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Deblending of marine simultaneous source seismic data via nonsubsamped contourlet transform and twin support vector machine

Kun Zou¹, Jianhua Wang^{2*}, Shuaibing Li³, Yu Zhong¹, Yong Liu¹, Yanan Zhang¹, Yandong Wang², and Hanming Gu⁴¹School of Artificial Intelligence, Hubei University of Automotive Technology, Shiyan, Hubei, China²CNOOC Research Institute Ltd., and the National Engineering Research Center of Offshore Oil and Gas Exploration, Beijing, China³School of Artificial Intelligence and Big Data, Sichuan University of Arts and Science, Dazhou, Sichuan, China⁴School of Geophysics and Geomatics, China University of Geosciences, Wuhan, Hubei, China

Abstract

In marine simultaneous source acquisition, although acquisition efficiency has significantly improved, the nearly concurrent firing of multiple sources introduces strong interference among source wavefields in the recorded data, which must be deblended before further processing. In this study, we adopt an iterative shrinkage thresholding deblending framework in a sparse transform domain. For such methods, designing an appropriate thresholding strategy to handle transform domain coefficients effectively is critical. To this end, the nonsubsamped contourlet transform (NSCT), which provides a multiscale, multidirectional, and shift-invariant representation, is employed. A refined shrinkage thresholding strategy tailored to the NSCT domain coefficients of blended seismic data is proposed. First, the blended data are decomposed into subbands at different scales and directions using the NSCT. Subsequently, an adaptive threshold estimation scheme is developed to accommodate the different distributions of useful signal and blending noise across scales and directions. However, coefficient heterogeneity exists not only across NSCT scales and directions but also within each directional subband. Therefore, after constructing scale- and direction-adaptive baseline thresholds, a twin support vector machine classifier is introduced to distinguish useful-signal-dominant coefficients from blending-noise-dominant coefficients, enabling class-dependent threshold refinement. Finally, shrinkage is applied to the NSCT coefficients using the refined coefficient-wise thresholds. Tests on synthetic and field datasets verify the effectiveness of the proposed method in separation.

Keywords: Marine simultaneous source; Deblending; Adaptive threshold; Nonsubsamped contourlet transform; Twin support vector machine

***Corresponding author:**Jianhua Wang
(Wangjh7@cnooc.com.cn)

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1. Introduction

Marine simultaneous source survey, by allowing multiple sources to fire nearly simultaneously,¹⁻³ can significantly improve acquisition efficiency, increase data

productivity, and reduce exploration costs without extending survey duration. This acquisition mode is highly compatible with modern high-density and wide-azimuth seismic acquisition systems⁴ and has therefore become an important strategy for improving the efficiency of offshore seismic exploration. However, the data acquired in this manner contain strong interference from the overlap of wavefields generated by different sources. Such records are commonly referred to as blended data. Conventional seismic data processing workflows are primarily designed for single-source acquisition.⁵ Therefore, they are difficult to apply directly to blended records. As a result, accurate separation of blended data before subsequent imaging, inversion, and interpretation has become a critical technical step and an active research topic in marine seismic exploration.

In marine simultaneous-source acquisition, random time dithering is commonly adopted to reduce coherent interference among sources. After applying time-shift correction to the primary source, a procedure often termed pseudo-deblending, the wavefields of the secondary source become incoherent in non-common shot gathers and are therefore typically regarded as blending noise. In contrast, the useful signal associated with the primary source remains coherent.^{6,7} This difference in coherence provides an important physical basis for deblending. By exploiting this property, a variety of deblending methods have been developed. Existing approaches can be broadly classified into three categories.

The first category consists of denoising-based methods. These methods treat the incoherent contributions of secondary sources as noise and suppress them using denoising techniques, thereby achieving data separation. Typical methods include median filtering and its improved variants.⁸⁻¹¹ Such methods are usually easy to implement and computationally efficient. However, because blending noise often overlaps with the useful signal in both time and space, denoising-based methods may suffer from limited separation accuracy, especially when the blending interference is strong or when weak signal events need to be preserved.

The second category comprises inversion-based methods, which formulate deblending as an inverse problem. Because this inverse problem is generally underdetermined, appropriate prior constraints or regularization terms must be introduced during the solution process. Seismic data exhibit strong local correlation and structural continuity, which enable sparse representation in suitable transform domains. Therefore, sparsity-promoting regularization has been widely employed to constrain the inverse deblending problem. Commonly used

sparse transforms include the Fourier transform,¹²⁻¹⁵ the curvelet transform,^{16,17} the Radon transform,¹⁸⁻²⁰ the seislet transform,²¹⁻²³ and the focal transform.^{24,25} When equipped with appropriate sparse representations and regularization strategies, inversion-based methods can generally provide higher separation accuracy than denoising-based approaches. Nevertheless, several limitations remain. First, many existing sparse transform algorithms still employ unified or scale-dependent thresholding strategies, which may not sufficiently account for the statistical variations among different scales and directional subbands. Second, even within the same directional subband, transform coefficients exhibit considerable heterogeneity: useful signal coefficients and blending noise coefficients may overlap in amplitude, although they exhibit distinct local structural continuity and spatial coherence. Therefore, thresholding solely based on coefficient amplitudes may either remove weak useful signal or retain strong blending noise.

The third category includes deep learning-based methods. In recent years, deep learning has been increasingly explored for blended data separation.²⁶⁻³¹ These methods have shown promising performance in some scenarios due to their strong nonlinear representation capabilities. Nevertheless, these methods generally require a large amount of high-quality training data, especially reliable, paired, labeled datasets, which are difficult to obtain in practical marine exploration.⁴ In addition, model generalization across different acquisition conditions, geological settings, and noise environments remains a challenge. These limitations restrict the practical applicability of purely data-driven methods in many real marine seismic processing tasks.

Motivated by the advantages of inversion-based methods and the need for a more effective transform-domain shrinkage strategy, we propose a deblending method based on sparsity-constrained inversion in the nonsubsampling contourlet transform (NSCT) domain. The NSCT,³² characterized by its multiscale, multidirectional, and shift-invariant properties, is well-suited for representing seismic data. Its multiscale decomposition helps separate coarse structures from fine details at different resolutions and enhances the representation of seismic events with varying dips. Its shift-invariance improves the stability of coefficient processing and reduces downsampling artifacts. These characteristics make the NSCT suitable for deblending, where preserving structural continuity and weak reflection events is of primary importance.

To address these issues, this study developed an NSCT-domain deblending method that combines scale- and direction-adaptive threshold estimation with twin support

vector machine (TSVM)-based coefficient classification and class-dependent threshold refinement. First, scale- and direction-adaptive thresholds (SDATs) were constructed for each directional subband by exploiting the statistical differences between useful signal and blending noise across different scales and directions. Then, to further distinguish useful-signal-dominant and blending-noise-dominant coefficients within the same directional subband, training samples were automatically constructed by using continuous coefficient structures and multi-attribute features. Based on the trained TSVM classifier, the thresholds were refined in a class-dependent manner to achieve coefficient-wise adaptive threshold adjustment, thereby progressively recovering weak useful signal while effectively suppressing blending noise.

Extensive experiments, including parameter sensitivity analysis, ablation studies, and comparison with seislet-domain inversion, were conducted to evaluate separation accuracy and computational efficiency.

2. Methods

2.1. Iterative deblending in the nonsubsamped contourlet transform domain

For a two-source blended acquisition, the blending process in the non-common shot domain can be expressed as Equation 1:

$$d = \Gamma_1 d_1 + \Gamma_2 d_2 \quad (1)$$

where d denotes the recorded blended data; d_1 and d_2 represent the unblended single-source data associated with source 1 and source 2, respectively; and Γ_1 and Γ_2 denote the corresponding time-delay operators, respectively. In the time domain, the action of the delay operator is equivalent to shifting each seismic trace according to the prescribed delay time. Based on Equation 1, deblending can be formulated as solving an underdetermined inverse problem to estimate the single-source data d_1 and d_2 from the blended data d . Applying the corresponding inverse time-delay operators Γ_1^{-1} and Γ_2^{-1} separately to the blended data d yields Equation 2:

$$\begin{cases} \Gamma_1^{-1} d = d_1 + \Gamma_1^{-1} \Gamma_2 d_2 \\ \Gamma_2^{-1} d = \Gamma_2^{-1} \Gamma_1 d_1 + d_2 \end{cases} \quad (2)$$

where $\Gamma_1^{-1} d$ and $\Gamma_2^{-1} d$ denote the pseudo-deblended data obtained by applying the inverse time-delay operators of source 1 and source 2 to the blended data, respectively. By combining the two equations in Equation 2, the deblending problem can be reformulated as the following augmented estimation problem (Equation 3):

$$\hat{d} = Fm \quad (3)$$

where

$$\begin{aligned} \hat{d} &= \begin{bmatrix} \Gamma_1^{-1} d \\ \Gamma_2^{-1} d \end{bmatrix} \\ F &= \begin{bmatrix} I & \Gamma_1^{-1} \Gamma_2 \\ \Gamma_2^{-1} \Gamma_1 & I \end{bmatrix} \\ m &= \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \end{aligned} \quad (4)$$

To solve this underdetermined inverse problem in Equation 3, the unknown model m is iteratively updated using the shaping regularization method²¹ (Equation 5):

$$m_{n+1} = S[m_n + B(\hat{d} - Fm_n)] \quad (5)$$

where $B = [I \ 0; 0 \ I]/2$ is the backward operator; S denotes the shaping operator, and its expression is given by Equation 6:

$$S = C^{-1}TC \quad (6)$$

where T represents the thresholding operator, detailed in the following section, C represents forward NSCT, and C^{-1} represents the inverse NSCT.

