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Source-independent elastic frequency-controllable envelope inversion

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Abstract

Elastic full-waveform inversion (EFWI) is a high-resolution inversion method that minimizes full-waveform misfit between synthetic and observed data under the seismic elastic wave equation. In practice, EFWI effectiveness depends strongly on the accuracy of the source wavelet and the initial models, as errors in either degrade the synthetic-observed match and drive the inversion toward local minima. To mitigate these issues, we develop a novel source-independent initial model building method for EFWI, termed source-independent elastic frequency-controllable envelope inversion (SIEFCEI), which enables accurate recovery of low-wavenumber P- and S-wave velocity models from multi-scale elastic frequency-envelope data without relying on an accurate source wavelet. The misfit function of SIEFCEI is formulated based on an amplitude-semblance function to eliminate the need for an accurate source wavelet during inversion, where the matching data in the misfit function are computed using the elastic frequency-controllable envelope operator. SIEFCEI exhibits two notable advantages. First, the data used in the inversion contain richer low-wavenumber information than conventional envelope data, owing to the frequency-controllable mechanism, which reduces the risk of convergence to local minima. Second, SIEFCEI avoids the need to select an optimized reference trace, as required by traditional source-independent approaches, which improves robustness under inconsistent source wavelets. Numerical tests on both synthetic and field data demonstrate the effectiveness of SIEFCEI when starting from poor initial models and using an inconsistent source wavelet.

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Citation: Du Q, Gao Z. Source-independent elastic frequency-controllable envelope inversion. *J Seismic Explor.*
doi: 10.36922/JSE026270121

Received: July 1, 2026**Revised:** August 13, 2026**Accepted:** August 13, 2026**Published online:** September 14, 2026**Copyright:** © 2026 Author(s).

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Keywords: Elastic full-waveform inversion; Elastic frequency-controllable envelope; Source-independent inversion; Multi-scale inversion

1. Introduction

Elastic full-waveform inversion (EFWI) is a highly nonlinear, multiparameter inversion technique that reconstructs high-resolution subsurface velocity models by minimizing the misfit between observed and synthetic seismic data, while fully exploiting both the kinematic and dynamic information of seismic wavefields.^{1,2} In practice, synthetic data are generated by convolving the source wavelet with the Green's function of the current parameter model. Consequently, errors in either the initial P- and S-wave velocity models or the source wavelet can significantly increase EFWI data misfit and ultimately affect inversion results.

To build accurate initial models for EFWI or FWI, researchers have developed several strategies. These include wave equation traveltime inversion, which primarily exploits kinematic information to reconstruct long-wavelength components of the velocity model;^{3-4,5} accumulated energy norm methods, which use the accumulated energy of seismic records to construct large-scale velocity models;⁶ envelope-based inversion methods that exploit envelope information with enriched low-frequency components to mitigate cycle skipping;⁷⁻¹⁰ nonlinear elastic reflection waveform inversion, which leverages reflected-wave information to recover background elastic properties;¹¹ and low-frequency extrapolation of multicomponent seismic data to compensate for the lack of true low-frequency information in typical acquisition settings.¹² Among existing strategies, envelope-based approaches have become one of the most widely adopted directions for initial model building in EFWI because they relax phase sensitivity and provide more stable, effective low-wavenumber updates during iterations. In particular, Gao *et al.*¹³ first introduced frequency-controllable inversion in acoustic full-waveform inversion, using envelope data from different frequency bands in a cascaded multi-scale workflow to construct large-scale P-wave velocity models. Building on this idea, Du *et al.*¹⁴ developed elastic frequency-controllable envelope inversion (EFCEI) and subsequently proposed structurally consistent EFCEI (SC-EFCEI).¹⁵ Nevertheless, these methods, including envelope-based approaches, can still be affected by source-wavelet inaccuracies, which ultimately limit their performance in practical applications.

