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Application of EBWO with chaotic mapping, reverse learning, and modified Lévy flight for joint inversion of seismic and VES data: Sea intrusion case

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Abstract

Joint inversion of Rayleigh-wave dispersion and vertical electrical sounding (VES) data can reduce the non-uniqueness of near-surface geophysical interpretation, but its performance strongly depends on the robustness of the optimization algorithm. In this study, an Enhanced Beluga Whale Optimization (EBWO) algorithm is proposed for joint seismic–electrical inversion. EBWO improves the original Beluga Whale Optimization (BWO) by incorporating Tent chaotic mapping and reverse learning for population initialization, together with a Lévy-flight-based golden sine strategy for offspring updating. Benchmark-function tests show that EBWO provided higher optimization accuracy and stability than the original BWO. Synthetic joint inversion experiments under both noise-free and noise-contaminated conditions further demonstrate that EBWO yielded more accurate and stable estimates than BWO and Particle Swarm Optimization. Application to field data from the Besirli region in Türkiye also produced lower fitting errors and better agreement with previous geophysical interpretations. These results indicate that EBWO is an effective and reliable optimization tool for joint Rayleigh-wave and VES inversion.

Keywords: Joint Inversion; Surface wave dispersion; Vertical electrical sounding; Metaheuristics; Near-surface investigation

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1. Introduction

In near-surface geophysical investigations, seismic and electrical methods are often integrated because they provide complementary information about subsurface structures.¹ Active-source surface-wave methods are widely used to estimate shear-wave velocity, which is a key parameter in geotechnical, environmental, and engineering studies.²⁻⁴ In contrast, vertical electrical sounding (VES) is sensitive to subsurface resistivity variations and has been applied to identify contamination zones, cavities, faulted structures, and water-related geological features.^{5,6} Combining these two datasets can therefore improve the reliability of subsurface characterization.

The inversion of seismic and electrical data remains challenging because geophysical inverse problems are generally nonlinear, ill-posed, and non-unique. Deterministic local optimization methods are computationally efficient, but their results may depend

strongly on the initial model and may become trapped in local minima. Machine-learning approaches, especially neural-network-based inversion methods, have also been introduced for surface-wave analysis.^{7,8} Although these methods can reduce dependence on initial models, they usually require representative training datasets and repeated forward simulations. Recent studies have attempted to reduce the required training size while maintaining inversion accuracy.^{9,10}

Joint inversion provides another way to reduce ambiguity by combining datasets with different physical sensitivities. Previous studies have demonstrated the advantages of integrating surface-wave dispersion, resistivity, seismic refraction, and other geophysical observations.¹¹⁻¹⁸ For Rayleigh-wave and VES data, joint inversion is particularly useful because shear-wave velocity and electrical resistivity respond differently to lithology, saturation, and pore-fluid conditions. Their combined interpretation can therefore provide more stable and geologically meaningful models than separate inversions.

Both local and global optimization strategies have been used in geophysical inversion. In recent years, many metaheuristic algorithms have been applied to nonlinear inverse problems, including Particle Swarm Optimization (PSO),¹⁹ the Shuffled Frog Leaping Algorithm,²⁰ Adaptive Chaotic Genetic PSO,²¹ the Grasshopper Optimization Algorithm,²² Adaptive Dragonfly Optimization,²³ the Barnacles Mating Optimizer,²⁴ the Lightning Attachment Procedure Optimizer,²⁵ the Sine Cosine Algorithm,²⁶ adaptive differential evolution methods,^{27,28} improved butterfly and wild horse optimizers,^{29,30} improved backtracking search optimization,³¹ and Hunger Games Search.³²⁻³⁵ However, the performance of these methods is problem-dependent and is strongly affected by the balance between global exploration and local exploitation.³⁶

The Beluga Whale Optimization (BWO) algorithm³⁷ is a recent swarm-intelligence method inspired by the swimming, predation, and whale-fall behaviors of beluga whales. Although BWO has shown promising performance in optimization applications, it may still suffer from insufficient population diversity and inefficient local search. To improve its applicability to geophysical joint inversion, this study develops an Enhanced Beluga Whale Optimization (EBWO) algorithm. EBWO combines Tent chaotic mapping, reverse-learning initialization, and a Lévy-flight-based golden sine offspring updating strategy to improve population diversity, search stability, and convergence behavior.

The main objective of this study is to determine whether EBWO improves the accuracy, stability, and robustness of joint Rayleigh-wave dispersion and VES inversion relative

to the original BWO and PSO. Accordingly, EBWO was first evaluated using benchmark functions, then tested with noise-free and noisy synthetic models, and finally applied to field data in the Besirli region of Türkiye. The comparisons focus on data fit, recovered model accuracy, run-to-run variability, and convergence behavior.

2. Methodology

2.1. Beluga Whale Optimization

The BWO algorithm is a population-based heuristic optimization technique inspired by the swimming, predation, and whale-fall behaviors of beluga whales. Compared to other heuristic algorithms, BWO demonstrates strong competitiveness, particularly in addressing large-scale optimization problems and complex nonlinear functions, offering high efficiency and robust convergence. BWO consists of three main evolutionary phases—Exploration, Exploitation, and Whale Fall—which simulate the swimming, predation, and post-mortem sinking behaviors of beluga whales, respectively.³⁷ The algorithm features a simple structure, an effective balance between global and local search, and excellent adaptability to real-world optimization tasks. Its mathematical formulation is summarized in this section (Figure 1).

Before introducing the mathematical formulation of BWO, we define the notation used in this section. The population at iteration t is denoted as $x_i^t = (x_{i,1}^t, x_{i,2}^t, \dots, x_{i,D}^t)$, where $i = 1, 2, \dots, N$ is the index of the search agent, $j = 1, 2, \dots, D$ is the index of the model-parameter dimension, N is the population size, D is the problem dimension, and $t = 0, 1, \dots, T$ is the iteration number. The lower and upper bounds of the j -th parameter are denoted by L_j and U_j , respectively. Unless otherwise stated, all random variables ξ_k are independently sampled from $U(0,1)$.

For initialization:

$$x_{i,j}^0 = L_j + \xi_{i,j} (U_j - L_j), i = 1, \dots, N, j = 1, \dots, D \quad (1)$$

where $x_{i,j}^0$ is the initial value of the j -th parameter of the i -th search agent, and $\xi_{i,j} \sim U(0,1)$. A balance factor B_f is introduced to regulate the transition between exploration and exploitation:

$$B_f = B_0 \left(1 - \frac{t}{T} \right), B_0 \sim U(0,1) \quad (2)$$

where B_0 is an independently generated random number, t is the current iteration number, and T is the maximum number of iterations.

During the exploration phase, BWO simulates beluga whale swimming behavior. The position update is written

as:

$$x_{i,j}^{t+1} = x_{i,p}^t + (x_{r,p}^t - x_{i,p}^t)(1 + \xi_1) \sin(2\pi\xi_2), j \text{ even} \quad (3)$$

$$x_{i,j}^{t+1} = x_{i,p}^t + (x_{r,p}^t - x_{i,p}^t)(1 + \xi_1) \cos(2\pi\xi_2), j \text{ odd} \quad (4)$$

where p is a randomly selected dimension from $\{1, \dots, D\}$, r is a randomly selected search-agent index different from i , and $\xi_1, \xi_2 \sim U(0,1)$.

During the exploitation phase, BWO simulates beluga whale predation behavior. The position update is given by:

$$x_i^{t+1} = \xi_3 x_{best}^t - \xi_4 x_i^t + C_1 LF(x_i^t - x_{best}^t) \quad (5)$$

$$C_1 = 2\xi_4 \left(1 - \frac{t}{T}\right) \quad (6)$$

where x_i^t , x_r^t , and x_{best}^t are the current individual, a randomly selected individual, and the best individual at iteration t , respectively; $\xi_3, \xi_4 \sim U(0,1)$; and C_1 is the Lévy-flight step-control coefficient. The Lévy-flight term is defined as:

$$LF = 0.05 \frac{\mu\sigma}{|v|^{1/\beta}} \quad (7)$$

$$\sigma = \left[\frac{\Gamma(1+\beta) \sin(\pi\beta/2)}{\Gamma((1+\beta)/2) \beta 2^{(\beta-1)/2}} \right]^{1/\beta} \quad (8)$$

where μ and ν are independent standard normal random variables, $\Gamma(\cdot)$ is the Gamma function, and $\beta = 1.5$.

The whale-fall phase is used to maintain population diversity and reduce premature convergence. The position update is:

$$x_i^{t+1} = \xi_5 x_i^t - \xi_6 x_r^t + \xi_7 x_{step}^t \quad (9)$$

$$x_{step}^t = \exp\left(-C_2 \frac{t}{T}\right)(U - L) \quad (10)$$

$$C_2 = 2W_f N \quad (11)$$

$$W_f = 0.1 - 0.05 \frac{t}{T} \quad (12)$$

where $\xi_5, \xi_6, \xi_7 \sim U(0,1)$, x_{step}^t is the whale-fall step vector, C_2 is the step factor, and W_f is the whale-fall probability.