2.2. Nonsubsamped contourlet transform domain adaptive threshold estimation

After transformation into the NSCT domain, the pseudo-deblended data in the non-common shot gathers can be written as Equation 7:

$$Y = X + N \quad (7)$$

where N , X , and Y represent the NSCT coefficients of the blending noise, useful signal, and pseudo-deblended data, respectively. In the NSCT domain, seismic data are decomposed into K scales, where $k = 1, 2, \dots, K$ denotes the k^{th} scale, and larger values of k correspond to higher frequencies. Each scale contains L directional subbands, where $l = 1, 2, \dots, L$ denotes the l^{th} direction.

For the transform coefficients of blended data, coefficients associated with useful signal typically have larger magnitudes, whereas those dominated by blending noise tend to have smaller magnitudes. Therefore, by setting a threshold, one can retain coefficients with magnitudes above it and suppress those below it, thereby separating the useful signal from blending noise. A commonly adopted threshold estimation method is the Bayesian threshold (Equation 8):³³

$$\lambda_{k,l} = \frac{\sigma_N^2}{\delta_{X(k,l)}} \quad (8)$$

where $\lambda_{k,l}$ is the threshold of the l^{th} directional subband of the k^{th} scale. σ_N^2 denotes the variance of the blending noise in the NSCT domain, which is typically estimated using Equation 9:

$$\sigma_N = \frac{\text{median}(\|Y_K\|)}{0.6745} \quad (9)$$

where Y_K denotes the coefficients of all directional subbands at the K^{th} scale in the NSCT domain. $\delta_{X(k,l)}$ is the standard deviation of the useful signal, which can be estimated by Equation 10:

$$\delta_{X(k,l)} = \sqrt{\max\left(\frac{1}{M} \sum_{i=1}^I \sum_{j=1}^J Y_{k,l}^2(i,j) - \sigma_N^2, 0\right)} \quad (10)$$

where $Y_{k,l}(i,j)$ denotes the coefficient at location (i,j) in the l^{th} directional subband at scale k ; $I \times J$ denotes the subband size; and M is the total number of coefficients in this subband. The conventional Bayesian threshold is not tailored to the distinct coefficient statistics of useful signal and blending noise across scales and directional subbands. Due to the distinct frequency bands associated with different scales, the distributions of useful signal and blending noise are not consistent across scales: the useful signal is typically concentrated at low scales, whereas blending noise is mainly distributed at high scales. Thus, we first introduced scale adaptivity into the Bayesian threshold, enabling it to vary adaptively according to the characteristics of different scales (Equation 11):

$$\lambda_{k,l}^s = \frac{\sigma_N^2}{\delta_{X(k,l)}} \cdot \beta_1 \quad (11)$$

where $\lambda_{k,l}^s$ denotes the scale-adaptive threshold (SAT), and β_1 is the scale factor, which was calculated using Equation 12:

$$\beta_1 = \left(\frac{\max(\|Y_K\|)}{\text{median}(\|Y_K\|)} \right)^{2^{k/K}} \quad (12)$$

The ratio $\max(\|Y_K\|)/\text{median}(\|Y_K\|)$ characterizes the peak-to-background contrast at the k^{th} scale. Since the exponent $2^{k/K}$ increases with k , β_1 provides progressively stronger threshold correction at finer scales, thereby enhancing high-frequency blending noise suppression while limiting excessive attenuation of useful signal at coarser scales.

When the direction of the seismic events is close to that of the NSCT basis functions, the corresponding directional

subbands tend to exhibit larger coefficients, resulting in higher energy concentration in those subbands. In contrast, subbands whose orientations are mismatched with the useful signal direction generally contain coefficients with smaller magnitudes. Based on this property, a direction-adaptive modification was further introduced on the SAT scheme (Equation 13):

$$\lambda_{k,l}^{s,d} = \frac{\sigma_N^2}{\delta_{X(k,l)}} \cdot \beta_1 \cdot \beta_2 \quad (13)$$

where $\lambda_{k,l}^{s,d}$ denotes the SDAT, and β_2 is the direction factor, which is calculated using Equation 14:

$$\beta_2 = \frac{\text{median}(\|Y_{k,l}\|_2)}{\|Y_{k,l}\|_2} \quad (14)$$

where $Y_{k,l}$ represents all the coefficients of the l^{th} directional subband at scale k , and $\|\cdot\|_2$ denotes the L_2 norm. The median coefficient norm across all directional subbands is used as a robust reference. Accordingly, the threshold is reduced for high-energy subbands aligned with coherent events to preserve useful signal, while it is increased for low-energy subbands to enhance blending noise suppression.

2.3. Twin support vector machine-guided refinement of the adaptive threshold

Although the above SDAT estimation improves coefficient processing, a unified threshold is still applied within each directional subband. Because the useful signal and blending noise coefficients follow different statistical distributions within the same subband, a single threshold cannot achieve high-precision separation. Given that useful signal coefficients typically exhibit stronger spatial continuity, we employed TSVM to classify subband coefficients into “useful-signal-dominant” and “blending-noise-dominant” classes. Based on the classification results, the adaptive thresholds were further adjusted to enable class-dependent thresholding. The following presents a detailed description of this method.

First, the directional subband coefficients $Y_{k,l}(i,j)$ were obtained (Figure 1A), from which an initial label map was constructed by assigning binary labels to all coefficients (Equation 15):

$$L_{k,l}(i,j) = \begin{cases} 1, & |Y_{k,l}(i,j)| \geq \tau_{k,l} \\ 0, & |Y_{k,l}(i,j)| < \tau_{k,l} \end{cases} \quad (15)$$

where $L_{k,l}(i,j)$ denotes the initial label map. For each directional subband, $\tau_{k,l}$ was set as the cutoff for selecting the coefficients with the largest $p\%$ absolute magnitudes.

When $L_{k,l}(i,j)=1$, the corresponding coefficient was preliminarily labeled as a useful-signal-dominant candidate, as shown in Figure 1B; such coefficients are highlighted in gray.

Next, since useful signal coefficients typically exhibit spatial continuity, we introduced the support value³⁴ on the initial label map $L_{k,l}(i,j)$ to quantify such continuity. Specifically, for any coefficient that was preliminarily labeled as a useful signal coefficient, if there existed at least one neighboring coefficient in its eight neighborhood (up, down, left, right, and the diagonal neighbors) that was also preliminarily classified as a useful signal coefficient, these coefficients were regarded as spatially connected and formed the same continuous structure. As shown in Figure 1B, the coefficient y_{21} is not connected to any preliminarily identified useful signal coefficient. The coefficients y_{33} and y_{44} belong to the same continuous structure. The coefficients y_{56} , y_{57} , y_{67} , y_{68} , y_{76} , y_{87} , and y_{88} belong to another continuous structure. The number of coefficients within the same continuous structure was defined as the support value of that structure. Figure 1C shows the support values corresponding to the coefficients preliminarily classified as useful signal in Figure 1B.

Subsequently, candidate useful signal coefficients were screened according to the support values of continuous structures: the top N_c continuous structures with the largest support values were selected, and the coefficients within these continuous structures were regarded as representative useful signal coefficients. To construct a class-balanced training set, we further randomly sampled the same number of coefficients from locations in the label map with $L_{k,l}(i,j)=0$ and treated them as representative blending noise coefficients. For the coefficient map of the directional subband, six feature maps were computed, including local entropy, local slope, Hessian-based principal

curvature,³⁵ phase congruency,³⁶ peak amplitude above average,³⁷ and instant amplitude.³⁷ The six feature values at each coefficient location formed a six-dimensional feature vector. The feature vectors from the selected representative locations constitute the training set, whereas those from all coefficient locations within the directional subband were used as the classification inputs.

