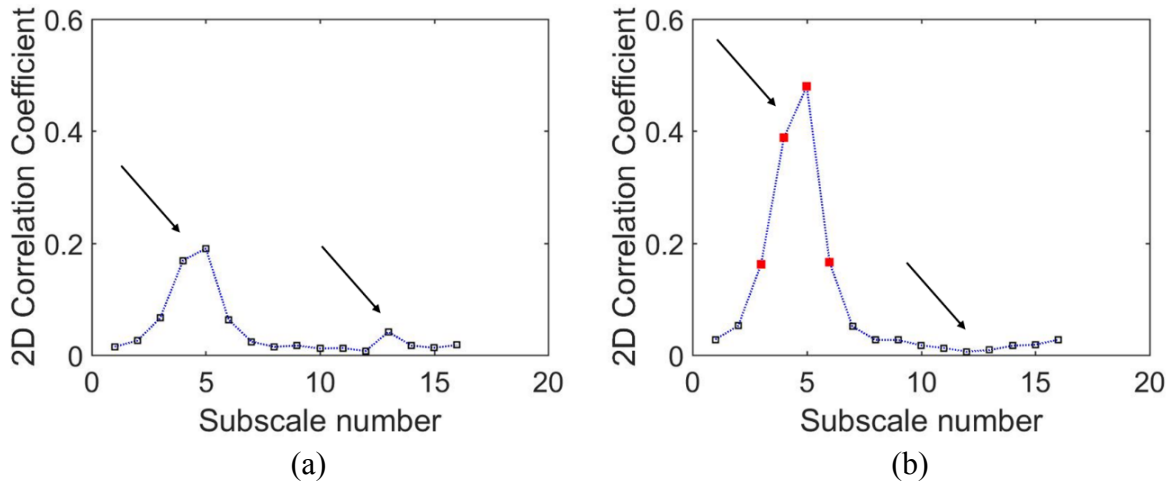


Fig. 7. The plot of 2D correlation coefficient between input and subscales (a) before applying directional differentiation, two black arrows pointing at peaks representing subscales embodying key features, and (b) after applying directional differentiation to reduce the effect of reflector features, black arrows pointing at the same subscales of part (a).

To classify output of CV-CN transform to fault and non-fault areas, multi-level thresholding method based on the principal of fuzzy entropy maximization, proposed by Sarkar et al. (2014) was applied on the output image. Therefore, we have used a method based upon fuzzy entropy which search for optimal threshold according to the clusters. This method works as first considering image histogram (Fig. 8), it fuzzifies the image utilizing trapezoidal membership function, second it calculates fuzzy entropy values using membership degrees by abovementioned equations, finally, the maximize fuzzy entropy and optimal threshold values, was obtained using a global optimization method, Differential Evolution. Since, the higher the value of fuzzy entropy in a certain region corresponds to the larger the ambiguity fluctuation near the pixel which belongs to the detail region, and the lower fuzzy entropy, the smaller fluctuations in the region which belongs to the flat region (Li et al., 2020), higher threshold values was selected to represent fault and isolating fault from non-fault. Fig. 6(d) illustrates the result of classification. According to the result, the proposed scheme can accurately locate fault in the data and separate fault from non-fault areas. By comparing Fig. 6(b) which is the variance section of seismic data as a common method for seismic fault detection to Fig. 6(d) as the final result of our proposed scheme, it clearly can be concluded that this paper's method works considerably better than variance attribute.