Accordingly, the success of EFWI also relies heavily on the accuracy of the estimated source wavelet. In practice, the wavelet is commonly extracted from observed seismic data under zero-phase or minimum-phase assumptions, or estimated using simple deconvolution-based procedures. Nevertheless, the extracted wavelet is usually only a reference approximation rather than the true source signature, because it is affected by noise, attenuation, source/receiver coupling, and limited bandwidth. As a result, additional post-processing procedures and wavelet correction steps are typically required to improve the accuracy of the estimated wavelet. These procedures can be computationally expensive and may become ineffective in some practical scenarios, ultimately degrading inversion effectiveness.¹⁶ In addition, the wavelet can be estimated within the EFWI framework by alternating updates of the velocity model and the source,¹⁷ or by performing simultaneous inversion in which both are treated as unknowns. However, errors in these two quantities can interact during inversion, especially when the initial model is poor, so the inversion may compensate for source errors

with incorrect model updates or vice versa, thereby hindering reliable convergence.¹⁸ Moreover, such strategies typically still require a reasonably accurate initial approximation of the source wavelet. A more practical alternative is to adopt a source-independent strategy to eliminate the influence of the source wavelet on EFWI. To this end, many algorithms have been developed using deconvolution or convolution. For example, Choi *et al.*¹⁹ defined a source-independent objective function using deconvolution in the frequency domain. Deconvolution-based trace normalization has also been adopted in the frequency domain to achieve source independence for both observed and modeled data.^{20,21} Cheong *et al.*²² suggested a convolution-based approach for source-independent waveform inversion, in which the wavefields are multiplied by a cross-reference wavefield in the frequency domain. Choi and Alkhalifah²³ developed a source-independent time-domain waveform inversion using convolved wavefields. To reduce artifacts introduced by convolution and cross-correlation, Zhang *et al.*²⁴ applied time windowing to the reference traces in the convolution-based objective function, suppressing noise and improving inversion stability. Ao *et al.*²⁵ proposed a source-independent envelope inversion method based on the conventional waveform Fréchet derivative to build an initial model, where the envelope is computed from the convolution results of seismic data with reference traces. Zhang *et al.*²⁶ further proposed a method based on the direct envelope Fréchet derivative, in which a source-independent objective function was constructed by convolving the envelope data with reference envelope traces. Despite these advances, a fundamental challenge shared by these methods is selecting an appropriate reference trace. Different reference-trace choices can lead to markedly different inversion behavior and results because the reference trace essentially affects data matching during inversion. Consequently, designing a reasonable and robust reference-trace selection strategy becomes a critical, often unavoidable step in practical applications.

To robustly construct accurate low-wavenumber P- and S-wave velocity models without relying on source-wavelet information, we propose a new source-independent envelope inversion method, termed source-independent EFCEI (SIEFCEI). Unlike existing source-independent strategies that require an optimized reference trace for data matching, SIEFCEI eliminates reference-trace selection by adopting an amplitude-semblance-based objective function.²⁷ During inversion, the matched data are computed using the elastic frequency-controllable envelope proposed in Du *et al.*,¹⁴ which provides richer and more focused low-frequency components than the conventional elastic envelope and is therefore more suitable for recovering large-scale background velocities.

Moreover, SIEFCEI is implemented within a cascaded multi-scale inversion framework, in which the inversion proceeds progressively from low- to high-frequency bands using stage-dependent envelope data. This design further improves the reliability of low-wavenumber velocity model updates and provides accurate initial models for subsequent EFWI.

2. Methodology

Successful P- and S-wave velocity model building with EFWI technology relies on accurate initial velocity models and a reasonably accurate source wavelet. However, commonly used initial model-building methods typically assume consistent wavelets between observed and synthetic data, an assumption that is often violated in field applications. Although some source-independent initial velocity modeling techniques have been proposed, their inversion performance still depends strongly on selecting a suitable reference trace, which may introduce instability or even lead to failure. Therefore, we propose SIEFCEI to reconstruct accurate low-wavenumber P- and S-wave velocity models without relying on an appropriate reference trace, providing accurate initial models for subsequent EFWI.

2.1. The review of elastic full-waveform inversion and elastic frequency-controllable envelope inversion

Elastic full-waveform inversion is a high-resolution multiparameter inversion technique in which subsurface parameters are updated by iteratively reducing the misfit between observed and modeled seismic data, under the constraint of the elastic wave equation. The misfit function of EFWI is written as **Equation 1**:

$$J_{EFWI} = \sum_k \sum_{i=1}^{N_s} \sum_{j=1}^{N_r} \|u_{ij}^k - d_{ij}^k\|^2 \quad (1)$$