In this study, the fast and robust TSVM (FR-TSVM) proposed by Gao *et al.*³⁸ was adopted as the binary classifier. This choice was motivated by the following considerations. First, the present task was essentially a binary classification problem, in which the subband coefficients were divided into useful-signal-dominant and blending-noise-dominant classes. Second, the training samples were automatically screened from blended data and thus inevitably contained certain uncertainty and noise. By introducing fuzzy membership into the TSVM framework, FR-TSVM improves classification robustness. Third, all coefficients in the considered subband need to be classified. The coordinate descent algorithm with a shrinking strategy adopted in FR-TSVM provides favorable computational efficiency.

Before training and classification, the training samples and classification inputs were normalized using the same scheme. A radial basis function kernel was adopted, with the kernel-width parameter set to 0.2. The regularization and fuzzy-error penalty parameters were both set to 10, while the fuzzy-membership balance parameter was set to 0.1. The optimization problem was solved using coordinate descent with heuristic shrinking, with a convergence tolerance of 10^{-3} and a maximum of 200 iterations. After training, FR-TSVM was applied to classify all coefficients in the considered directional subband, yielding the predicted label map $\hat{L}_{k,l}(i,j)$. The classification performance of FR-TSVM and the influences of p and N_c were quantitatively

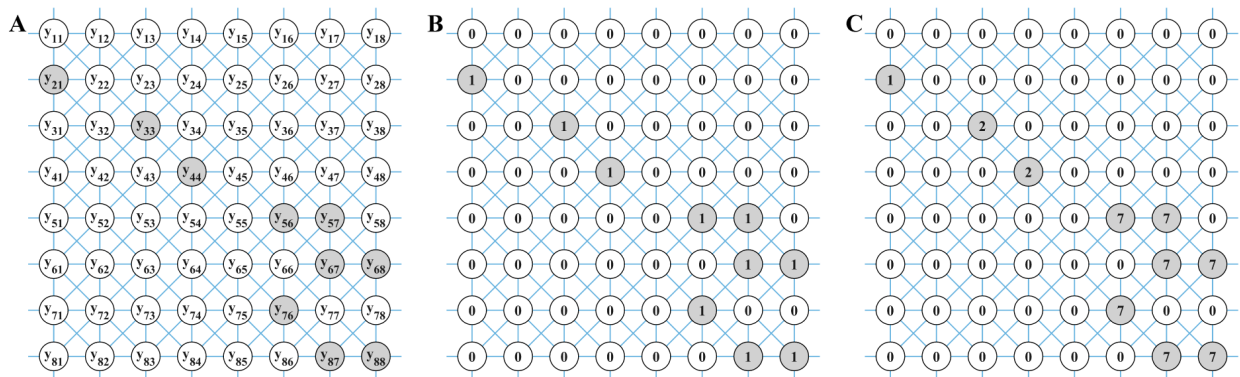


Figure 1. Schematic illustration of initial candidate selection and continuous structure support calculation: (A) Coefficient map of a nonsubsampling contourlet transform directional subband; (B) corresponding initial binary label map, in which the gray nodes denote candidate coefficients; (C) support-value map, in which each candidate coefficient is assigned the number of coefficients belonging to its continuous structure

evaluated using synthetic data in Section 3.1.1.

Based on the predicted label map $\hat{L}_{k,l}(i, j)$, the SDAT $\lambda_{k,l}^{s,d}$ was further refined in a coefficient-wise manner as **Equation 16**:

$$\lambda_{k,l}^{s,d}(i, j) = \begin{cases} \frac{\sigma_N^2}{\delta_{X(k,l)}} \cdot \beta_1 \cdot \beta_2 \cdot \alpha^n, & \text{if } \hat{L}_{k,l}(i, j) = 1 \\ \frac{\sigma_N^2}{\delta_{X(k,l)}} \cdot \beta_1 \cdot \beta_2, & \text{if } \hat{L}_{k,l}(i, j) = 0 \end{cases} \quad (16)$$

where $\alpha \in (0,1)$ is the attenuation parameter and n denotes the current iteration index. For coefficients classified as useful-signal-dominant ($\hat{L}_{k,l}(i, j) = 1$), the baseline adaptive threshold was multiplied by α^n . Since α^n progressively decreases with n , the threshold was gradually relaxed to facilitate the recovery of weak useful signal. For coefficients classified as blending-noise-dominant ($\hat{L}_{k,l}(i, j) = 0$), the baseline threshold remained unchanged to maintain blending noise suppression. The influence of different values of α on the deblending performance was evaluated using synthetic data in Section 3.1.2, and the selected value was used in all subsequent experiments. The refined threshold estimation scheme, together with the hard thresholding function, constitutes the thresholding operator in **Equation 6**. The hard thresholding function was adopted to preserve the amplitudes of the retained coefficients and avoid the amplitude bias introduced by soft thresholding. **Figure 2** presents the workflow of the proposed deblending method.

3. Results

Experiments were conducted on both synthetic and field data to evaluate the proposed method. The synthetic data were used for parameter-sensitivity analysis and ablation experiments, whereas the field data were used to assess the practical applicability of the proposed method through

comparison with an existing seislet-transform-based method.

3.1. Synthetic seismic data

The proposed method was first evaluated using a synthetic seismic dataset comprising 64 shots, each recorded by 64 receivers, with 256 samples per trace and a temporal sampling interval of 4 ms. **Figure 3A** shows the first common receiver gather of the original unblended data. Numerical blending was subsequently performed using a blending factor of 2, and **Figure 3B** shows the corresponding pseudo-deblended common receiver gather. All parameter-sensitivity analyses and ablation experiments were conducted in the common receiver gather domain using this synthetic dataset.

3.1.1. Sensitivity analysis of p and N_c

The parameters p and N_c control the proportion of initial candidate coefficients and the number of continuous structures retained for constructing the representative useful-signal-dominant training samples, respectively. To investigate their influence on the FR-TSVM classification performance, p was varied over {1%, 2%, 4%, 6%, 10%}, while N_c was varied over {3, 4, 6, 8, 10}. All other parameters were kept unchanged during the analysis. For each combination of p and N_c , the classification performance was evaluated separately for every directional subband of the 64 common receiver gathers. For quantitative evaluation, the corresponding original unblended data were transformed using the same NSCT configuration as the blended data. The resulting coefficient maps were used to construct the reference labels, with useful-signal-dominant coefficient locations assigned a label of 1 and the remaining locations assigned a label of 0. The predicted labels were then compared with these reference labels, with label 1 treated as the positive class and label 0 as the negative class.

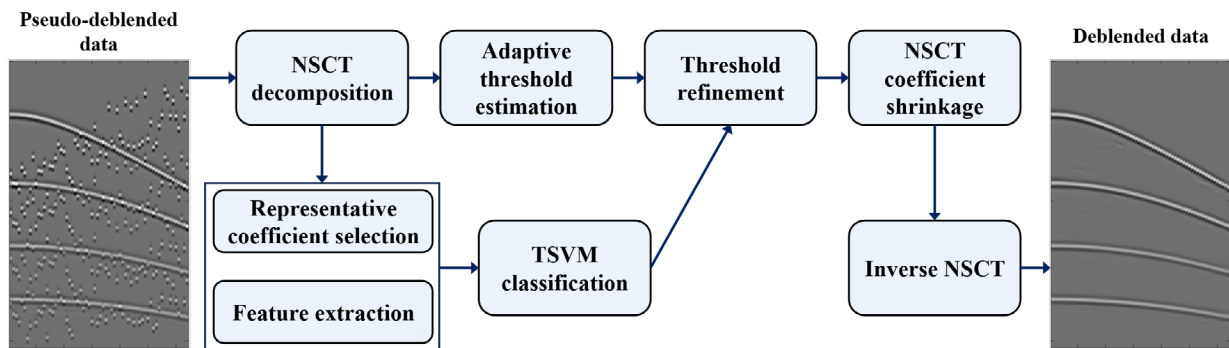


Figure 2. Workflow of the proposed deblending algorithm

Abbreviations: NSCT: Nonsubsampled contourlet transform; TSVM: Twin support vector machine.