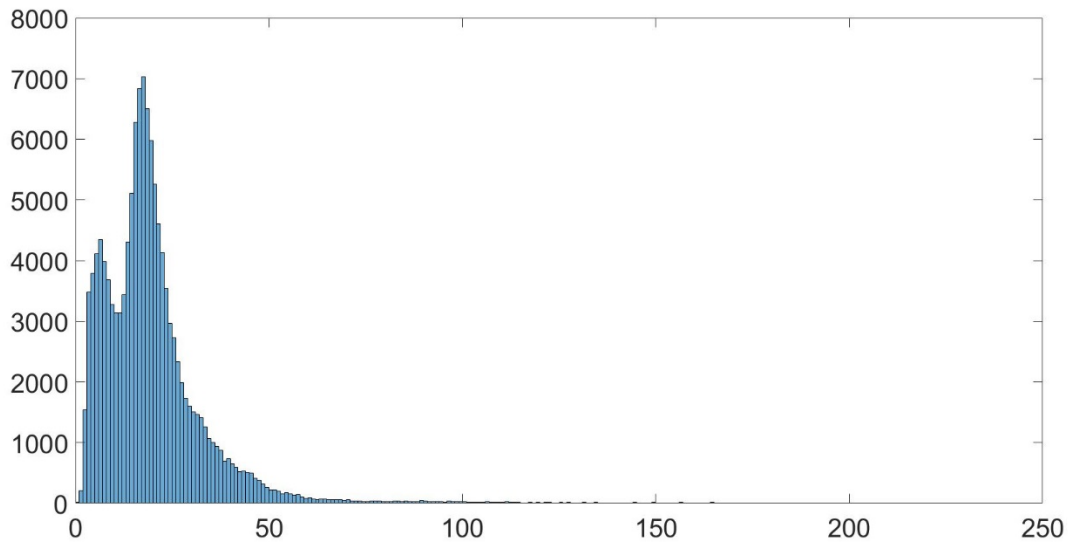


Fig. 8. The histogram of output of CV-CN transform filtering, as input to fuzzy entropy classification.

Similarly, Fig. 9 to Fig. 11 explain the same procedure for the second dataset. Fig. 9(a) and Fig. 9(b) show the second real seismic section containing fault data and its variance section, respectively. Similar to previous dataset, a four-level CV and CN decomposition were applied on variance section and the number of each direction sub-band are 1-8-16-16,

respectively. The plot of 2D correlation coefficient between input and each subscale before and after applying directional differentiation is shown in Fig.10. In the same way as previous example, two general peaks in the plot pointed by black arrows, corresponding to subscales holding reflectors and faults. Fig. 10(b) is the result of applying horizontal differentiation prior to correlation coefficient calculation to highlight the effect of fault (as a more vertical phenomena) and reduce the effect of reflectors. Then, by setting a threshold to isolate higher correlation coefficient, zeroing remaining subscales and reconstructing, fault features are extracted from the dataset adaptively. Reconstructed images from both domain containing both detail and smooth information are added up. Fig. 9(c) is the added output of automatic feature selection in CV-CN domain prior to classification.