where the subscript k represents the index of different components of seismic data, for example, $k = x, z$ in the two-dimensional situation; the subscripts i and j are the indices of sources and receivers, respectively; N_s and N_r are the number of sources and receivers, respectively; u denotes the synthetic seismic data in the time domain simulated by the elastic wave equation, and d is the observed seismic data in the time domain. The gradient of the inversion parameters $m = [v_p, v_s, \rho]$ can be calculated using the adjoint state method and is commonly obtained as the zero-lag cross-correlation between the forward wavefield and the back-propagated residual (adjoint) wavefield. Based on the calculated parameter gradients, inversion parameters can be updated using a local optimization approach, such as the steepest descent method, which is

widely used in waveform inversion. The update formula is written as **Equation 2**:

$$m^{n+1} = m^n - \alpha g_m^n \quad (2)$$

where n denotes the index of iterations, α is the step length in the n th iteration, and g_m^k corresponds to the direction of update, that is, the gradient of the parameter. A key challenge of EFWI is its strong dependence on initial velocity models. When the initial P- and S-wave velocity models are inaccurate, the calculated and observed waveforms can be misaligned by more than half a cycle. In this case, the inversion tends to converge to a local minimum rather than the true model, causing the inversion to become trapped in cycle skipping and leading to unreliable reconstruction of the velocity model.

As mentioned above, accurate initial velocity models are essential for successful EFWI. EFCEI is a practical approach to building accurate initial velocity models for EFWI, implemented within a cascaded multi-scale envelope inversion workflow. Its results yield accurate low-wavenumber velocity models, which effectively mitigate cycle skipping in EFWI. The misfit function of EFCEI is formulated as **Equation 3**:

$$J_{EFCEI} = \sum_k \sum_{i=1}^{N_s} \sum_{j=1}^{N_r} \|G(u_{ij}^k) - G(d_{ij}^k)\|^2 \quad (3)$$

where $G(\cdot)$ represents the frequency-controllable envelope, which is expressed as **Equation 4**:

$$\begin{cases} G(x) = \sqrt{[D(x) + \eta]^2 + H^2[D(x) + \eta]} \\ D(x) = x(t) * g(t) \end{cases} \quad (4)$$

where $*$ denotes the convolution operation; $g(t)$ is the filter that enables the controllable-frequency mechanism. $D(x)$ is the filtered result of the seismic data $x(t)$, H denotes the Hilbert operator, and η is a correction factor that helps calculate a more accurate adjoint source. The data matched during the inversion are the envelopes of elastic seismic data within a specified frequency band, which contain richer low-frequency information than the conventional envelope. These more focused low-frequency components provide a more informative basis for recovering the large-scale background components of the velocity models and help improve inversion stability in the early stages. The gradients of inversion parameters for EFCEI were also calculated using the adjoint-state method, which is formulated as **Equations 5 and 6**:

$$\nabla_{v_p} J(m) = -2\rho v_p \sum_{N_s} \left\langle \frac{\partial p_x(t)}{\partial x} + \frac{\partial p_z(t)}{\partial z} \right\rangle \left\langle \frac{\partial \Psi_x(t)}{\partial x} + \frac{\partial \Psi_z(t)}{\partial z} \right\rangle \quad (5)$$

$$\begin{aligned} \nabla_{v_s} J(m) = & 4\rho v_s \sum_{N_s} \left\langle \frac{\partial p_x(t)}{\partial x} + \frac{\partial p_z(t)}{\partial z} \right\rangle \left\langle \frac{\partial \Psi_x(t)}{\partial x} + \frac{\partial \Psi_z(t)}{\partial z} \right\rangle \\ & - 2\rho v_s \sum_{N_s} \left\langle \frac{\partial p_x(t)}{\partial z} + \frac{\partial p_z(t)}{\partial x} \right\rangle \left\langle \frac{\partial \Psi_x(t)}{\partial z} + \frac{\partial \Psi_z(t)}{\partial x} \right\rangle \\ & + 2 \left\langle \frac{\partial p_x(t)}{\partial x} \frac{\partial \Psi_x(t)}{\partial x} \right\rangle \left\langle \frac{\partial p_z(t)}{\partial z} \frac{\partial \Psi_z(t)}{\partial z} \right\rangle \end{aligned} \quad (6)$$

where $p_x(t)$ and $p_z(t)$ are the synthetic wavefields in the x - and z -directions, respectively. $\Psi_x(t)$ and $\Psi_z(t)$ are the corresponding adjoint wavefields. ρ denotes the density. Since EFCEI exploits the envelope information of seismic data during the inversion, it is less sensitive to phase errors and can tolerate source inconsistencies to some extent. However, when the source mismatch becomes too large, EFCEI may converge to an incorrect solution and become trapped in a local minimum. Therefore, source-wavelet accuracy remains a key factor limiting EFCEI's effectiveness.