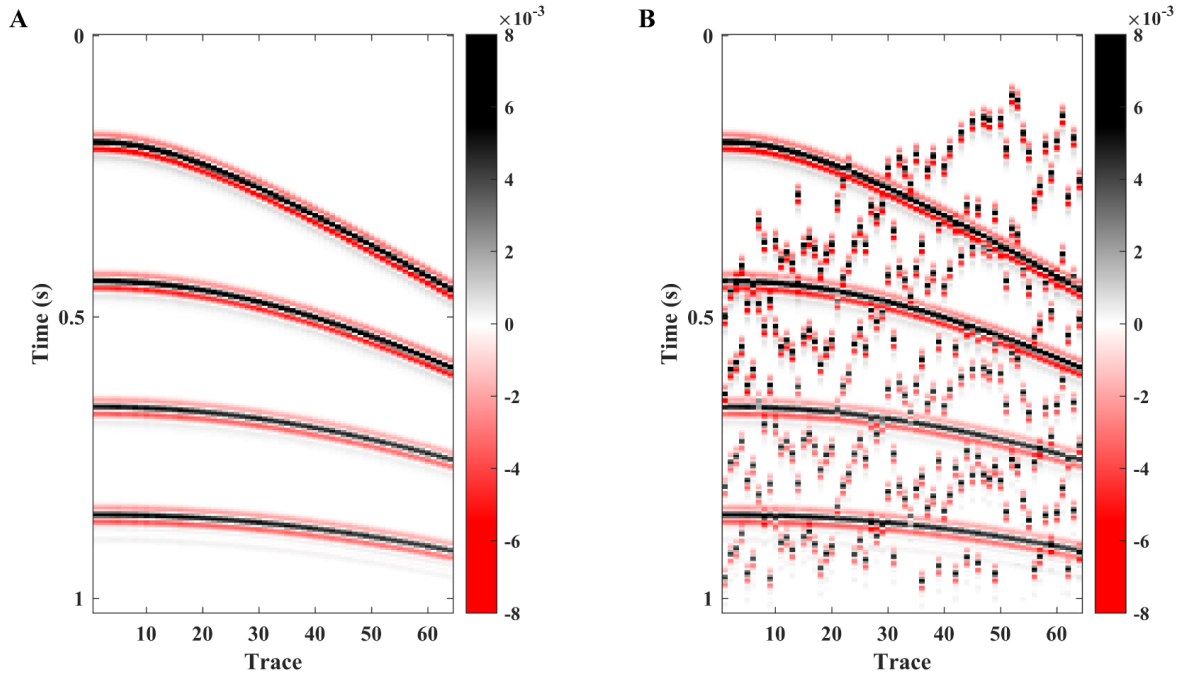


Figure 3. First common receiver gather of the synthetic dataset: (A) Original unblended data; (B) corresponding pseudo-deblended data generated using a blending factor of 2

Accuracy, precision, recall, and F1-score were adopted as the evaluation metrics and calculated using **Equation 17**:

$$\begin{aligned}
 \text{Accuracy} &= \frac{TP + TN}{TP + TN + FP + FN} \\
 \text{Precision} &= \frac{TP}{TP + FP} \\
 \text{Recall} &= \frac{TP}{TP + FN} \\
 \text{F1-score} &= \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}
 \end{aligned} \quad (17)$$

where TP and TN denote the numbers of correctly classified useful-signal-dominant and blending-noise-dominant coefficients, respectively, while FP and FN denote the corresponding numbers of incorrectly classified coefficients.

Figure 4 presents the mean F1-scores obtained with different combinations of p and N_c . Among the tested values, $p = 2\%$ consistently produced relatively high F1-scores for all values of N_c , indicating that retaining the coefficients with the largest 2% absolute magnitudes provides representative and reliable initial samples. The five parameter combinations with the highest mean F1-scores are summarized in **Table 1**, all of which correspond to $p = 2\%$. Their precision values remained consistently high, whereas accuracy and recall exhibited greater variation

with N_c . Although $N_c = 10$ yielded the highest precision, its relatively low recall indicates that more useful-signal-dominant coefficients were missed. In contrast, $p = 2\%$ and $N_c = 6$ provided the best overall balance. Therefore, this parameter combination was selected for the subsequent experiments.

3.1.2. Sensitivity analysis of α

After determining p and N_c , the influence of the attenuation parameter α was investigated using the first common receiver gather as a representative example. To ensure a consistent comparison, the maximum number of iterations was fixed at 50 for all tested values of α , while all other parameters were kept unchanged. The signal-to-noise ratio (SNR) of the deblended result was calculated using **Equation 18**:

$$\text{SNR} = 10 \log_{10} \frac{\|d_u\|_2^2}{\|d_u - d_s\|_2^2} \quad (18)$$

where d_s represents the separated data and d_u represents the original unblended data. A higher SNR indicates better recovery of the original unblended data.

Figure 5 shows the SNR variations over 50 iterations for different values of α . The SNR initially increased rapidly and subsequently improved more gradually for all tested values. A relatively small α produced faster initial

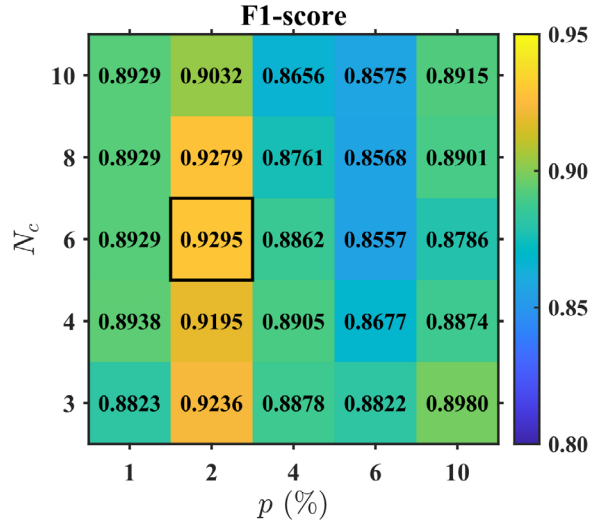


Figure 4. Mean F1-scores for different combinations of p and N_c averaged over all directional subbands of the 64 common receiver gathers

Table 1. Mean classification metrics for the five parameter combinations with the highest mean F1-scores

Rank	p (%)	N_c	Accuracy	Precision	Recall	F1-score
1	2	6	0.8855	0.9447	0.9147	0.9295#
2	2	8	0.8830	0.9451	0.9112	0.9279
3	2	3	0.8731	0.9422	0.9057	0.9236
4	2	4	0.8689	0.9461	0.8944	0.9195
5	2	10	0.8431	0.9550	0.8568	0.9032

Note: All metrics are averaged over the directional subbands of the 64 common receiver gathers. p denotes the percentage of initial candidate coefficients, and N_c denotes the number of retained continuous structures. The highest F1-score is indicated by #.

improvement because the threshold applied to useful-signal-dominant coefficients decreased rapidly; however, the SNR subsequently leveled off at a relatively low value. As α approached 1, the threshold attenuation became weaker, resulting in slower recovery of useful signal. In contrast, $\alpha = 0.85$ provided sustained SNR improvement and achieved the highest SNR after 50 iterations. These results indicate that $\alpha = 0.85$ provides a balance between useful signal preservation and blending noise suppression. Therefore, $\alpha = 0.85$ was adopted in the subsequent experiments.

3.1.3. Ablation study

To quantify the contributions of the individual components, an ablation study was conducted using four variants: (i) the initial threshold alone (IT); (ii) SAT, obtained by incorporating the scale factor β_1 into the initial threshold; (iii) SDAT, obtained by further incorporating the direction factor β_2 ; and (iv) FR-TSVM-guided SDAT (FT-SDAT), which refines SDAT in a coefficient-wise manner according to the coefficient labels predicted by FR-TSVM. All four

variants employed the same NSCT configuration and hard-thresholding operator, and all other applicable parameters were kept unchanged. Each variant was independently applied to the 64 common receiver gathers of the synthetic dataset. The deblended results were evaluated against the corresponding original unblended data using SNR, root mean square error (RMSE), and correlation coefficient (CC). The SNR was calculated using Equation 18, while RMSE and CC were calculated using Equations 19 and 20:

$$\text{RMSE} = \sqrt{\frac{1}{N} \| \mathbf{d}_u - \mathbf{d}_s \|^2} \quad (19)$$

$$\text{CC} = \frac{\sum_{i,j} [\mathbf{d}_u(i,j) - \bar{\mathbf{d}}_u] [\mathbf{d}_s(i,j) - \bar{\mathbf{d}}_s]}{\sqrt{\sum_{i,j} [\mathbf{d}_u(i,j) - \bar{\mathbf{d}}_u]^2} \sqrt{\sum_{i,j} [\mathbf{d}_s(i,j) - \bar{\mathbf{d}}_s]^2}} \quad (20)$$

where N is the total number of samples in each common receiver gather, and $\bar{\mathbf{d}}_u$ and $\bar{\mathbf{d}}_s$ denote the mean amplitudes of the original unblended and separated data, respectively. A lower RMSE and a higher CC indicate better deblending

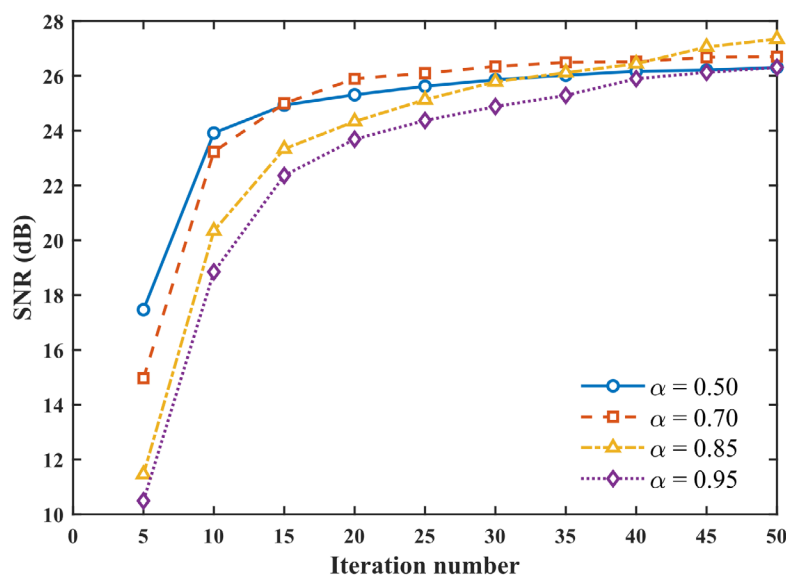


Figure 5. Evolution of the deblending signal-to-noise ratio (SNR) over 50 iterations for different values of α , evaluated on the first common receiver gather

performance.