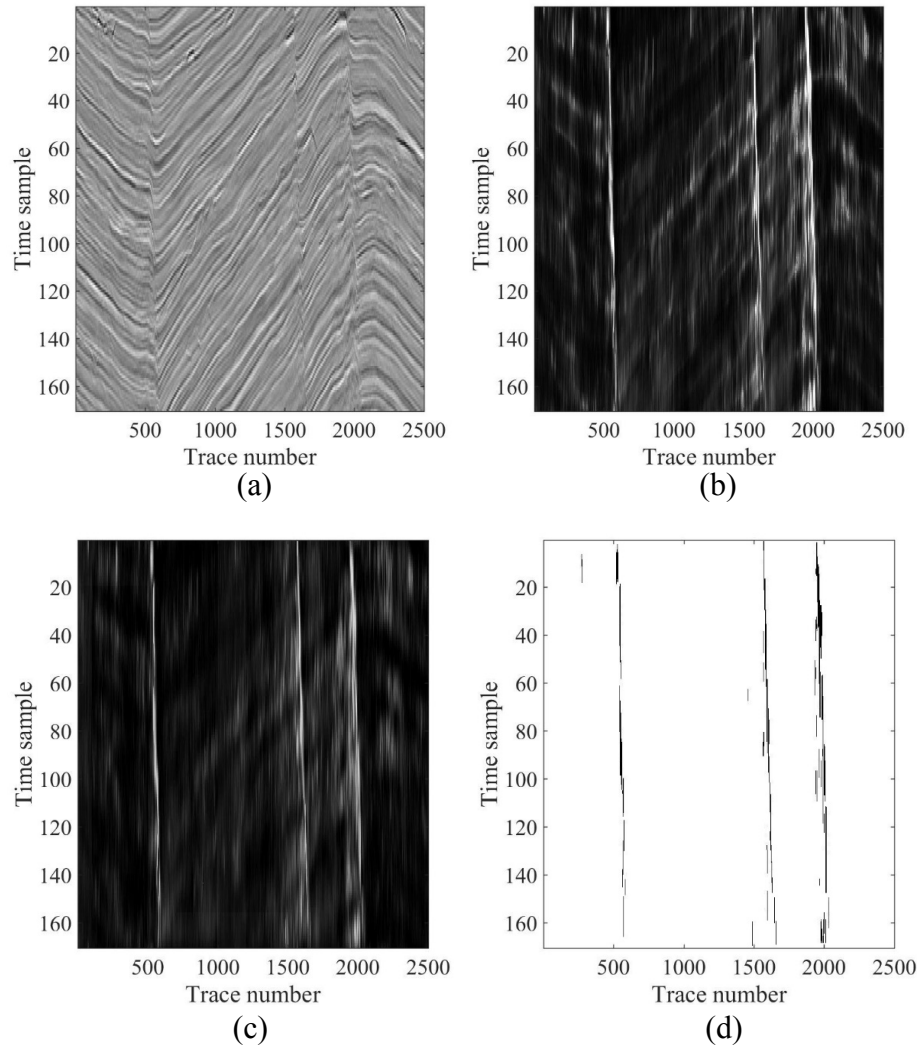


Fig. 9. (a) Real seismic section containing fault feature, (b) variance section of part (a), (c) output of CV-CN domain feature selection before employing fuzzy entropy classification, (d) final output of our designed method (after fuzzy entropy classification) which has located fault accurately.

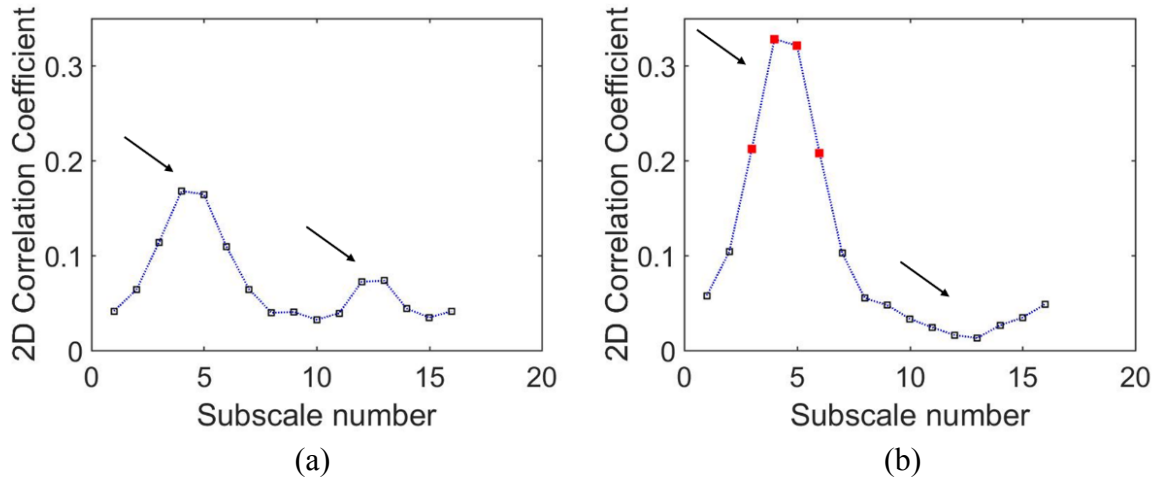


Fig. 10. The plot of 2D correlation coefficient between input and subscales (a) before applying directional differentiation, two black arrows pointing at peaks representing subscales embodying key features, and (b) after applying directional differentiation to reduce the effect of reflector features, black arrows pointing at the same subscales of part (a).