2.2. Enhanced Beluga Whale Optimization

The proposed EBWO modifies the original BWO by introducing three strategies: Tent chaotic mapping for initialization, reverse learning for improving the initial population, and a Lévy-flight-based golden sine strategy for offspring updating.

2.2.1. Tent chaotic mapping strategy

Instead of using purely random initialization, EBWO first

generates a chaotic sequence using the Tent map:³⁸

$$z_{i,j}^{n+1} = \frac{z_{i,j}^n + \varepsilon_{i,j}^n / N}{a}, 0 \leq z_{i,j}^n < a \quad (13)$$

$$z_{i,j}^{n+1} = \frac{1 - z_{i,j}^n + \varepsilon_{i,j}^n / N}{1 - a}, a \leq z_{i,j}^n \leq 1 \quad (14)$$

where n is the internal iteration index of the Tent map (distinct from the EBWO iteration index t). The initial chaotic seed $z_{i,j}^0$ and the perturbation term are independently sampled from $U(0,1)$, and $a = 0.5$. **Equation 13** is applied recursively to generate the chaotic sequence. The superscript n is not a user-selected control parameter in the main EBWO algorithm: initialization continues only until $N \times D$ chaotic values have been generated, one for each population coordinate (i,j) , after which the values are mapped to the physical parameter bounds using **Equation 14**. Thus, no separate tuning or convergence criterion is required for n .

The chaotic variable is then mapped to the search space by

$$x_{i,j}^0 = L_j + (U_j - L_j)z_{i,j} \quad (15)$$

where L_j and U_j are the lower and upper bounds of the j -th model parameter.

2.2.2. Reverse learning strategy

To improve the quality of the initial population, EBWO generates an opposite candidate solution for each chaotic initial solution:³⁹

$$p_{i,j}^0 = L_j + U_j - x_{i,j}^0 \quad (16)$$

where $p_{i,j}^0$ is the opposite candidate corresponding to $x_{i,j}^0$. The chaotic initial population and the reverse-learning population are then merged, ranked by fitness, and the best N individuals are retained as the initial population.

2.2.3. Lévy-flight-based golden sine offspring updating strategy

In EBWO, the original BWO exploitation update in **Equation 5** is replaced by a Lévy-flight-based golden sine update:⁴⁰

$$x_i^{t+1} = x_i^t \sin(\xi_1) + L(\beta) \xi_2 \sin(\xi_1) |g_1 x_{best}^t - g_2 x_i^t| \quad (17)$$

$$g_1 = -\pi + (1 - \lambda)2\pi, g_2 = -\pi + \lambda 2\pi \quad (18)$$

where g_1 and g_2 are golden sine coefficients, $\lambda = (\sqrt{5} - 1)/2$ is the golden-section coefficient, $L(\beta)$ is the Lévy-flight term, and $\xi_1, \xi_2 \sim U(0,1)$. The symbols g_1 and g_2 are used instead of x_1 and x_2 to avoid confusion with the position vector x . BWO with these three modifications is upgraded to EBWO.

3. Results

3.1. Benchmark function test

Table 1 summarizes the performance of BWO, three single-strategy BWO variants, and the final EBWO on four 30-dimensional benchmark functions following the procedure in a previous study.³³ Each method was run independently 30 times. For each function, the table reports the mean final objective function value and its standard deviation (SD), which quantifies run-to-run variability. Thus, the previously reported Ackley value $6.06140727997691 \times 10^{-5}$ was a mean objective value rather than a 15-digit physical measurement; it is now reported together with its SD as $(6.06 \pm 1.16) \times 10^{-5}$. All results were rounded to three significant figures to avoid false precision. EBWO generally yielded smaller residual objective values and/or lower variability than the original BWO, although the relative performance remained function-dependent.

3.2. Synthetic tests

To evaluate the feasibility of applying the EBWO to joint inversion of Rayleigh wave dispersion curves and VES data, a synthetic four-layer geological model was constructed to reflect typical stratigraphic characteristics encountered in practical engineering applications.⁴¹ The model parameters are summarized in Table 2. The primary factors controlling variations in Rayleigh wave dispersion curves are shear wave velocity and layer thickness, whereas

other parameters have a comparatively minor influence.⁴² To improve computational efficiency, this study limited the inversion of surface wave data to shear wave velocity and layer thickness, while other parameters were fixed using empirical relationships.^{43,44} For the VES data, both resistivity and layer thickness were included in the inversion, with the parameter search ranges provided in Table 2. The joint inversion objective function $\text{Obj}(h, V_s, \rho)$ is defined as:

$$\text{Obj}(h, V_s, \rho) = w_1 \sum_{i=1}^{M_1} \sum_{r=1}^R \frac{[V_{\text{obs}}^r(f_i) - V_{\text{cal}}^r(f_i, h, V_s)]^2}{[V_{\text{obs}}^r(f_i)]^2} + w_2 \sum_{i=1}^{M_2} \frac{[\rho_{\text{obs}}(x_i) - \rho_{\text{cal}}(x_i, h, \rho)]^2}{[\rho_{\text{obs}}(x_i)]^2} \quad (19)$$

where h is the layer-thickness vector, V_s is the shear-wave velocity vector, and ρ is the resistivity vector. $V_{\text{obs}}^r(f_i)$ and $V_{\text{cal}}^r(f_i, h, V_s)$ are the observed and calculated Rayleigh-wave phase velocities of the r -th mode at frequency f_i , respectively. $\rho_{\text{obs}}(x_i)$ and $\rho_{\text{cal}}(x_i, h, \rho)$ are the observed and calculated apparent resistivities at electrode spacing x_i , respectively. M_1 and M_2 are the numbers of dispersion and VES data points, R is the number of considered Rayleigh-wave modes, and w_1 and w_2 are weighting factors ($w_1 = w_2$ in this study).

Rayleigh wave dispersion was computed following a previously described method,³⁵ while forward modeling of

Table 1. Final objective-function values (mean and SD) over 30 independent runs for four benchmark functions

Function	Global optimum (f^*)	Statistic	BWO	BWO + Tent	BWO + reverse learning	BWO + Lévy–golden sine	EBWO
Ackley	0	Mean	6.06×10^{-5}	2.14×10^{-8}	8.01×10^{-10}	6.36×10^{-13}	7.42×10^{-16}
		SD	1.16×10^{-5}	2.80×10^{-5}	0	0	0
Rastrigin	0	Mean	6.28×10^{-3}	2.46×10^{-1}	9.10×10^{-1}	0	0
		SD	4.66×10^{-3}	2.11×10^{-1}	0	0	0
Eggholder	-960	Mean	-457	-686	-435	-878	-989
		SD	37.0	6.14	3.62	2.16	7.61
Schwefel	0	Mean	68.5	6.51×10^{-1}	4.72×10^{-2}	6.47×10^{-3}	6.56×10^{-1}
		SD	35.8	1.11×10^{-2}	7.87×10^{-1}	8.72×10^{-1}	9.69×10^{-1}

Notes: Values are rounded to three significant figures. Mean and SD denote the average final objective-function value and the standard deviation across 30 independent runs, respectively. SD quantifies run-to-run variability; it is not a measurement error or the standard error of the mean. For example, the BWO Ackley result is $(6.06 \pm 1.16) \times 10^{-5}$. Population size $N = 300$, problem dimension $D = 30$, and maximum iterations $T = 100$. Function definitions and search domains follow Ai *et al.*³³

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization.