For each ablation variant, the three metrics were calculated separately for each of the 64 common receiver gathers, and their distributions were summarized using the boxplots shown in Figure 6. As shown in Figure 6, the quantitative performance improved progressively as the individual components were incorporated. Compared with IT, SAT shifted the SNR and CC distributions upward while reducing RMSE, demonstrating the effectiveness of SAT adjustment. SDAT provided further improvement in all three metrics, confirming the contribution of

direction-adaptive threshold adjustment. With the further incorporation of FR-TSVM-guided coefficient-wise refinement, FT-SDAT achieved the highest median SNR and CC and the lowest median RMSE. The consistent shifts in the metric distributions across the 64 common receiver gathers demonstrate that scale adaptivity, direction adaptivity, and FR-TSVM-guided refinement each contribute to the overall deblending performance.

To complement the quantitative evaluation, the first common receiver gather was selected for visual comparison. Figure 7 presents the deblending results

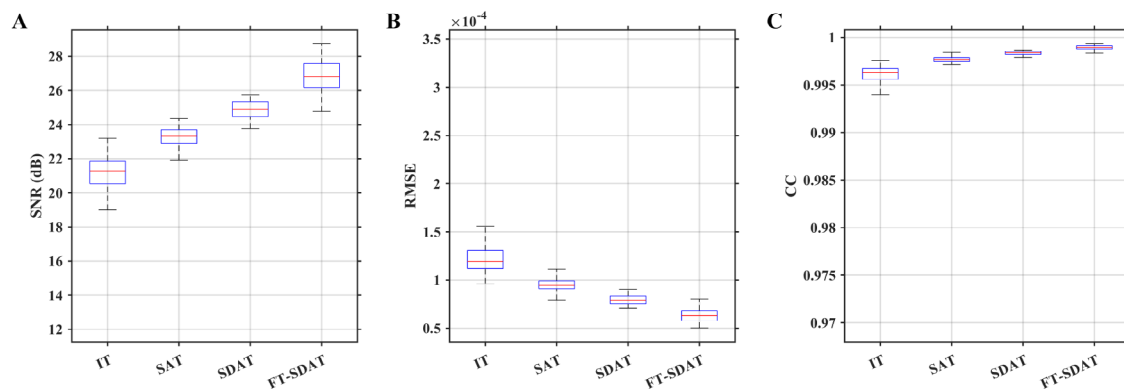


Figure 6. Boxplots of the quantitative metrics obtained using the four ablation variants over the 64 common receiver gathers: (A) SNR, (B) RMSE, and (C) CC

Abbreviations: CC: Correlation coefficient; FT-SDAT: Fast and robust twin support vector machine-guided scale- and direction-adaptive threshold; IT: Initial threshold; RMSE: Root mean square error; SAT: Scale-adaptive threshold; SDAT: Scale- and direction-adaptive threshold; SNR: Signal-to-noise ratio.

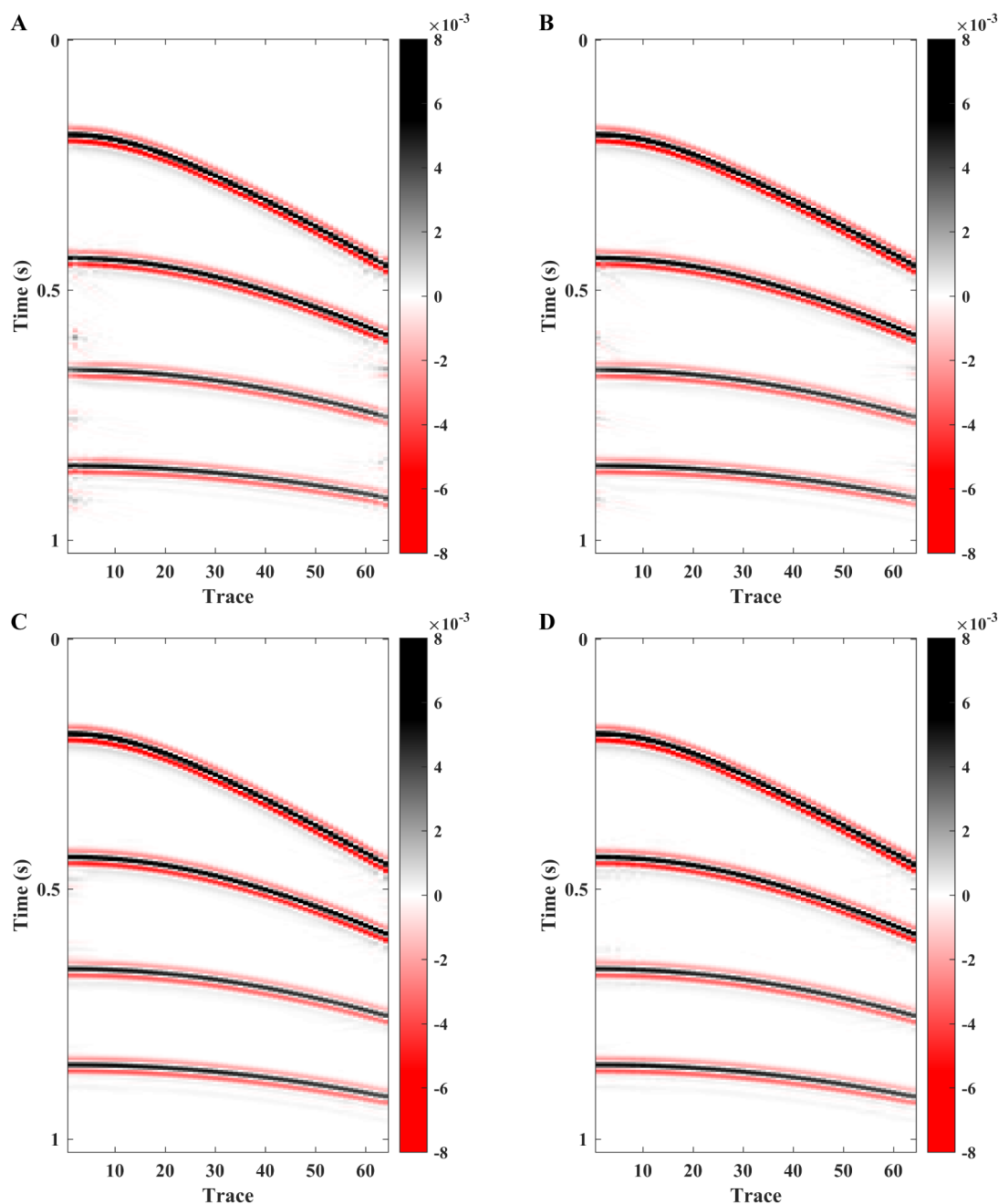


Figure 7. Deblending results for the first common receiver gather obtained using the four ablation variants: (A) IT, (B) SAT, (C) SDAT, and (D) FT-SDAT. Abbreviations: FT-SDAT: Fast and robust twin support vector machine-guided scale- and direction-adaptive threshold; IT: Initial threshold; SAT: Scale-adaptive threshold; SDAT: Scale- and direction-adaptive threshold.

obtained using IT, SAT, SDAT, and FT-SDAT, while Figure 8 shows their corresponding errors relative to the original unblended data. All four variants recovered the main coherent events, but IT retained relatively pronounced artifacts and event-related errors. These errors were progressively reduced by incorporating scale adaptivity in

SAT and direction adaptivity in SDAT. FT-SDAT produced the cleanest result and the weakest coherent error energy, indicating improved event preservation and reduced signal leakage. These visual observations were consistent with the quantitative results in Figure 6 and confirm the contributions of the three adaptive components.