For classification, the same procedure as mentioned above was applied on output of CV-CN transform. Therefore, owing to the image histogram (Fig. 11), it fuzzifies the image using trapezoidal membership function, then fuzzy entropy values was computed using the estimated membership degrees by abovementioned equations. At last, the maximize fuzzy entropy and optimal threshold values, was obtained using a global optimization method (Differential Evolution).

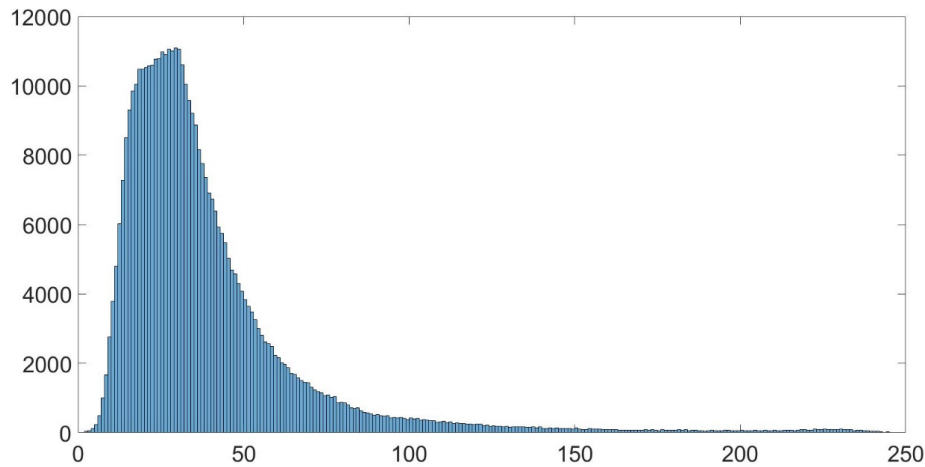


Fig. 11. The histogram of output of CV-CN transform filtering, as input to fuzzy entropy classification.

Correspondingly, higher threshold values were selected as fault and the rest as non-fault. Fig. 9(d) illustrates the result of classification. According to the result, and comparing it to variance section of part (b) as the common method for fault detection, our proposed scheme performs successfully and considerably better than variance attribute in locating fault in seismic data.

DISCUSSION

One of the important parameters in multidirectional multiresolutional domains is the number of decomposition levels and directional subscales in each level. It is noteworthy that most of data energy is concentrated in the first level of these domains. This energy can be distributed to other levels and subscales by increasing the number of levels, vice versa, by selecting high number of levels, there will remain less feature in each subscale and is not suitable for our analysis. Therefore, optimal number of levels and subscales is one of issues we need to take care of. As mentioned above, 4 was selected as optimum number of level for this study for both subscales. Also, each level was decomposed into 1-8-16-16 subscales in both domains.

CN focuses on the concept of directional filter banks which have higher capability to capture linear singularities in the specified directions. This transform has the potential to develop smooth object boundaries and to efficiently represent images made of smooth region. CV transform can represent curves successfully. It has superior ability to capture curve like features and provide more detailed content of the image. Using both transform provides a powerful feature space to extract target features. These features will be used in image classification.

To be able to select fault features automatically and discriminate between faults and reflectors, it was decided to apply a horizontal differentiation prior to correlation coefficient calculation, in order to highlight the effect of fault and reduce the effect of reflectors. By setting a threshold, holding subscales with higher correlation coefficient than threshold, and zeroing other subscales, and finally applying reconstruction, the automatic feature selection is carried out. One of parameter influencing the performance of this filter in this step is the value of threshold that we choose. By selecting higher values for threshold, less subscales will be selected and leads to not selecting whole length of fault or select fault with less amplitude. This will cause error in classification task as a histogram based method. Also, by choosing less values of thresholds, more subscale will be selected and more features will be in the output image, causing error in classification task and non-fault area may classify as fault. After detecting the subscales of interest, other subscales were zeroes in all levels and the remaining subscales were reconstructed.

In the classification step, as the last stage of this technique, the number of threshold we choose is an impacting factor within the execution of the planned approach. By choosing more numbers of thresholds, it will divide data to more than two areas with less probability of being fault. Therefore, choosing the number of thresholds for multi-thresholding approach is an influencing parameter in the result.