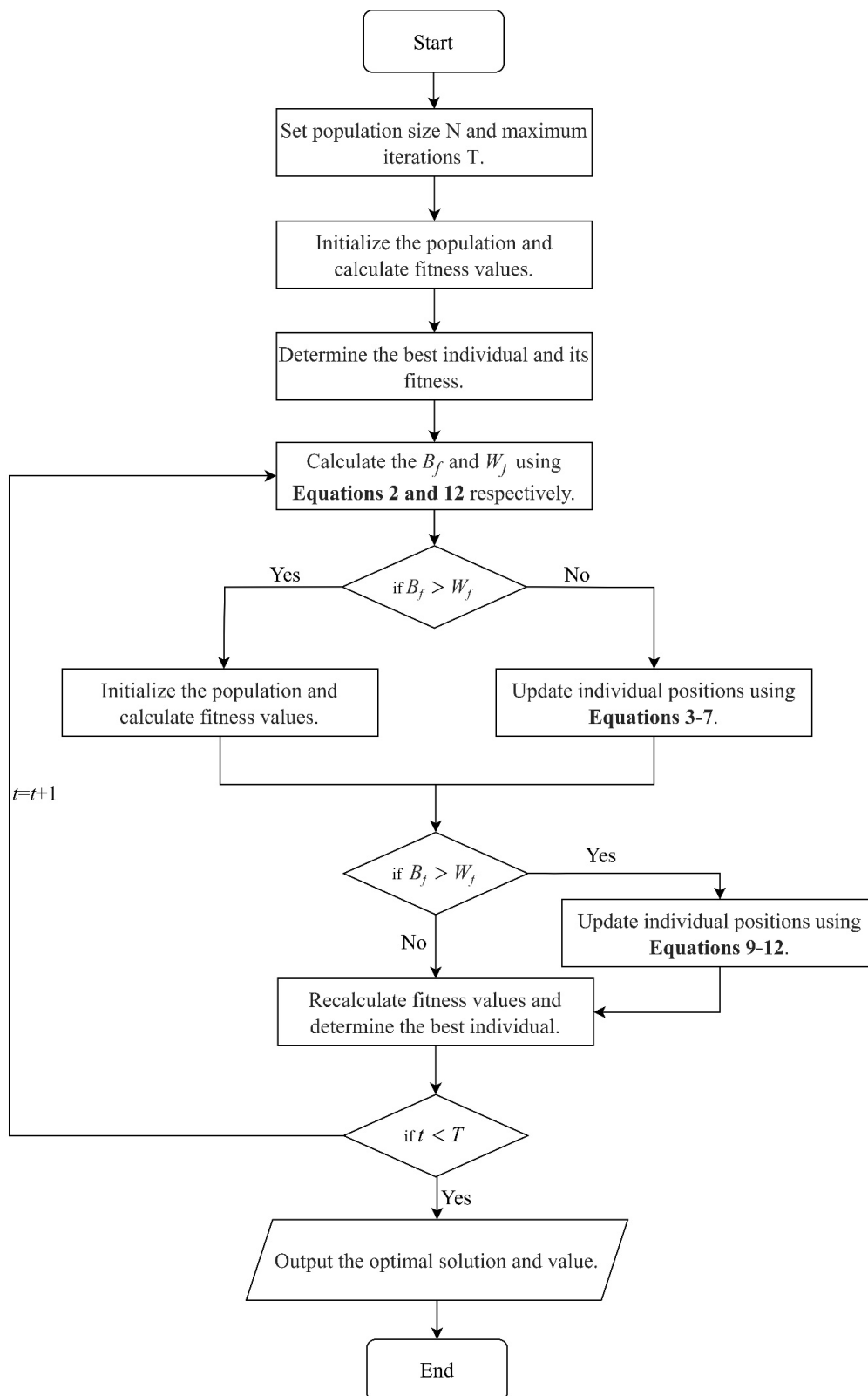


Figure 1. Flowchart of the Beluga Whale Optimization

Table 2. Model parameters of the synthetic model and model space defined for joint inversion

Layer number	Model parameters					Model space		
	v_s (m/s)	σ	ρ_d (g/cm ³)	h (m)	ρ_r (Ω -m)	v_s (m/s)	h (m)	ρ_r (Ω -m)
1	200	0.20	1.8	3	150	80–320	1–5	50–250
2	400	0.25	1.7	11	50	160–640	2–20	20–80
3	250	0.35	1.8	4	150	100–400	1–7	60–300
Half-space	700	0.20	1.9	∞	600	280–11,200	∞	200–1,000

Note: σ is Poisson's ratio.

VES data was conducted according to Senkaya and Karşlı.¹⁷

3.2.1. Synthetic test without noise contamination

Next, BWO, EBWO, and PSO ($w = 0.729$, $c_1 = 2.041$, $c_2 = 0.948$; w is the inertia weight and c_1 , c_2 are the individual and social learning factors, respectively³⁵) were applied to the joint inversion of noise-free Rayleigh wave dispersion curves and VES data. The surface wave dispersion data spanned a frequency range of 5–80 Hz with a 1 Hz interval, while the VES measurements used 30 logarithmically spaced AB/2 points between $10^{0.5}$ m and $10^{2.8}$ m. For all three algorithms, the population size was set to 300 and the maximum number of iterations to 100. To minimize the impact of random variability on the inversion results, each synthetic experiment was performed 30 independent times. The final inverted model was obtained by averaging all runs, and the corresponding standard deviation was used to assess the stability of the optimization algorithm. To further demonstrate the advantages of joint inversion, the same algorithms were also tested using separate inversions in which the objective function included only either the surface-wave dispersion data or the VES observations, with all remaining inversion settings kept identical.

The inversion results for the noise-free datasets are illustrated in Figure 2. The figure is organized by optimizer because its purpose is to compare joint and individual data fits within each algorithm; the cross-algorithm comparison was evaluated based on recovered models (Figure 3), convergence histories (Figure 4), and statistical results (Table 3). For all three optimization methods, both the joint inversion results (pink curves) and the separate inversion results (yellow curves) provided satisfactory fits to the observed data. However, comparing the recovered shear-wave velocity and resistivity structures in Figure 3 reveals that EBWO (blue and yellow solid curves in Figure 3A and 3B) reproduced the reference models (black solid curves) more accurately than the other algorithms, regardless

of whether joint or individual inversion was employed. By comparison, the BWO and PSO results showed more obvious deviations from the synthetic model parameters. In particular, the improvement achieved by joint inversion was clearly visible in the EBWO results, whereas the distinction between joint and separate inversions was less evident for BWO and PSO. These findings suggest that although all methods can adequately fit the observed data, significant differences remain in the recovered subsurface parameters because of the intrinsic non-uniqueness of geophysical inversion problems. Among the tested approaches, EBWO demonstrates the strongest overall inversion capability.

Figure 4 presents the convergence curves of BWO, EBWO, and PSO for the joint inversion case, with logarithmic scaling applied to the horizontal axes. Although the convergence speeds of the three algorithms were generally comparable, EBWO consistently achieved smaller final objective function values. In contrast, the relatively rapid convergence accompanied by larger residual errors in the other algorithms indicates premature convergence, where the optimization process becomes trapped in local minima. This behavior further demonstrates the improved robustness and global-search capability of EBWO compared with BWO and PSO.

The statistical summary of the joint inversion results for the noise-free Rayleigh-wave and VES datasets is listed in Table 3, including the mean values, standard deviations, and relative errors of the recovered parameters. EBWO yielded smaller relative errors and lower parameter uncertainties than both BWO and PSO. Moreover, the consistently small standard deviations across the 30 independent EBWO runs confirm the high stability and repeatability of the algorithm.

In summary, the results indicate that EBWO can reliably and stably invert noise-free Rayleigh-wave

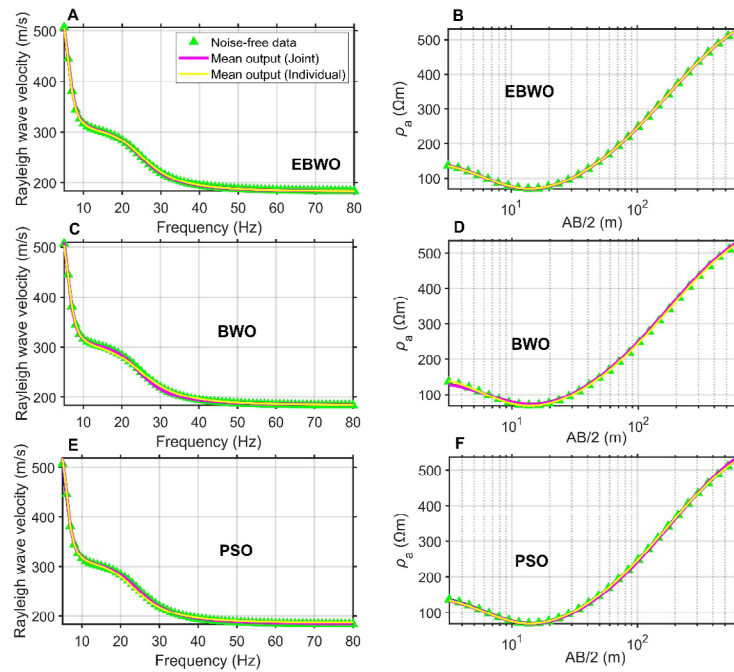


Figure 2. Joint and individual inversion results of noise-free Rayleigh wave dispersion and VES data across algorithms. Joint and individual inversion results of (A) Rayleigh wave dispersion and (B) VES data of EBWO; (C) Rayleigh wave dispersion and (D) VES data of BWO; and (E) Rayleigh wave dispersion and (F) VES data of PSO.