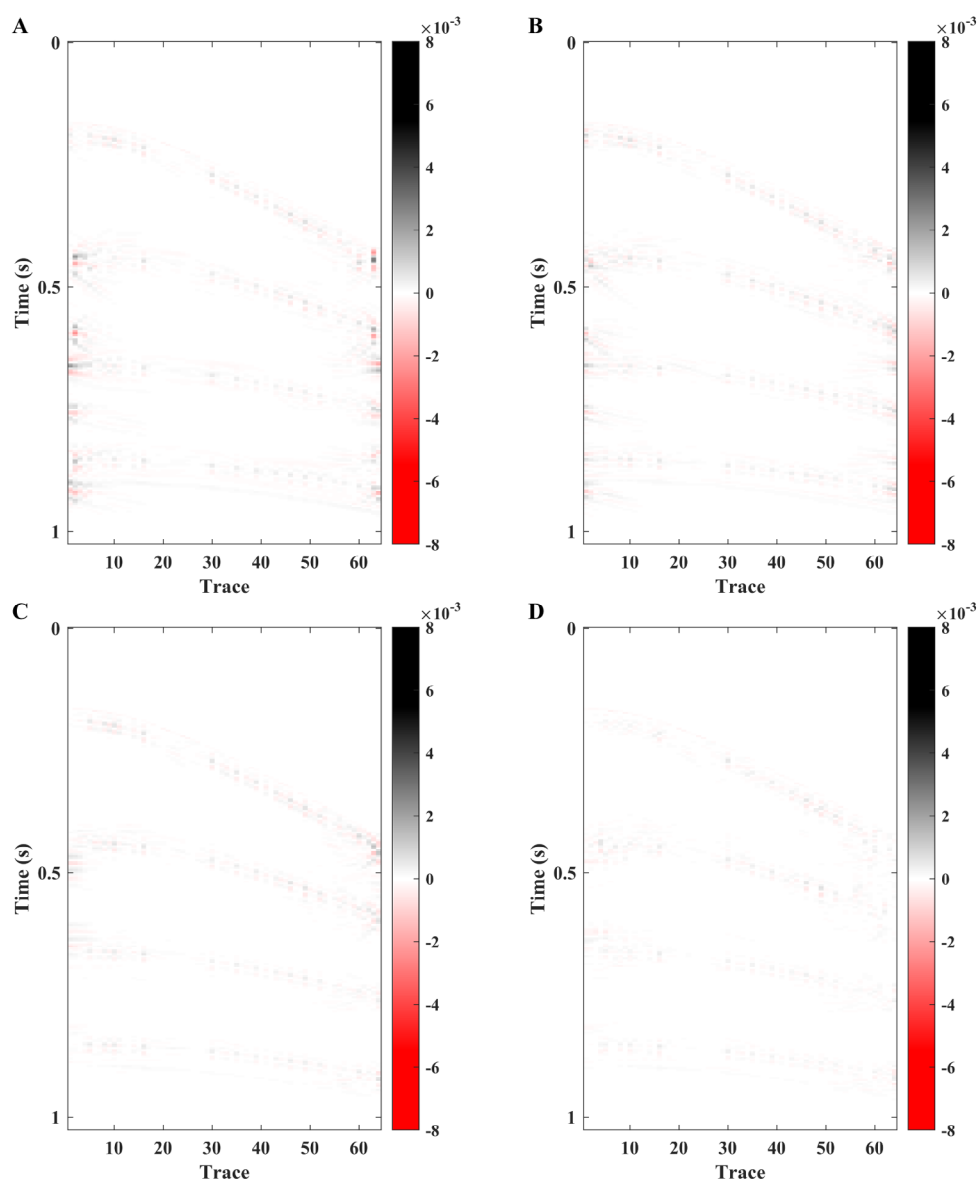


Figure 8. Errors between the original unblended data and the deblending results obtained using (A) IT, (B) SAT, (C) SDAT, and (D) FT-SDAT. Abbreviations: FT-SDAT: Fast and robust twin support vector machine-guided scale- and direction-adaptive threshold; IT: Initial threshold; SAT: Scale-adaptive threshold; SDAT: Scale- and direction-adaptive threshold.

3.2. Field seismic data

To further evaluate its practical applicability, the proposed method was tested on two field datasets and compared with an existing seislet-transform-based deblending method.

3.2.1. Field data example I

The proposed method was first evaluated using the GM8805 field dataset obtained from the Marine Geoscience Data System.³⁹ The dataset comprises 100 shots, each recorded by 150 receivers, with a record length of 3.2 seconds per

trace. Numerical blending was performed using a blending factor of 2, after which the data were rearranged into the common offset domain for deblending. Figure 9A and 9B show the original unblended data and the corresponding pseudo-deblended data for the 10th common offset gather, respectively.

Both the seislet-transform-based and proposed methods were applied to all common offset gathers. When averaged over all gathers, the proposed method achieved an SNR of 20.3147 dB, an RMSE of 0.01394, and a CC

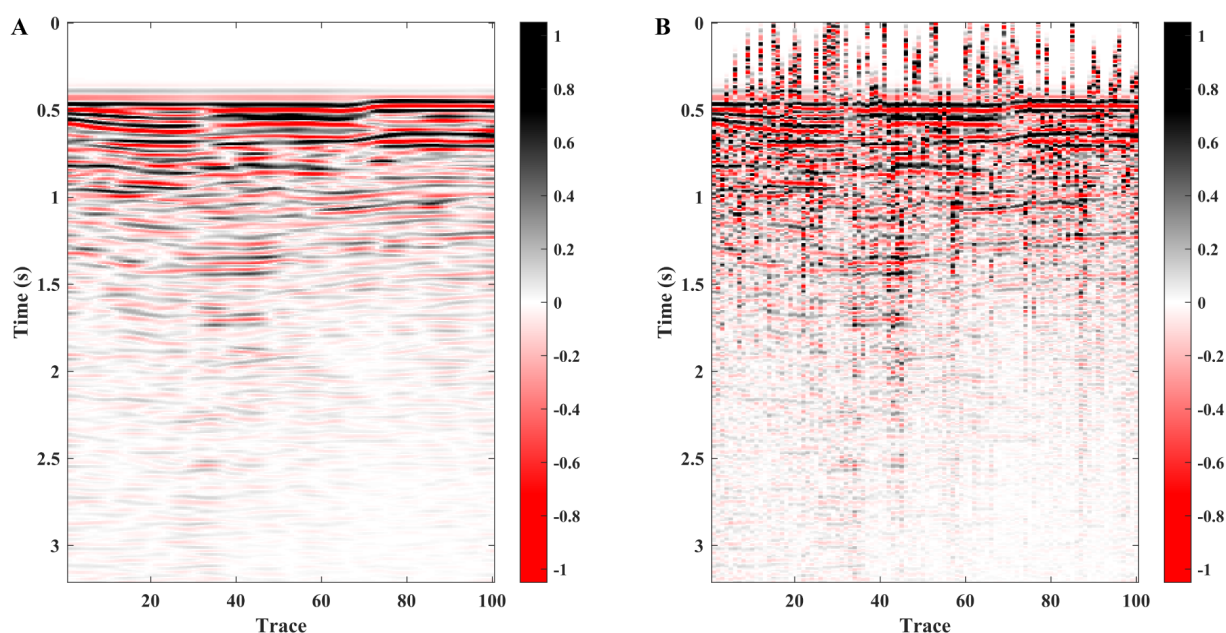


Figure 9. The 10th common offset gather of the first field dataset: (A) Original unblended data; (B) corresponding pseudo-deblended data generated using a blending factor of 2

of 0.9843, whereas the seislet-transform-based method yielded 17.8435 dB, 0.01930, and 0.9814, respectively. Thus, the proposed method achieved higher mean SNR and CC and lower mean RMSE, indicating improved deblending accuracy. Both methods were implemented in MATLAB R2023b and executed on the same workstation equipped with a 13th-generation Intel Core i9-13900K CPU at 3.00 GHz and 128 GB of RAM, running 64-bit Windows 11. Under this configuration, the average processing times per common offset gather were 182 seconds and 869 seconds for the seislet-transform-based and proposed methods, respectively. Although the proposed method required a longer runtime, it provided better overall deblending performance, indicating a trade-off between separation accuracy and computational cost.

Figure 10 compares the deblending results obtained using the seislet-transform-based and proposed methods for the 10th common offset gather, together with their corresponding errors relative to the original unblended data. Both methods recovered the main coherent reflections; however, the proposed method provided improved event continuity and amplitude preservation. Moreover, its error section contained weaker residual energy and fewer coherent artifacts, indicating reduced signal leakage and amplitude distortion. These visual observations are consistent with the quantitative results reported above.

After deblending in the common offset domain, the

results were re-sorted into the common shot domain for further evaluation. Figure 11A and 11B show the original unblended data and the corresponding pseudo-deblended data for the 5th common shot gather, respectively. Figure 12 compares the results obtained using the two methods for this gather. Compared with the seislet-transform-based method, the proposed method suppressed more residual blending interference while better preserving the continuity of coherent seismic events, demonstrating improved deblending performance.