2.2. Source-independent elastic frequency-controllable envelope inversion

To eliminate the need for accurate source-wavelet information in EFCEI, we incorporated source independence and formulated a corresponding misfit function in the frequency domain based on the amplitude-semblance function, whose definition at the selected frequency points is given as **Equation 7**:

$$J_{SIEFCEI}(\omega) = \frac{1}{2} \sum_k \sum_{i=1}^{N_s} [1 - \Phi_i^k(\omega)]^2 \quad (7)$$

where ω is the frequency, k represents the index of different directions, N_s is the number of shots, and Φ is the amplitude-semblance function, which is defined as **Equation 8**:

$$\Phi_i^k(\omega) = \frac{\sum_{j=1}^{N_r} |G(u_{ij}^k(\omega))| \cdot |G(d_{ij}^k(\omega))|}{\|G(u_i^k(\omega))\| \|G(d_i^k(\omega))\|} \quad (8)$$

where N_r is the number of receivers, $|\cdot|$ denotes the magnitude operation, and $\|G_{ij}^{syn}(\omega)\| \|G_{ij}^{obs}(\omega)\|$ is a normalization factor defined as **Equation 9**:

$$\|G(u_i^k(\omega))\| \|G(d_i^k(\omega))\| = \sqrt{\sum_{j=1}^{N_r} |G(u_{ij}^k(\omega))|^2} \sqrt{\sum_{j=1}^{N_r} |G(d_{ij}^k(\omega))|^2} \quad (9)$$

The amplitude-semblance function Φ provides a quantitative measure of the similarity between the synthetic and observed data. It is typically bounded between 0 and 1, where 1 indicates perfect agreement between synthetic and observed seismic data, and lower values indicate increasing differences between them. Since the inversion aims to minimize an objective function,

the misfit is defined as “1 – Φ ,” which decreases as the similarity between the two datasets increases. Notably, this formulation does not involve any reference trace, thereby avoiding the sensitivity and potential instability introduced by reference-trace selection. Notably, SIEFCEI was implemented as a hybrid time–frequency domain method: forward wavefield modeling was performed in the time domain, whereas the objective function and gradients were calculated in the frequency domain. The source-independent property of SIEFCEI is further demonstrated in the following subsection.

The parameter gradient of the misfit function in **Equation 7** can be computed using the adjoint-state method. Specifically, with the frequency variable ω omitted for notational simplicity, the gradient of $J_{SIEFCEI}$ with respect to the n -th model parameter m_n is given by **Equation 10**:

$$\frac{\partial J_{SIEFCEI}}{\partial m_n} = \sum_k \sum_{i=1}^{N_s} (1 - \Phi_i^k) \frac{\partial (1 - \Phi_i^k)}{\partial m_n} = - \sum_k \sum_{i=1}^{N_s} (1 - \Phi_i^k) \frac{\partial \Phi_i^k}{\partial m_n} \quad (10)$$

where

$$\frac{\partial \Phi_i^k}{\partial m_n} = \frac{1}{\|G(d_i^k)\|} \left[\frac{\sum_{j=1}^{N_r} \frac{\partial |G(u_{ij}^k)|}{\partial m_n} |G(d_{ij}^k)|}{\|G(u_i^k)\|} - \frac{\frac{\partial \|G(u_i^k)\|}{\partial m_n} \sum_{j=1}^{N_r} |G(u_{ij}^k)| |G(d_{ij}^k)|}{\|G(u_i^k)\|^2} \right] \quad (11)$$

where $\frac{\partial \|G(u_i^k)\|}{\partial m_n}$ can be further expanded as **Equation 12**:

$$\begin{aligned} \frac{\partial \|G(u_i^k)\|}{\partial m_n} &= \frac{1}{2} \left[\sum_{j=1}^{N_r} |G(u_{ij}^k)|^2 \right]^{-1/2} \cdot 2 \sum_{j=1}^{N_r} |G(u_{ij}^k)| \frac{\partial |G(u_{ij}^k)|}{\partial m_n} \\ &= \frac{1}{\|G(u_i^k)\|} \sum_{j=1}^{N_r} |G(u_{ij}^k)| \frac{\partial |G(u_{ij}^k)|}{\partial m_n} \end{aligned} \quad (12)$$