Notes: Green: Noise-free data; Pink: Joint inversion; Yellow: Individual inversion.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

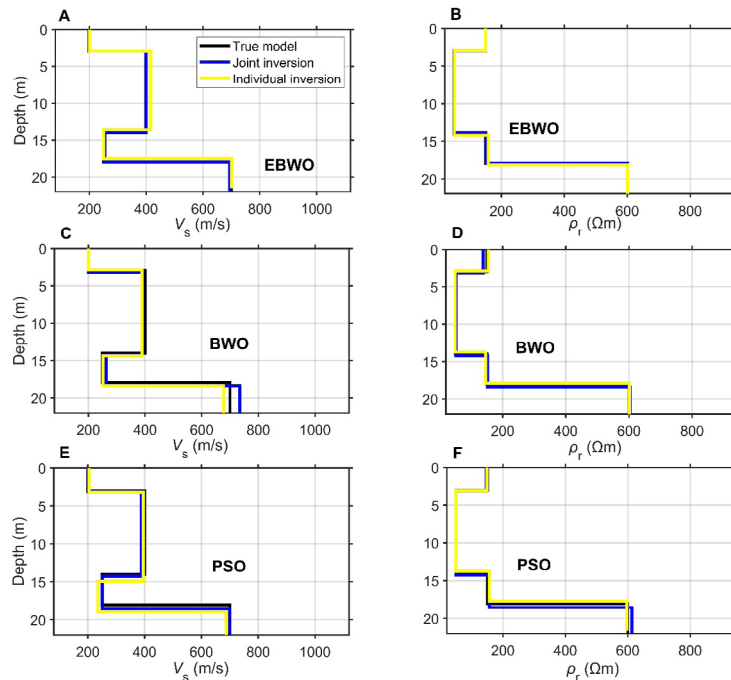


Figure 3. Joint and individual inversion results of noise-free synthetic data using EBWO, BWO, and PSO. (A) V_s profiles recovered using EBWO. (B) ρ_r profiles recovered using EBWO. (C) V_s profiles recovered using BWO. (D) ρ_r profiles recovered using BWO. (E) V_s profiles recovered using PSO. (F) ρ_r profiles recovered using PSO.

Notes: Black: True model; Blue: Joint inversion; Yellow: Individual inversion.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization.

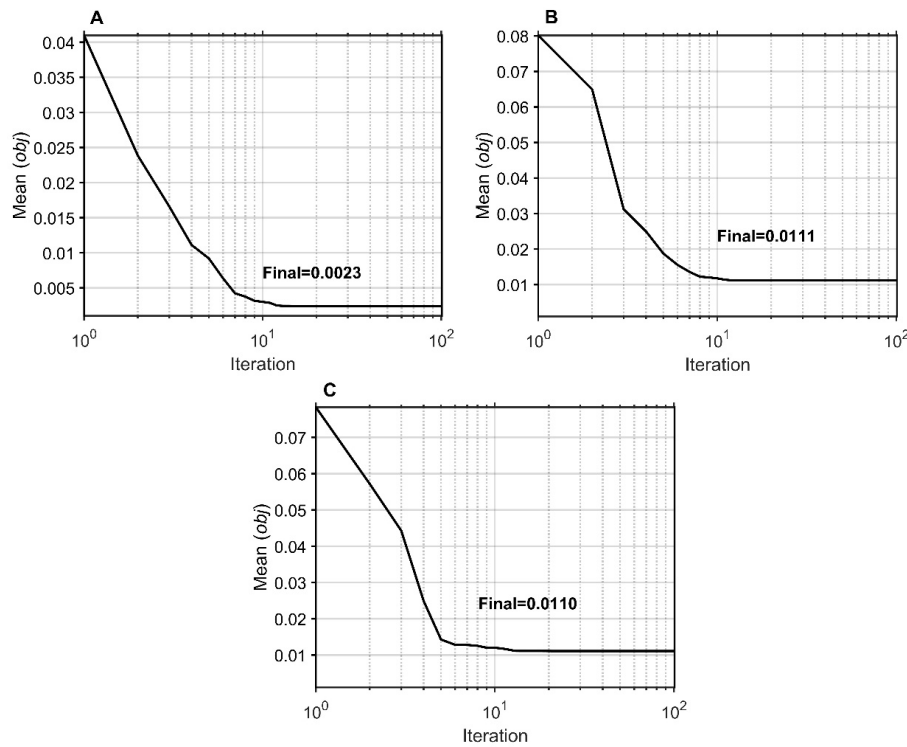


Figure 4. Iteration process of joint inversion of noise-free Rayleigh wave dispersion curves and VES data using (A) EBWO, (B) BWO, and (C) PSO. Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

dispersion and VES data. Compared with BWO and PSO, EBWO exhibited superior optimization efficiency and more effectively approaches the global optimum. The results also demonstrate that EBWO-based joint inversion outperformed separate inversion, highlighting the advantage of combining complementary geophysical datasets to better constrain subsurface structures.

3.2.2. Synthetic tests with noise

To simulate a more realistic scenario with measurement noise, 10% random noise was added to the theoretical model's Rayleigh wave dispersion curves and VES data following Equation 12:³⁵

$$d_{\text{noise}} = d + NR \cdot \bar{d} \cdot (\xi_1 - \xi_2), NR=10\% \quad (20)$$

where d is the original noise-free theoretical data, d_{noise} is the noise-contaminated data, NR is the noise ratio, and $\bar{d}$ is the mean value of d . ξ_1 and ξ_2 are random vectors with elements independently sampled from $U(0,1)$. In this study, $NR = 10\%$.

Using the same inversion settings adopted for the noise-free experiments, the BWO, EBWO, and PSO algorithms

were implemented for both separate and joint inversions of the noise-corrupted datasets. The corresponding inversion results are displayed in Figure 5. Despite the presence of added noise, all three optimization methods still produced acceptable fits to the theoretical responses for both the individual and combined inversion schemes. Nevertheless, noticeable discrepancies appeared in the recovered shear-wave velocity and resistivity models, particularly for the BWO and PSO results, which deviated considerably from the true subsurface structure (Figure 6). These differences mainly arise from noise in the observed data. Among the tested algorithms, EBWO remained the most effective, recovering subsurface models that most closely resemble the true structure. It is also observed that the mismatch between the inverted and reference models was slightly larger in the joint inversion results compared with the separate inversions.

Figure 7 presents the convergence histories of BWO, EBWO, and PSO during the joint inversion of noisy datasets, with the horizontal axis plotted on a logarithmic scale. As anticipated, the introduction of noise led to larger convergence errors for all algorithms compared with the

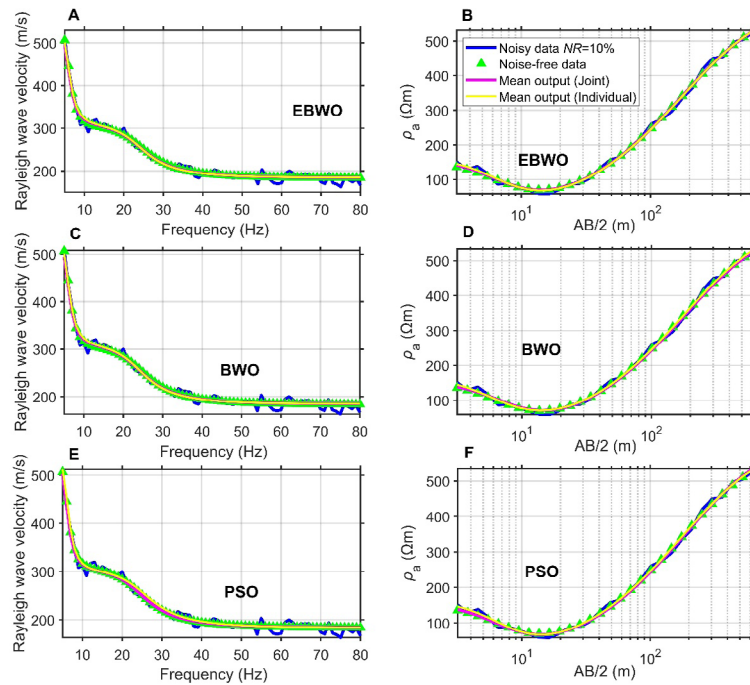


Figure 5. Joint and individual inversion of noise-contaminated Rayleigh wave dispersion and VES data across algorithms. Joint and individual inversion results of (A) Rayleigh wave dispersion and (B) VES data of EBWO; (C) Rayleigh wave dispersion and (D) VES data of BWO; and (E) Rayleigh wave dispersion and (F) VES data of PSO.