3.2.2. Field data example II

The proposed method was further evaluated using the EW9203 field dataset.⁴⁰ The dataset comprises 200 shots, each recorded by 200 receivers, with a record length of 5.0 seconds per trace. The two-source numerical blending scheme was applied. Both the seislet-transform-based and proposed methods were applied in the common offset domain, after which the deblended results were re-sorted into the common shot domain.

The quantitative performance on the second field dataset was evaluated in the common shot domain. The SNR, RMSE, and CC were calculated separately for each of the 200 common shot gathers and then averaged over all gathers. The proposed method achieved a mean SNR of 13.8779 dB, an RMSE of 1.1107×10^{-6} , and a CC of 0.9709, whereas the seislet-transform-based method yielded 12.2181 dB, 1.3259×10^{-6} , and 0.9567, respectively. These results demonstrate that the proposed method provides

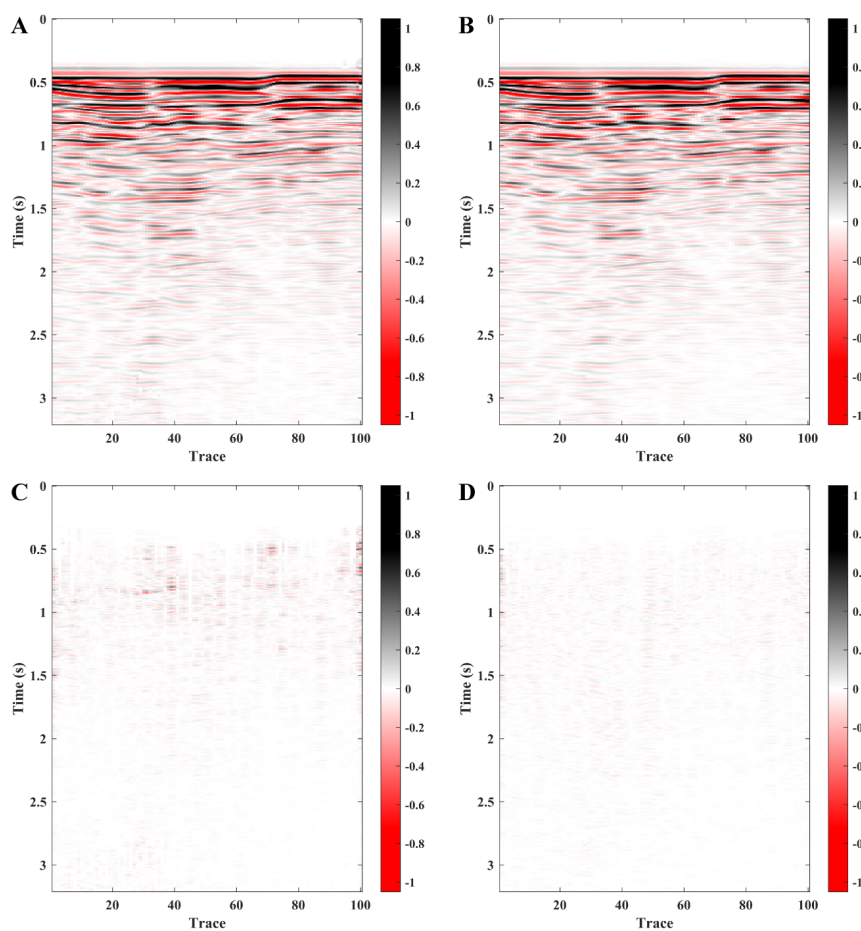


Figure 10. Deblending comparison for the 10th common offset gather of the first field dataset: (A) Result of the seislet-transform-based method; (B) result of the proposed method; (C) error of the seislet-transform-based method; and (D) error of the proposed method

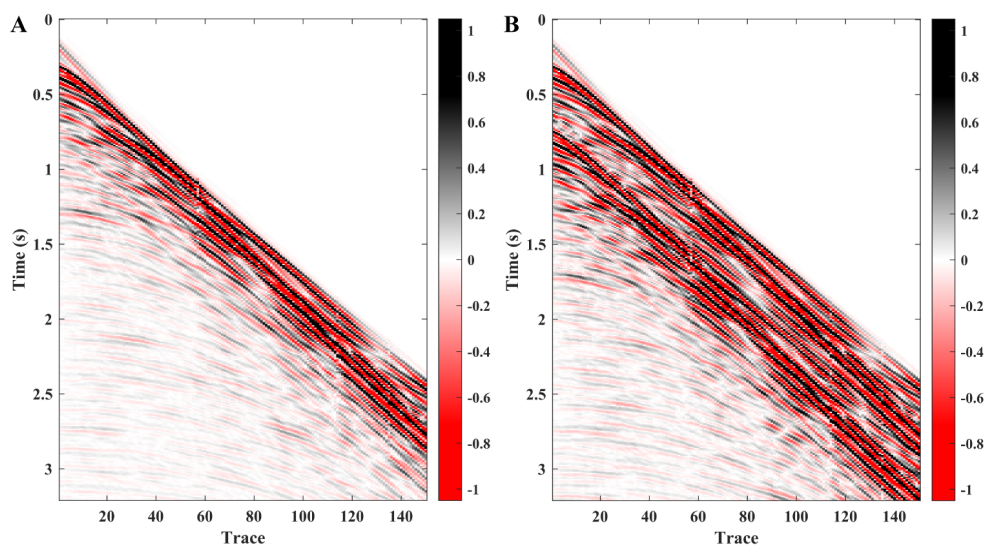


Figure 11. The 5th common shot gather of the first field dataset: (A) Original unblended data; (B) corresponding pseudo-deblended data generated using a blending factor of 2

improved deblending accuracy. Figure 13A and 13B show the original unblended data and the corresponding pseudo-deblended data for the 80th common shot gather, respectively. Figure 14 compares the deblended results obtained using the seislet-transform-based and proposed methods, together with their corresponding errors relative to the original unblended data. Compared with the seislet-transform-based method, the proposed method produced weaker residual interference while better preserving the continuity of coherent seismic events. These visual results are consistent with the quantitative evaluation presented above.

4. Discussion

The proposed method integrates SDAT and FR-TSVM-

guided coefficient-wise threshold refinement into a shaping-regularized iterative inversion framework in the NSCT domain. The ablation results show progressive improvements from IT to SAT, SDAT, and FT-SDAT, demonstrating the respective contributions of scale adaptivity, direction adaptivity, and classification-guided refinement. The proposed method also achieved higher mean SNR and CC and lower mean RMSE than the seislet-transform-based method on both field datasets, consistent with the weaker residual interference observed in the error sections.

The effectiveness of the method arises from its treatment of coefficient heterogeneity at two levels. Across scales and directional subbands, the scale and direction factors adjust the baseline threshold according to the corresponding

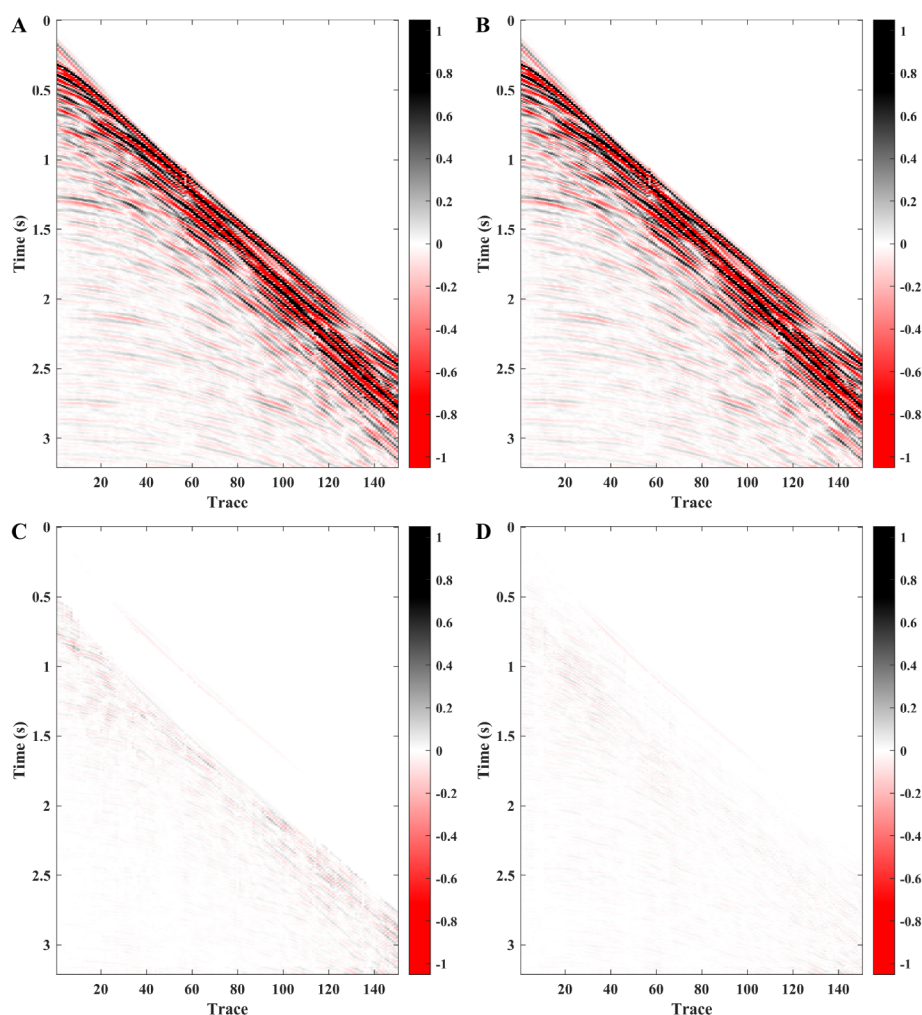


Figure 12. Deblending comparison for the 5th common shot gather of the first field dataset: (A) Results obtained using the seislet-transform-based method; (B) results obtained using the proposed method; (C) error of the seislet-transform-based result relative to the original unblended data; and (D) error of the proposed result relative to the original unblended data