Substituting **Equation 12** into **Equation 11** yields **Equation 13**:

$$\frac{\partial \Phi_i^k}{\partial m_n} = \sum_{j=1}^{N_r} \left[\frac{|\hat{G}(d_{ij}^k)| |G(u_i^k)|^2 - |G(u_{ij}^k)| \sum_{j=1}^{N_r} |G(u_{ij}^k)| |G(d_{ij}^k)|}{\|\hat{G}(d_i^k)\| \|G(u_i^k)\|^3} \right] \frac{\partial |\hat{G}(u_{ij}^k)|}{\partial m_n} \quad (13)$$

because $|G(u_{ij}^k)| = \sqrt{\hat{G}(u_{ij}^k) (\hat{G}(u_{ij}^k))^*}$, $\frac{\partial |\hat{G}(u_{ij}^k)|}{\partial m_n}$ can be further expanded as **Equation 14**:

$$\begin{aligned} \frac{\partial |\hat{G}(u_{ij}^k)|}{\partial m_n} &= \frac{1}{2 |\hat{G}(u_{ij}^k)|} \left[(\hat{G}(u_{ij}^k))^* \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} + \hat{G}(u_{ij}^k) \frac{\partial (\hat{G}(u_{ij}^k))^*}{\partial m_n} \right] \\ &= \frac{\text{Re} \left((\hat{G}(u_{ij}^k))^* \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} \right)}{|\hat{G}(u_{ij}^k)|} \end{aligned} \quad (14)$$

$$= \left| \hat{G}(u_{ij}^k) \right| \text{Re} \left(\frac{1}{\hat{G}(u_{ij}^k)} \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} \right)$$

Therefore, **Equation 13** can be further written as **Equation 15**:

$$\begin{aligned} \frac{\partial \Phi^k}{\partial m_n} &= \sum_{j=1}^{N_s} \left[\frac{\left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2 - \left\| G(u_j^k) \right\|^2 \sum_{j=1}^{N_s} \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2}{\left\| \hat{G}(d_j^k) \right\| \left\| G(u_j^k) \right\|^3} \right] \left| \hat{G}(u_{ij}^k) \right| \text{Re} \left(\frac{1}{\hat{G}(u_{ij}^k)} \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} \right) \\ &= \sum_{j=1}^{N_s} \text{Re} \left(\frac{\left| \hat{G}(u_{ij}^k) \right| \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2 - \left\| G(u_j^k) \right\|^2 \sum_{j=1}^{N_s} \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2}{\left\| \hat{G}(d_j^k) \right\| \left\| G(u_j^k) \right\|^3} \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} \right) \end{aligned} \quad (15)$$

where

$$\begin{aligned} \frac{\partial \hat{G}(u_{ij}^k)}{\partial m_n} &= \frac{\partial \mathcal{F}\{G(u_{ij}^k(t))\}}{\partial m_n} \\ &= \mathcal{F} \left\{ \frac{\partial \sqrt{(u_{ij}^k(t) * g(t))^2 + [H(u_{ij}^k(t) * g(t))]^2}}{\partial m_n} \right\} \\ &= \mathcal{F} \left\{ \frac{(u_{ij}^k(t) * g(t)) \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right) + H(u_{ij}^k(t) * g(t)) H \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right)}{G(u_{ij}^k(t))} \right\} \end{aligned} \quad (16)$$

Substituting **Equation 16** into **Equation 15** yields **Equation 17**:

$$\frac{\partial \Phi^k}{\partial m_n} = \sum_{j=1}^{N_s} \text{Re} \left\{ \mathcal{F} \left\{ \frac{\left| \hat{G}(u_{ij}^k) \right| \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2 - \left\| G(u_j^k) \right\|^2 \sum_{j=1}^{N_s} \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2}{\left\| \hat{G}(d_j^k) \right\| \left\| G(u_j^k) \right\|^3} \hat{G}(u_{ij}^k) \right\} \left(\frac{(u_{ij}^k(t) * g(t)) \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right) + H(u_{ij}^k(t) * g(t)) H \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right)}{G(u_{ij}^k(t))} \right) \right\} \quad (17)$$