Notes: Blue: Noisy data; Pink: Joint inversion; Yellow: Individual inversion; Green triangles: Noise-free data.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

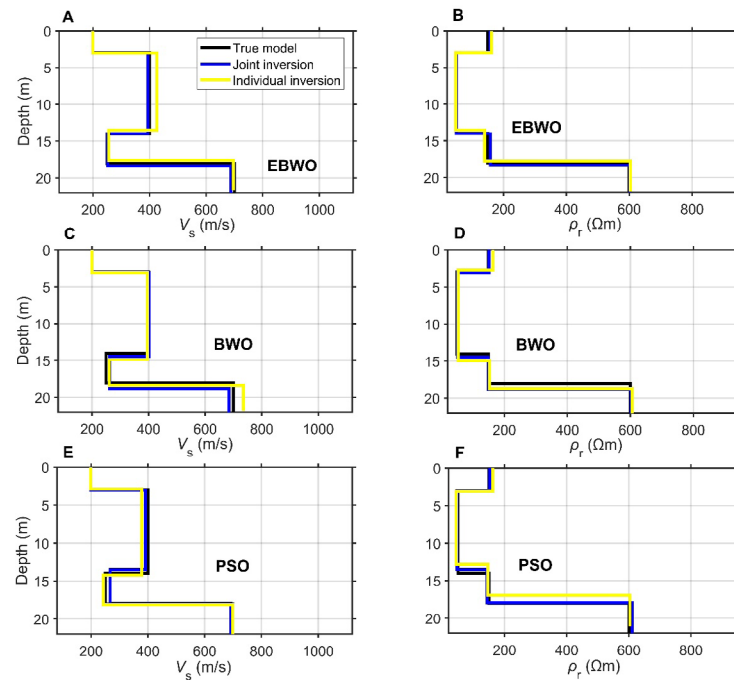


Figure 6. Joint and individual inversion of noise-contaminated synthetic data using EBWO, BWO, and PSO. (A) V_s profiles recovered using EBWO. (B) ρ_r profiles recovered using EBWO. (C) V_s profiles recovered using BWO. (D) ρ_r profiles recovered using BWO. (E) V_s profiles recovered using PSO. (F) ρ_r profiles recovered using PSO. Black: True model; Blue: Joint inversion; Yellow: Individual inversion.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization.

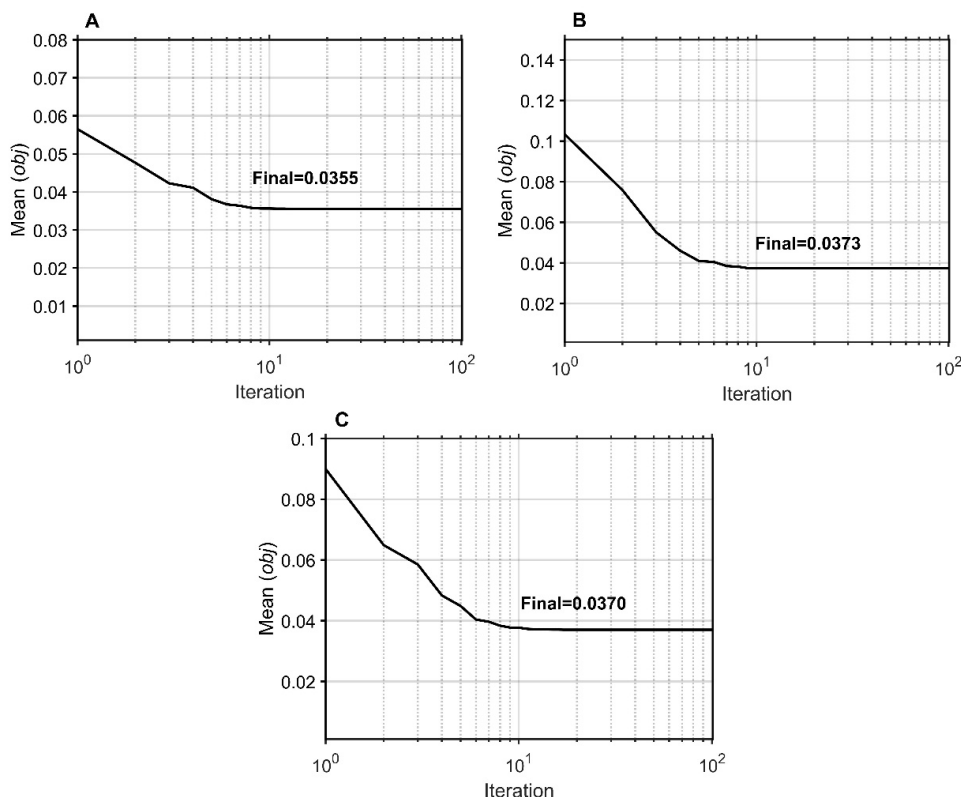


Figure 7. Iteration process of joint inversion of noise-contaminated Rayleigh wave dispersion curves and VES data using (A) EBWO, (B) BWO, and (C) PSO

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

noise-free case. Even so, EBWO consistently produced the smallest objective function values throughout the inversion process, while the overall convergence behavior of the three algorithms remained broadly similar.

Table 3 summarizes the noisy joint-inversion statistics for EBWO, BWO, and PSO. Although noise increased both relative errors and parameter variability, EBWO generally retained smaller errors and/or standard deviations than the two comparison algorithms. The reformatted table reports each estimate as mean $\pm$ SD so that the average and its run-to-run variability can be distinguished unambiguously.

3.3. Field data application

A previous multichannel surface-wave and VES investigation was carried out in the Besirli district of Ortahisar, Trabzon, Türkiye.¹⁸ Figure 8 presents the location of the survey area. Because the site is located adjacent to the Black Sea coastline, the subsurface is likely affected by seawater intrusion, which can significantly influence both

electrical and seismic properties.

As reported by Senkaya *et al.*,¹⁸ their seismic survey employed a 24-channel PASI 16S24-U acquisition system (Italy) coupled with 4.5 Hz vertical-component geophones. Data acquisition was performed with a sampling interval of 1 ms and a recording duration of 1,024 ms, using a 2 m receiver spacing and a total spread length of 46 m. An 8 m source offset was selected to reduce near-offset effects while preserving an adequate signal-to-noise ratio. Seismic energy was generated using a 10 kg hammer impacting a rigid plastic plate, and each shot location was repeated five times with subsequent stacking to improve signal quality. The extracted dispersion curve is illustrated by the blue solid line in the panels on the left of Figure 9.

The direct-current resistivity measurements were collected using an ABEM Terrameter LS system (Sweden) configured with a Schlumberger electrode array. The maximum half-current electrode spacing (AB/2) reached

Table 3. Joint-inversion results obtained with EBWO, BWO, and PSO for the noise-free and noisy synthetic models

Algorithm	Model parameter	True value	Noise-free case		Noisy case	
			Mean $\pm$ SD	Relative error	Mean $\pm$ SD	Relative error
EBWO	vs1 (m/s)	200	199.63 $\pm$ 0.69	0.0024	198.25 $\pm$ 3.99	0.0141
	vs2 (m/s)	400	399.33 $\pm$ 5.72	0.010	393.61 $\pm$ 17.73	0.0313
	vs3 (m/s)	250	254.30 $\pm$ 4.31	0.0172	252.19 $\pm$ 17.12	0.0484
	vs4 (m/s)	700	694.74 $\pm$ 0.15	0.0075	687.49 $\pm$ 7.03	0.0178
	h1 (m)	3	3.00 $\pm$ 0.04	0.0109	2.96 $\pm$ 0.003	0.0128
	h2 (m)	11	10.89 $\pm$ 0.05	0.0099	11.04 $\pm$ 0.52	0.0340
	h3 (m)	4	4.08 $\pm$ 0.008	0.0220	4.36 $\pm$ 0.27	0.0919
	pr1 (Ω -m)	150	150.36 $\pm$ 1.06	0.0050	157.49 $\pm$ 6.83	0.0499
	pr2 (Ω -m)	50	49.48 $\pm$ 0.32	0.0103	49.60 $\pm$ 1.85	0.0262
	pr3 (Ω -m)	150	150.88 $\pm$ 6.16	0.0290	158.31 $\pm$ 13.57	0.0639
	pr4 (Ω -m)	600	600.68 $\pm$ 0.41	0.0011	602.31 $\pm$ 3.34	0.0039
BWO	vs1 (m/s)	200	200.17 $\pm$ 0.22	0.0008	199.72 $\pm$ 0.549	0.0019
	vs2 (m/s)	400	390.59 $\pm$ 11.002	0.0235	401.28 $\pm$ 20.50	0.0362
	vs3 (m/s)	250	262.54 $\pm$ 6.37	0.0501	261.91 $\pm$ 12.09	0.0476
	vs4 (m/s)	700	735.08 $\pm$ 44.61	0.0501	683.44 $\pm$ 48.28	0.0487
	h1 (m)	3	3.21 $\pm$ 0.08	0.0717	3.09 $\pm$ 0.10	0.0308
	h2 (m)	11	11.02 $\pm$ 0.42	0.0275	11.36 $\pm$ 0.91	0.0587
	h3 (m)	4	4.17 $\pm$ 0.14	0.0449	4.32 $\pm$ 0.82	0.1450
	pr1 (Ω -m)	150	138.56 $\pm$ 8.02	0.0762	151.67 $\pm$ 6.65	0.0313
	pr2 (Ω -m)	50	52.18 $\pm$ 2.22	0.0437	50.84 $\pm$ 2.54	0.0359
	pr3 (Ω -m)	150	152.18 $\pm$ 9.52	0.0449	148.31 $\pm$ 22.73	0.1071
	pr4 (Ω -m)	600	603.13 $\pm$ 9.57	0.0112	602.67 $\pm$ 8.79	0.0103
PSO	vs1 (m/s)	200	200.81 $\pm$ 0.98	0.0040	200.08 $\pm$ 4.22	0.0149
	vs2 (m/s)	400	387.64 $\pm$ 8.91	0.0308	391.27 $\pm$ 27.17	0.0480
	vs3 (m/s)	250	250.65 $\pm$ 4.08	0.0115	266.73 $\pm$ 8.13	0.0669
	vs4 (m/s)	700	700.49 $\pm$ 80.46	0.0812	692.61 $\pm$ 35.23	0.0355
	h1 (m)	3	3.04 $\pm$ 0.14	0.0335	3.02 $\pm$ 0.07	0.0166
	h2 (m)	11	11.26 $\pm$ 0.88	0.0566	10.46 $\pm$ 0.04	0.0486
	h3 (m)	4	4.26 $\pm$ 0.11	0.0659	4.44 $\pm$ 0.04	0.1112
	pr1 (Ω -m)	150	146.55 $\pm$ 5.13	0.0242	152.57 $\pm$ 3.20	0.0171
	pr2 (Ω -m)	50	49.55 $\pm$ 4.44	0.0628	46.77 $\pm$ 2.35	0.0645
	pr3 (Ω -m)	150	156.91 $\pm$ 4.70	0.0461	144.34 $\pm$ 7.80	0.0377
	pr4 (Ω -m)	600	613.06 $\pm$ 2.74	0.0217	610.61 $\pm$ 11.19	0.0176