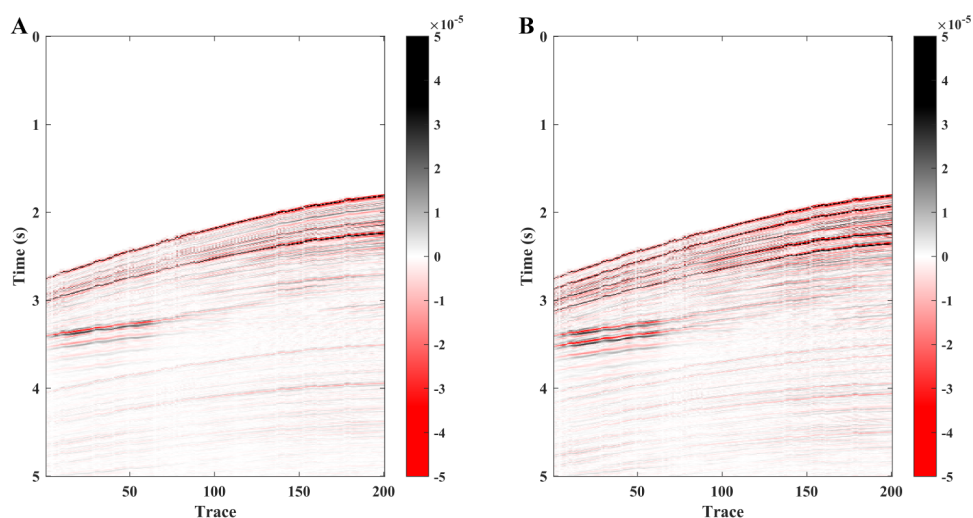


Figure 13. The 80th common shot gather of the second field dataset: (A) Original unblended data; (B) corresponding pseudo-deblended data generated using a blending factor of 2

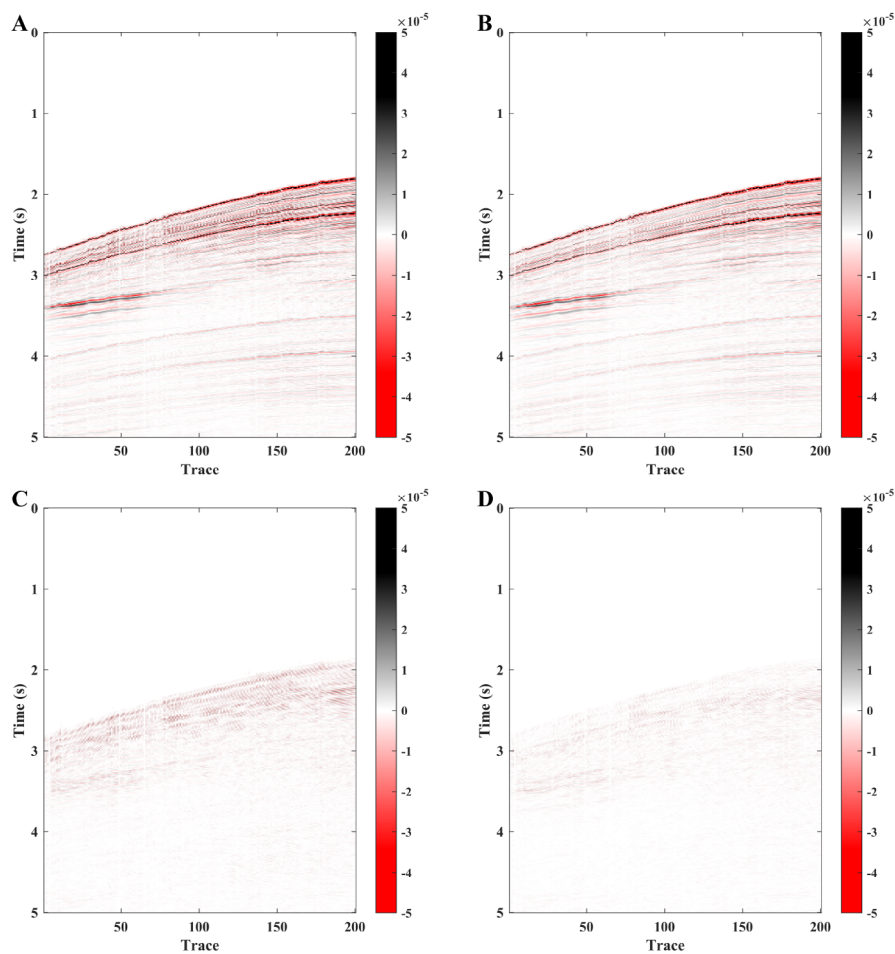


Figure 14. Deblending comparison for the 80th common shot gather of the second field dataset: (A) Result obtained using the seislelet-transform-based method; (B) result obtained using the proposed method; (C) error of the seislelet-transform-based result relative to the original unblended data; and (D) error of the proposed result relative to the original unblended data

coefficient distributions, thereby improving blending noise suppression while avoiding excessive attenuation of useful signal. Within each directional subband, FR-TSVM distinguishes useful-signal-dominant coefficients from blending-noise-dominant coefficients and refines their thresholds accordingly. This coefficient-wise refinement further improves useful signal preservation compared with applying a single threshold to the entire subband.

It is worth noting that the training samples used in this study do not rely on external labeled data but were instead automatically constructed from the blended data. Based on the initial label map, the support values of continuous structures, and representative coefficient selection, typical useful-signal samples and typical blending-noise samples can be extracted from the subband coefficients and then used to train the classifier. This strategy avoids the reliance of conventional supervised learning methods on large amounts of manually labeled training data, while allowing the classification process to better reflect the intrinsic local continuity of seismic data. For marine simultaneous source data deblending, where obtaining high-quality, paired, labeled samples is difficult, this characteristic is of considerable practical significance.

The improved deblending accuracy was accompanied by increased computational cost compared with the seislet-transform-based method. The additional cost mainly arises from NSCT decomposition and reconstruction, feature calculation, and FR-TSVM training and classification. Therefore, the proposed method involves a trade-off between deblending accuracy and computational efficiency.

5. Conclusion

This study proposes an FR-TSVM-guided SDAT method within an NSCT-domain shaping-regularized iterative inversion framework for simultaneous-source deblending. Scale- and direction-adaptive baseline thresholds were first constructed to account for coefficient heterogeneity across scales and directional subbands. FR-TSVM was then used to distinguish useful-signal-dominant coefficients from blending-noise-dominant coefficients, enabling coefficient-wise threshold refinement. The training samples were automatically selected from the blended data, without requiring manual annotations or external training datasets. Experiments on synthetic data and two numerically blended field datasets show that the proposed method achieved higher SNR and CC and lower RMSE than the comparison method, while producing weaker residual interference and better preserving coherent seismic events. Although this improvement was accompanied by increased computational cost, the results

demonstrate the effectiveness of the proposed method for simultaneous source deblending.

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Conflict of interest

The authors declare they have no competing interests.

Author contributions

Conceptualization: Kun Zou, Jianhua Wang

Formal analysis: Kun Zou, Yong Liu, Yanan Zhang, Yandong Wang

Investigation: Kun Zou, Shuaibing Li, Yu Zhong

Methodology: Kun Zou, Jianhua Wang, Hanming Gu

Writing—original draft: Kun Zou

Writing—review & editing: Kun Zou

Availability of data

The data associated with this paper can be obtained from the corresponding author upon reasonable request.

Further disclosure

During the preparation of this manuscript, the authors used DeepSeek for text translation. The authors have reviewed and edited the generated content and take full responsibility for the content of this publication.

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