Thus, **Equation 10** becomes **Equation 18**:

$$\frac{\partial J_{\text{SIEFCEI}}}{\partial m_n} = - \sum_{k=1}^{N_s} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \text{Re} \left\{ \mathcal{F} \left\{ \frac{(1 - \phi_i^k) \left| \hat{G}(u_{ij}^k) \right| \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2 - \left\| G(u_j^k) \right\|^2 \sum_{j=1}^{N_s} \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2}{\left\| \hat{G}(d_j^k) \right\| \left\| G(u_j^k) \right\|^3} \hat{G}(u_{ij}^k) \right\} \left(\frac{(u_{ij}^k(t) * g(t)) \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right) + H(u_{ij}^k(t) * g(t)) H \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right)}{G(u_{ij}^k(t))} \right) \right\} \quad (18)$$

Let

$$R^1 = (1 - \phi_i^k) \frac{\left| \hat{G}(u_{ij}^k) \right| \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2 - \left\| G(u_j^k) \right\|^2 \sum_{j=1}^{N_s} \left| \hat{G}(d_j^k) \right| \left\| G(u_j^k) \right\|^2}{\left\| \hat{G}(d_j^k) \right\| \left\| G(u_j^k) \right\|^3} \hat{G}(u_{ij}^k) \quad (19)$$

$$\mathcal{F} \left\{ \mathcal{X} \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} \right) \right\} = \mathcal{F} \left\{ \frac{(u_{ij}^k(t) * g(t)) \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right) + H(u_{ij}^k(t) * g(t)) H \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} * g(t) \right)}{G(u_{ij}^k(t))} \right\} \quad (20)$$

Equation 18 can be simply expressed as **Equation 21**:

$$\frac{\partial J_{\text{SIEFCEI}}}{\partial m_n} = - \text{Re} \sum_k \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \left\langle R^1, \mathcal{F} \left\{ \mathcal{X} \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} \right) \right\} \right\rangle_{\omega} \quad (21)$$

By the definition of the adjoint Fourier transform, the frequency-domain inner product can be equivalently rewritten in the time domain. Therefore, **Equation 21** can be further written as **Equation 22**:

$$\begin{aligned} \frac{\partial J_{\text{SIEFCEI}}}{\partial m_n} &= - \text{Re} \sum_k \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \left\langle \mathcal{F}^* \{R^1\}, \mathcal{X} \left(\frac{\partial u_{ij}^k(t)}{\partial m_n} \right) \right\rangle_t \\ &= - \text{Re} \sum_k \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \left\langle \mathcal{X}^* (\mathcal{F}^* \{R^1\}), \frac{\partial u_{ij}^k(t)}{\partial m_n} \right\rangle_t \end{aligned} \quad (22)$$

where $\mathcal{F}^*$ denotes the adjoint operator of $\mathcal{F}$, and $\mathcal{X}^*$ denotes the adjoint operator of $\mathcal{X}$. Based on the above derivation, the adjoint source of SIEFCEI can be obtained as **Equation 23**:

$$r_{ij} = - \mathcal{X}^* (\mathcal{F}^* \{R^1\}) \quad (23)$$

Accordingly, the adjoint source of SIEFCEI is given by **Equation 23**, while the gradient expressions with respect to model parameters remain the same as in **Equations 5** and **6**. As indicated by the above derivation, a manual code implementation of the adjoint source is rather cumbersome. Therefore, the residual source of SIEFCEI was computed via automatic differentiation. Given the resulting parameter gradients, the steepest-descent gradient method was used to update the inversion parameters. In addition, we adopted a fixed step length rather than estimating a variable step size via parabolic fitting. **Figure 1** illustrates the overall implementation process of SIEFCEI.

2.3. Source independence analysis

In this subsection, we provide a detailed theoretical derivation of the wavelet independence of the SIEFCEI misfit function. Before demonstrating the source independence of the misfit function J_{SIEFCEI} , we firstly reformulated the elastic frequency-controllable envelope data as a convolution expression.^{26,28} One component of the recorded seismic data at the surface can be formulated as **Equations 24** and **25**:

$$U(t, x_s, x_r) \approx P(t, x_s, x_r) r_w(t) \quad (24)$$

$$r_w(t) = \sum_i w(t) * r_i(t, x_i) \quad (25)$$

where U represents the seismic data recorded at the receiver at x_r from a source located at x_s , t denotes time, P is the propagator from the source to the scattering point and back to the receiver, and $r_w(t)$ is the convolution of the source wavelet $w(t)$ and the reflection series $r_i(t, x_i)$. Since D is the filtered seismic data, it can be formulated as:

$$D(t, x_s, x_r) \approx P(t, x_s, x_r) r_{\text{ew}}(t) \quad (26)$$

$$r_{\text{ew}}(t) = \sum_i s(t) * w(t) * r_i(t, x_i) = \sum_i w_e(t) * r_i(t, x_i) \quad (27)$$