CONCLUSION

We have designed an adaptive seismic fault detection framework by making the use of CV-CN feature space benefiting from both transform domain, and fuzzy entropy thresholding algorithm. By applying this filter to a variance section, first subscales embodying fault feature had been adaptively detected in both domains, and reconstructed and categorized as fault and non-fault using fuzzy entropy thresholding. This filter was applied to two real seismic datasets. Results indicate that this method is very efficient in locating faults accurately. The principal effective parameters for this filter is the number of level and subscales in each level, the threshold value for selecting higher correlation coefficient, and selecting number of threshold for classification.

REFERENCES

- Amirpour Asl, A. and Shad Manaman, N., 2020. Application of the curvelet transform to synthetic and real magnetic data. *Explor. Geophys.*, 51: 285-299.
- Alparone, L., Baronti, S., Garzelli, A. and Nencini, F., 2006. The curvelet transform for fusion of very-high resolution multi-spectral and panchromatic images. In: *Proc. 25th EARSel Symp.*, Porto, Portugal. Millpress Science Publishers, Amsterdam.
- Al-Sharhan, S., Karray, F., Gueaieb, W. and Basir, O., 2001. Fuzzy entropy: a brief survey. *10th IEEE Internat. Conf. Fuzzy Syst. (Cat. No. 01CH37297)*, 3: 1135-1139.
- Bahorich, M. and Farmer, S., 1995. 3-D seismic discontinuity for faults and stratigraphic features: The coherence cube. *The Leading Edge*, 14: 1053-1058.
- Bamberger, R.H. and Smith, M.J., 1992. A filter bank for the directional decomposition of images: Theory and design. *IEEE Transact. Signal Process.*, 40: 882-893.
- Candès, E.J., 2003. What is... a curvelet? *Notic. Am. Mathemat. Soc.*, 50: 1402-1403.
- Candès, E.J. and Demanet, L., 2003. Curvelets and Fourier integral operators. *Compt. Rendus Mathemat.*, 336: 395-398.
- Candès, E.J. and Guo, F., 2002. New multiscale transforms, minimum total variation synthesis Applications to edge preserving image reconstruction. *Sign. Process.*, 82: 1519-1543.
- Candès, E.J. and Donoho, D.L., 2000. Curvelets: A surprisingly effective nonadaptive representation for objects with edges. *Stanford Univ. Dept. of Statistics, Stanford, CA*.