Note: Values are mean $\pm$ SD over 30 independent runs; relative error is dimensionless.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization.

60 m, and the observed apparent-resistivity data are shown as the blue solid line in the panels on the right of Figure 9. In a previous study,¹⁸ inversion was performed using a gradient-based local search. The initial estimate of the layer number was inferred from the VES response, whereas the maximum investigation depth was constrained by the longest wavelength in the dispersion data. Preliminary velocity and resistivity parameters were selected from the characteristics of the measured dispersion and apparent-resistivity curves.

Given the coastal environment of the Besirli area, seawater penetration is hypothesized to decrease both resistivity and shear-wave velocity in deeper sections of the subsurface. In addition, weak and water-saturated formations may introduce considerable complexity into the dispersion behavior of surface waves,^{44,45} making the dataset well suited for evaluating the robustness of joint inversion techniques. Using the same control parameters adopted in the synthetic experiments, the BWO, EBWO, and PSO algorithms were applied to both separate and combined inversions of the field data. The predefined model space, established from prior geophysical information, is summarized in Table 4, whereas the inversion results are presented in Figures 9 and 10.

Figure 9 displays the fitting performance of EBWO, BWO, and PSO for both individual and joint inversion schemes. Although field noise and environmental interference were present, all three approaches reproduced the observed data reasonably well, and the visual differences between joint and separate inversions remained relatively small. Nevertheless, the recovered

subsurface models differed substantially from the earlier interpretation reported in Senkaya *et al.*¹⁸ (Figure 10). Among the tested approaches, the EBWO-based joint inversion produced results that are most compatible with the previous geological and geophysical interpretation while simultaneously yielding the smallest misfit values.

Interpretation of the inverted models indicates that the shallow subsurface beneath the Besirli site can be divided into five principal layers (Figure 10). At depths greater than approximately 6 m, a low-velocity and low-resistivity zone was observed, interpreted as evidence of seawater intrusion. Figure 11 compares the convergence histories of the three optimization algorithms during the joint inversion process, with logarithmic scaling applied to the horizontal axes, consistent with the noisy synthetic experiments. Although the overall convergence trends were similar, EBWO consistently attained lower objective function values than BWO and PSO. The average values and corresponding standard deviations of the inversion results are listed in Tables 5–7, further demonstrating the improved accuracy and stability of EBWO.

In summary, EBWO outperforms both BWO and PSO in terms of inversion reliability and stability. Even in the presence of significant noise contamination, EBWO retrieves model parameters that agree closely with previous interpretations, while also producing lower standard deviations and better overall agreement with both the dispersion and VES observations.

The definition of the objective function is another important aspect of joint inversion. In the present work,

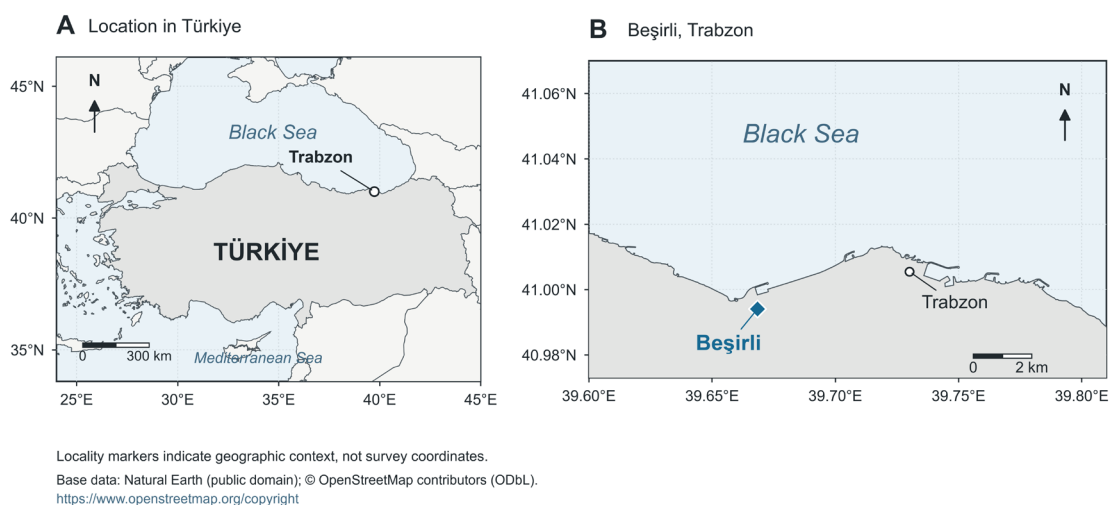


Figure 8. Simplified map of the study area. (A) Location in Türkiye. (B) Besirli, Trabzon. (source: OpenStreetMap, © OpenStreetMap contributors, CC BY-SA 2.0).

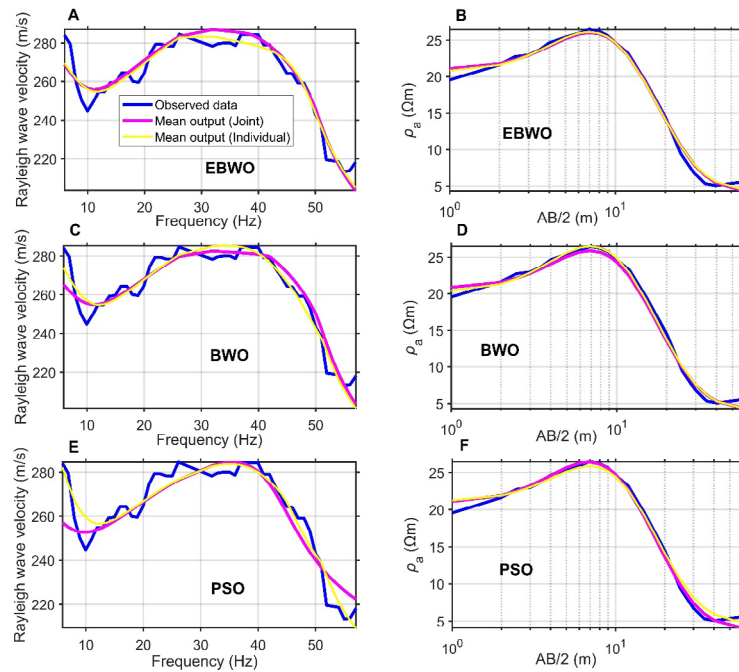


Figure 9. Joint and individual inversion results for Rayleigh wave and VES data in the Besirli area across algorithms. Joint and individual inversion results of (A) Rayleigh wave dispersion and (B) VES data of EBWO; (C) Rayleigh wave dispersion and (D) VES data of BWO; and (E) Rayleigh wave dispersion and (F) VES data of PSO.