The convolution expression of elastic frequency-controllable envelope data can be derived from **Equations 26 and 27**. The square of the envelope G can be written as **Equation 28**:

$$G(t)^2 = [P(t; x_s, x_r) r_{\text{ew}}(t)]^2 + \{H[P(t; x_s, x_r) r_{\text{ew}}(t)]\}^2 \quad (28)$$

By the Bedrosian–Brown product theorem,^{29,30} and since the propagator P is a low-pass function under the forward-scattering approximation, **Equation 28** can be rewritten as **Equation 29**:

$$\begin{cases} G(t)^2 = P^2(w_e * r)^2 + P^2[H(w_e * r)]^2 \\ r = \sum_i r_i(t, x_i) = \sum_i a_i \delta(t - t_i) \end{cases} \quad (29)$$

Based on the expression of the Hilbert transform, $G(t)^2$ can be further expanded as follows:

$$\begin{aligned} G(t)^2 &= P^2(w_e * r)^2 + P^2\left[w_e * r * \frac{1}{\pi t}\right]^2 \\ &= P^2(w_e * r)^2 + P^2[w_e^H * r]^2 \end{aligned} \quad (30)$$

Expanding the first term in **Equation 30** yields **Equation 31**:

$$\begin{aligned} P^2(w_e * r)^2 &= P^2[w_e(t) * r_1 + w_e(t) * r_2 + \dots + w_e(t) * r_n]^2 \\ &= P^2[a_1 w_e(t - t_1) + a_2 w_e(t - t_2) + \dots + a_n w_e(t - t_n)]^2 \\ &\approx P^2[a_1^2 w_e(t - t_1)^2 + a_2^2 w_e(t - t_2)^2 + \dots + a_n^2 w_e(t - t_n)^2] \end{aligned} \quad (31)$$

Applying the same procedure, the second term in

Equation 30 becomes **Equation 32**:

$$P^2(w_e^H * r)^2 \approx P^2[a_1^2 w_e^H(t - t_1)^2 + a_2^2 w_e^H(t - t_2)^2 + \dots + a_n^2 w_e^H(t - t_n)^2] \quad (32)$$

By substituting **Equations 31 and 32** into **Equation 30**, we obtain **Equation 33**:

$$G(t)^2 = [P(t)^2 r(t)^2] * E_{\text{ew}}(t)^2 \quad (33)$$

where $E_{\text{ew}}(t)$ is the envelope of the source $w_e(t)$. Assuming that the model reflectivity is sparse, the envelopes of individual reflection events are non-overlapping.³¹ Consequently, **Equation 33** takes the form of **Equation 34**:

$$G(t) = [P(t) \| r(t) \|] * E_{\text{ew}}(t) = C(t) * E_{\text{ew}}(t) \quad (34)$$

Equation 34 gives the convolutional representation of one component of the elastic frequency-controllable envelope data, where $C(t)$ represents an envelope reflection response governed by the reflectivity structure of the velocity model.

Next, we reformulated the amplitude-semblance objective function Φ in the frequency domain as **Equation 35**:

$$\Phi_i(\omega) = \frac{\sum_{j=1}^{N_r} C_{ij}^{\text{syn}} E_{\text{ew}_i}^{\text{syn}} \cdot C_{ij}^{\text{obs}} E_{\text{ew}_i}^{\text{obs}}}{E_{\text{ew}_i}^{\text{syn}} \|C_{ij}^{\text{syn}}\| \|C_{ij}^{\text{obs}}\| E_{\text{ew}_i}^{\text{obs}}} = \frac{\sum_{j=1}^{N_r} C_{ij}^{\text{syn}} \cdot C_{ij}^{\text{obs}}}{\|C_{ij}^{\text{syn}}\| \|C_{ij}^{\text{obs}}\|} \quad (35)$$

Equation 35 indicates that the source effect, including amplitude and phase differences between the synthetic and observed data, is effectively removed from the objective function. Consequently, the misfit becomes less sensitive to source-wavelet inaccuracies and is governed primarily