- Candès, E.J. and Donoho, D.L., 2004. New tight frames of curvelets and optimal representations of objects with piecewise C2 singularities. *Commun. Pure Appl. Mathemat.*, 57: 219-266.
- Cao, J., De Luca, A. and Termini, S., 1972. A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory. *Informat. Control*, 20: 301-312.
- Di, H. and Gao, D., 2014. A new algorithm for evaluating 3D curvature and curvature gradient for improved fracture detection. *Comput. Geosci.*, 70: 15-25.
- Do, M.N. and Vetterli, M., 2001. Pyramidal directional filter banks and curvelets. *Proc. Internat. Conf. Image Process.*, (Cat. No. 01CH37205), 3: 158-161.
- Dong, Y. and Ma, J., 2011. Bayesian texture classification based on contourlet transform and BYY harmony learning of Poisson mixtures. *IEEE Transact. Image Process.*, 21: 909-918.
- Femy, F.F. and Victor, S.P., 2014. Feature extraction using contourlet transform for glaucomatous image classification. *Internat. J. Comput. Applicat.*, 95: 18.
- Golpardaz, M., Helfroush, M.S., Danyali, H. and Ghaffari, R., 2021. Fully statistical, wavelet-based conditional random field (FSWCRF) for SAR image segmentation. *Expert Syst. Applicat.*, 168: 114370.
- Hameed, A.S., 2021. A high secure speech transmission using audio steganography and duffing oscillator. *Wireless Person. Commun.*, 120: 1-15.
- Li, L., Ma, L., Jiao, L., Liu, F., Sun, Q. and Zhao, J., 2020. Complex contourlet-CNN for polarimetric SAR image classification. *Patt. Recogn.*, 100: 107110.
- Li, W., Lin, Q., Wang, K. and Cai, K., 2021. Improving medical image fusion method using fuzzy entropy and nonsubsampling contourlet transform. *Internat. J. Imag. Syst. Technol.*, 31: 204-214.
- Richard, J.L., 1994. Detection of zones of abnormal strains in structures using Gaussian curvature analysis. *AAPG Bull.*, 78: 1811-1819.
- Liu, G., Kang, H., Wang, Q., Tian, Y. and Wan, B., 2021. Contourlet-CNN for SAR image despeckling. *Remote Sens.*, 13: 764.
- Lu, P., Guo, A. and Li, Y., 2021. Seismic resolution enhancement by the curvelet transform. *Explor. Geophys.*, 52(6): 1-15.
- Mankar, R., Gajjala, C., Foroozandeh, F., Prasad, S., Mayerich, D. and Reddy, R., 2021. Multi-modal image sharpening in Fourier Transform Infrared (FTIR) microscopy. *Analyst.*, 146: 4822-4834.
- Marfurt, K.J., Kirlin, R.L., Farmer, S.L. and Bahorich, M.S., 1998. 3-D seismic attributes using a semblance-based coherency algorithm. *Geophysics*, 63: 1150-1165.
- Matos, M.C.D. and Osório, P.L.M., 2002. Wavelet transform filtering in the 1D and 2D for ground roll suppression. *Expanded Abstr.*, 72nd Ann. Internat. SEG.Mtg., Salt Lake City: 2245-2248.
- Orujov, F., Maskeliūnas, R., Damaševičius, R. and Wei, W., 2020. Fuzzy based image edge detection algorithm for blood vessel detection in retinal images. *Appl. Soft Comput.*, 94: 106452.
- Pepper, R. and Van Bommel, P., 2011. U.S. Patent No. 6,757,614. Washington, DC: U.S. Patent and Trademark Office.
- Rajini, N., 2019. A two-dimensional image segmentation method based on hybrid genetic algorithm with particle swarm optimization and entropy. *Internat. J. Appl. Engineer. Res.*, 14: 2270-2274.
- Roden, R., Smith, T. and Sacrey, D., 2015. Geologic pattern recognition from seismic attributes: Principal component analysis and self-organizing maps. *Interpretation*, 3(4): SAE59-SAE83.

- Sang, Y., Pan, X. and Yan, C., 2019. Random noise reduction of seismic data in NSCT domain. In: 2019 IEEE 25th Internat. Conf. Parallel Distrib. Syst. (ICPADS): 933-936.
- Sarkar, S., Paul, S., Burman, R., Das, S. and Chaudhuri, S.S., 2014. A fuzzy entropy based multi-level image thresholding using differential evolution. In: Internat. Conf. Swarm, Evolutionary and Memetic Computing. Springer Verlag, Berlin: 386-395.
- Starck, J.L., Donoho, D.L. and Candès, E.J., 2003. Astronomical image representation by the curvelet transform. *Astron. Astrophys.*, 398: 785-800.
- Van Bemmelen, P.P. and Pepper, R.E., 2000. U.S. Patent No. 6,151,555. Washington, DC: U.S. Patent and Trademark Office.
- Versaci, M. and Morabito, F.C., 2021. Image edge detection: A new approach based on fuzzy entropy and fuzzy divergence. *Internat. J. Fuzzy Syst.*, 23(4): 1-19.
- Yu, M. and Yan, Z., 2011. Flexible surface multiple attenuation using the curvelet transform. *Expanded Abstr.*, 81st. Ann. Internat. SEG Mtg., San Antonio: 3485-3489.
- Zhang, H., Chen, X.-H. and Zhang, L.-Y., 2017. 3D simultaneous seismic data reconstruction and noise suppression based on the curvelet transform. *Appl. Geophys.*, 14: 87-95.
- Zhao, J., 2016. Simultaneous seismic interpolation and denoising based on sparse inversion with a 3D low redundancy curvelet transform. *Explor. Geophys.*, 48: 422-429.