Notes: Blue: Field data; Pink: Joint inversion; Yellow: Individual inversion.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

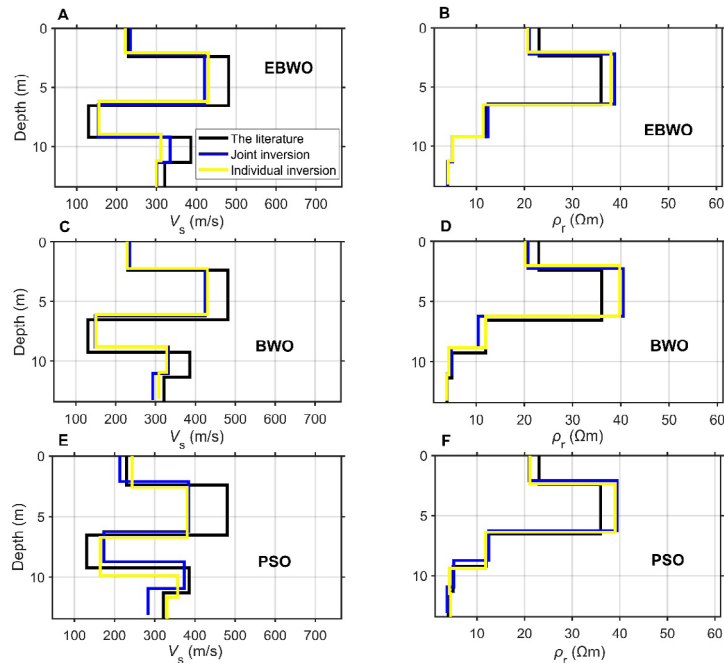


Figure 10. Joint and individual inversion results of Rayleigh wave and VES data from the Besirli area using EBWO, BWO, and PSO. (A) V_s profiles recovered using EBWO. (B) ρ_r profiles recovered using EBWO. (C) V_s profiles recovered using BWO. (D) ρ_r profiles recovered using BWO. (E) V_s profiles recovered using PSO. (F) ρ_r profiles recovered using PSO.

Notes: Black: Previous inversion from Senkaya *et al.*¹⁸; Blue: Joint inversion; Yellow: Individual inversion.

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization.

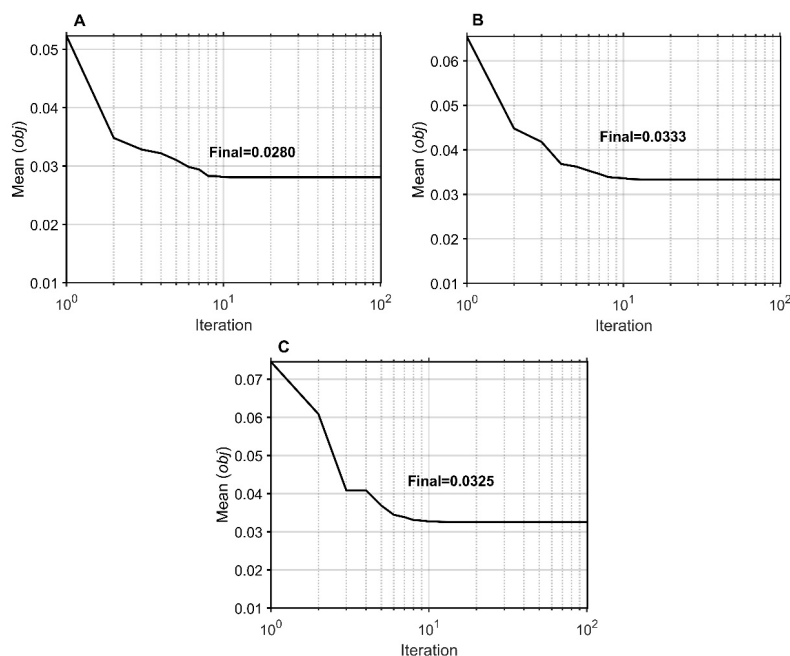


Figure 11. Iteration process of joint inversion of Rayleigh wave dispersion curves and VES data from the Besirli area using (A) EBWO, (B) BWO, and (C) PSO

Abbreviations: BWO: Beluga Whale Optimization; EBWO: Enhanced Beluga Whale Optimization; PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

identical weights (0.5:0.5) were assigned to the surface-wave and resistivity datasets. To investigate the influence of weighting, an additional EBWO inversion was conducted using randomly assigned weighting coefficients. The corresponding results are presented in Figure 12 and Table 8. Only minor differences were observed between the two weighting strategies; however, the equal-weight configuration produced slightly more stable and accurate inversion results. Although the weighting effect was not particularly significant for this dataset, multi-objective inversion approaches are still recommended. Such methods avoid the need for subjective weighting selection and offer a more adaptive, balanced, and robust strategy for integrating multiple geophysical datasets.

Table 4. Model space constructed from available geophysical results

Layer number	v_s (m/s)	h (m)	ρ_r ($\Omega\cdot\text{m}$)
1	50–400	0.5–5	5–40
2	100–800	1.0–7	10–70
3	20–300	0.5–5	2–25
4	100–600	0.5–5	1–10
Half-space	50–550	–	1–10

Table 5. Mean values and SDs from the EBWO-based joint inversion of Rayleigh-wave dispersion and VES data for the Besirli area

Layer number	v_s (m/s)		h (m)		ρ_r ($\Omega\cdot\text{m}$)	
	Mean	SD	Mean	SD	Mean	SD
1	235.07	0.80	2.24	0.05	20.99	0.18
2	420.49	3.07	4.20	0.04	38.81	0.54
3	153.89	4.27	2.78	0.06	12.39	0.40
4	334.20	17.06	2.10	0.07	4.94	0.17
Half-space	300.00	0.91	–		3.94	0.01

Abbreviations: EBWO: Enhanced Beluga Whale Optimization; VES: Vertical electrical sounding.

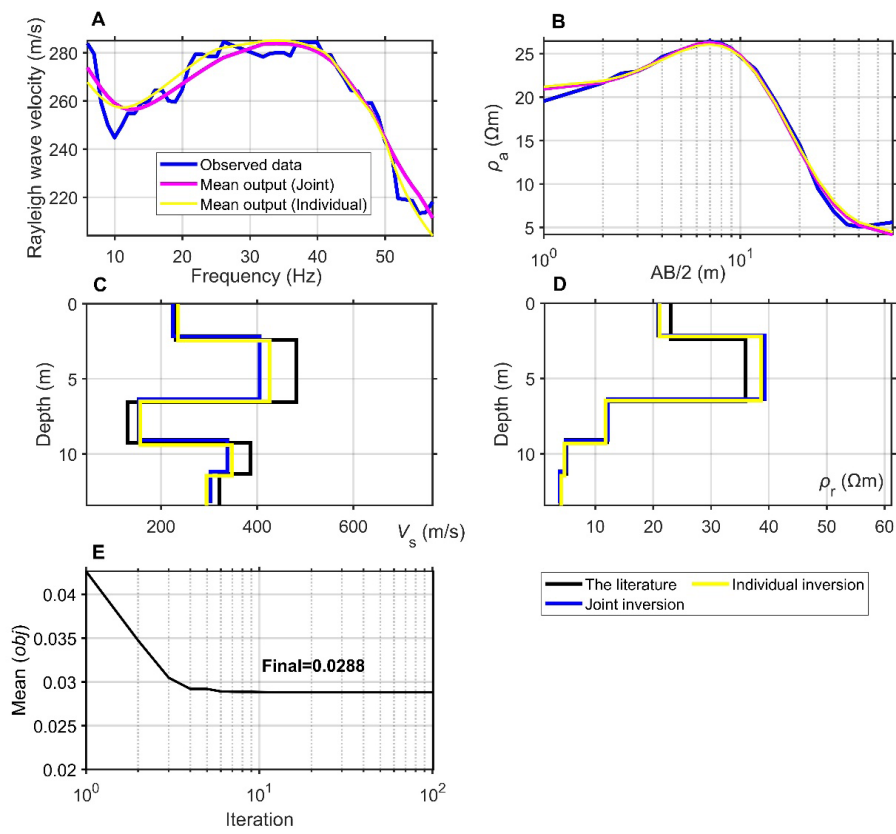


Figure 12. Joint inversion results for the Besirli area obtained using the EBWO algorithm with a modified objective function incorporating random weighting factors. (A) Fitting of the Rayleigh-wave dispersion data. (B) Fitting of the VES data. (C–D) Comparison of the (C) inverted shear-wave velocity profile and (D) inverted resistivity profile with previously reported results.¹⁸ (E) Convergence behavior during the inversion iterations. Abbreviations: EBWO: Enhanced Beluga Whale Optimization; VES: Vertical electrical sounding.

Table 6. Mean values and SDs from the BWO-based joint inversion of Rayleigh-wave dispersion and VES data for the Besirli area

Layer number	v_s (m/s)		h (m)		ρ_r ($\Omega\cdot m$)	
	Mean	SD	Mean	SD	Mean	SD
1	235.09	4.42	2.25	0.14	20.74	0.88
2	424.85	14.50	3.96	0.08	40.54	4.93
3	149.13	6.64	2.63	0.20	10.42	2.42
4	332.09	2.05	2.19	0.09	4.93	0.31
Half-space	292.66	9.39	–		3.93	0.08

Abbreviations: BWO: Beluga Whale Optimization; VES: Vertical electrical sounding.

Table 7. Mean values and SDs from the PSO-based joint inversion of Rayleigh-wave dispersion and VES data for the Besirli area

Layer number	v_s (m/s)		h (m)		ρ_r ($\Omega\cdot m$)	
	Mean	SD	Mean	SD	Mean	SD
1	212.58	2.86	2.11	0.08	20.95	1.08
2	384.27	25.71	4.14	0.25	39.45	0.03
3	173.46	3.60	2.48	0.16	12.44	0.04
4	373.47	36.76	2.19	0.31	5.13	0.17
Half-space	283.23	1.72	–		3.73	0.37

Abbreviations: PSO: Particle Swarm Optimization; VES: Vertical electrical sounding.

Table 8. Mean values and SDs from the EBWO-based joint inversion with randomly assigned objective function weights for the Besirli area

Layer number	v_s (m/s)		h (m)		ρ_r (Ω -m)	
	Mean	SD	Mean	SD	Mean	SD
1	224.07	11.68	2.14	0.10	20.81	0.52
2	405.29	3.51	4.20	0.007	39.26	0.24
3	153.89	2.13	2.70	0.02	12.18	0.36
4	337.67	25.68	2.09	0.01	4.89	0.006
Half-space	303.27	8.42	–		3.81	0.04

Abbreviation: EBWO: Enhanced Beluga Whale Optimization.

4. Discussion

A major difficulty in the inversion of surface-wave data for near-surface shear-wave velocity characterization is the determination of an appropriate subsurface layering scheme. The selected number of layers has a direct influence on the stability and accuracy of the inverted model. Several studies have attempted to overcome this limitation using different methodologies. For example, Killingbeck *et al.*⁴⁶ applied a transdimensional Markov Chain Monte Carlo (MCMC) framework to simultaneously estimate both the layer number and the associated model parameters, whereas Wang *et al.*⁴⁷ incorporated layer-related parameters into a global optimization scheme for joint inversion. Nevertheless, considering the number of layers as an additional unknown significantly increases the dimensionality of the inversion problem, thereby complicating the search space and making it more challenging for optimization algorithms to maintain an effective balance between global exploration and local exploitation.

In many existing studies, relatively broad parameter bounds are assigned to variables such as layer number, shear-wave velocity, and compressional-wave velocity, while the strong physical-property contrasts that may exist between neighboring layers are often insufficiently considered. As a consequence, the admissible model space becomes excessively large, which further complicates stochastic optimization. To alleviate this issue, Ai *et al.*⁴⁸ proposed a transdimensional inversion strategy based on multiple model spaces, enabling simultaneous estimation

of layer number while improving the efficiency of parameter-space exploration. In the present work, this issue was not examined in detail because previous geophysical investigations in the study region had already provided reliable information on the subsurface layering. Future studies will focus on the direct and efficient inversion of layer numbers. One potential strategy is to discretize the subsurface into finely sampled and uniformly distributed layers, allowing the inversion procedure to adaptively identify the effective layering structure without explicitly prescribing the number of layers beforehand.⁴⁹

Another important issue in surface-wave analysis is the mode-kissing phenomenon, which may result in incorrect mode identification and inaccurate phase-velocity picking. Although this problem was not specifically addressed in the current study, it remains an important subject for future research. Several approaches have been developed to reduce mode-kissing. For example, high-resolution transform techniques, such as the vector wavenumber transformation method,⁵⁰ can produce clearer dispersion-energy images, thereby improving mode separation. In addition, Song *et al.*⁵¹ introduced a deep learning-based framework for automatic extraction of dispersion curves, reducing subjective human intervention and partially mitigating mode-kissing effects. Zhang *et al.*⁵² proposed a sequential joint inversion approach based on a generalized misfit function that simultaneously incorporates fundamental and higher-mode dispersion information, thereby simplifying the complexity associated with mode numbering. Likewise, Yan *et al.*⁵³ developed an inversion workflow that avoids manual mode identification during dispersion-curve inversion, while Le *et al.*⁵⁴ modified the PSO algorithm to directly invert Rayleigh-wave frequency-velocity spectra, eliminating the need for explicit dispersion-curve extraction.

Furthermore, although the present study introduces three modifications to the original BWO algorithm, namely Tent chaotic mapping, reverse-learning initialization, and Lévy-flight-based golden sine updating, the main objective of this work is not to provide a complete theoretical analysis of global optimizers. According to the No-Free-Lunch theorem, no single optimization algorithm can be universally superior for all types of problems. Therefore, the effectiveness of an improved metaheuristic method should be evaluated in relation to the specific inverse problem being solved. In this study, EBWO is developed and tested mainly as a practical optimization tool for joint inversion of Rayleigh-wave dispersion and VES data, rather than as a general-purpose optimizer for all benchmark or engineering problems. Nevertheless, the effects of the three improvement strategies can still be interpreted in terms

of exploration and exploitation. Tent chaotic mapping improves the diversity and spatial coverage of the initial population, helping the algorithm explore the model space more effectively at the early stage. Reverse learning further enriches the initial candidate solutions by considering opposite positions in the search space, thereby reducing the possibility that the initial population is concentrated in a limited region. The modified Lévy-flight-based golden sine update enhances the offspring generation process by combining long-range perturbation with nonlinear local search, helping the algorithm escape local optima while maintaining convergence toward promising regions. The partial comparison results in Table 1 also support this interpretation. The BWO variants using Tent chaotic mapping, reverse learning, and modified Lévy flight individually showed varying degrees of improvement over the original BWO, indicating that each strategy contributes to the optimization process. However, the final EBWO, which integrates all three strategies, generally provided better optimization accuracy and stability than the single-strategy variants. This suggests that the three components are complementary: chaotic mapping and reverse learning mainly improve the initial exploration capability, whereas the modified Lévy golden sine update strengthens the search and convergence behavior during subsequent iterations. Their combination therefore provides a more balanced exploration-exploitation mechanism than any individual modification alone. It should also be noted that a rigorous theoretical derivation of the coupling mechanism among these strategies would require a dedicated analysis of stochastic search dynamics, convergence behavior, and parameter sensitivity. Such an analysis is beyond the scope of the present geophysical inversion study. Instead, this paper focuses on whether the proposed EBWO can improve the accuracy, stability, and robustness of Rayleigh-wave and VES joint inversion. The benchmark tests, synthetic inversion experiments, and field-data application collectively demonstrate that EBWO is effective for this purpose. Future work may further investigate the theoretical convergence properties and adaptive parameter-control mechanisms of EBWO, especially for high-dimensional and multimodal geophysical inverse problems.

Finally, beyond algorithmic developments, the integration of multiple independent datasets can further improve inversion reliability and reduce ambiguities associated with mode identification and subsurface layering. Joint inversion and multi-source data fusion methods that incorporate geological, geotechnical, and petrophysical constraints have shown considerable potential for cross-validating inversion results and enhancing the robustness

of the final subsurface model.⁵⁵⁻⁵⁹

5. Conclusion

This study investigated the applicability of EBWO to the joint inversion of Rayleigh-wave dispersion and VES data. The method was evaluated using benchmark functions, synthetic models with and without noise, and field data from the Besirli region in Türkiye. The main conclusions are as follows:

- (i) BWO as an effective nonlinear optimization approach for joint inversion

The EBWO algorithm provides an effective optimization framework for seismic-electrical joint inversion. By combining Rayleigh-wave and VES data, the inversion benefits from the complementary sensitivities of shear-wave velocity and electrical resistivity, leading to more reliable estimates of layer thickness, velocity, and resistivity than separate inversions.

- (ii) Improved global exploration and convergence characteristics

The three EBWO modifications, namely Tent chaotic mapping, reverse-learning initialization, and Lévy-flight-based golden sine updating, improve population diversity and convergence behavior relative to the original BWO. The synthetic tests show that EBWO produced more stable and accurate inversion results than BWO and PSO, especially under noise-contaminated conditions.

- (iii) Superior performance in field-data inversion

The field-data application demonstrates that EBWO can recover subsurface models that agree well with previous geophysical interpretations while producing lower data-misfit values. This suggests that EBWO is suitable for practical near-surface inversion problems involving complex and non-unique model spaces.

Future work should further improve EBWO in two directions. First, adaptive parameter-control strategies can be introduced to adjust the population size, Lévy-flight intensity, and exploration-exploitation balance during iteration. Second, EBWO can be extended to transdimensional or multimodal inversion frameworks, enabling more automatic estimation of layer numbers, mode identification, and model parameters. These developments would make EBWO more flexible for complex geophysical applications, such as self-potential, magnetotelluric, and seismic-source inversion.

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Conflict of interest

The authors declare no potential conflict of interest.

Author contributions

Conceptualization: All authors

Formal analysis: Daiguang Fu

Investigation: Fagang Wang, Liming Zhou

Methodology: Fagang Wang, Daiguang Fu

Writing–original draft: Fagang Wang

Writing–review & editing: Fagang Wang, Liming Zhou

Availability of data

Field data sets from the Besirli site in Türkiye were acquired from the study of Senkaya *et al.*¹⁸ Data are available from the corresponding author on reasonable request.

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