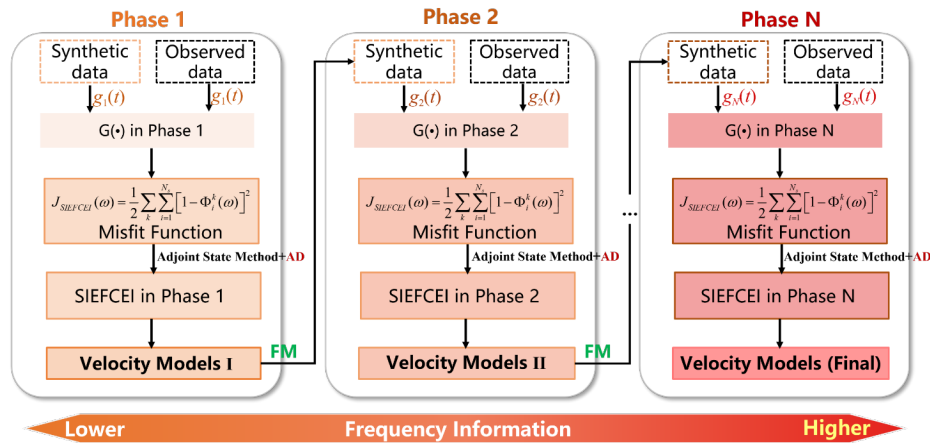


Figure 1. Illustration of the source-independent elastic frequency-controllable envelope inversion (SIEFCEI) process, which is performed in a cascaded manner. Benefiting from the effect of G , the inversion starts in a lower-frequency band in the initial phase and gradually expands the frequency content as the inversion progresses. In the figure, “AD” denotes automatic differentiation, and “FM” represents forward modeling.

by the medium response. In other words, the objective function is essentially governed solely by the reflection response $C(t)$, without any explicit source term, which eliminates source-related factors during inversion.

3. Results

In this section, we demonstrate the advantages of SIEFCEI using a synthetic example based on the elastic overthrust model, together with Volve field data. We present the results of these two experiments separately.

3.1. Synthetic example based on the elastic overthrust model

Figure 2A and C show the true P- and S-wave velocity models, respectively. The initial P- and S-wave velocity models are simple linear models that increase with depth, as shown in Figure 2B and D, and therefore lack the key lateral variations and fault-related features of the true models. The density is fixed at $1,929 \text{ kg/m}^3$ and is not treated as an inversion parameter. The experimental model contains 801×170 grid points with a uniform grid spacing of 6 m. A total of 51 shots were used to simulate the elastic wavefields, which were evenly distributed along a horizontal line at a depth of 12 m with a spacing of 80 m. In addition, 170 receivers were deployed at 20 m spacing at the same depth as the shots to record multicomponent seismic data. Forward wavefield simulation was conducted by solving the two-dimensional elastic wave equation in the velocity-stress formulation using a staggered-grid finite-difference method. An eighth-order finite-difference scheme was

used for spatial discretization, while a second-order finite-difference scheme was adopted for time discretization. The total recording time is 2.5 s with a sampling interval of 0.5 ms, ensuring sufficient temporal resolution for subsequent inversion and satisfying the stability requirement of the wavefield simulation. The source wavelets used for the observed and synthetic seismic data are plotted in Figure 3, where the black line is the observed wavelet with a dominant frequency of 8 Hz and the red line is the synthetic wavelet with a dominant frequency of 10 Hz. In addition to the dominant-frequency difference, the two wavelets exhibit noticeable amplitude discrepancies and a clear time shift, which together represent a challenge for initial-model building and subsequent EFWI.

First, we conducted EFCEI in three phases, assuming the source wavelets are consistent between the observed and synthetic seismic data. The frequency spectra of the filter $g(t)$ used in different inversion phases are plotted in Figure 4, where the passband was progressively broadened as the inversion proceeded, from 0–5 Hz in Phase 1, to 0–8 Hz in Phase 2, and to 0–11 Hz in Phase 3. The inversion was performed from low- to high-frequency data in a multi-scale manner. The corresponding inverted low-wavenumber P- and S-wave velocity models at different inversion stages are shown in Figures 5–7. As the inversion progresses, the left thrust faults and the layered structures on the right were gradually reconstructed, and the final inverted models clearly delineated the dominant large-scale features of the true models. The data misfit curves for the three EFCEI phases are shown in Figure 8. In all

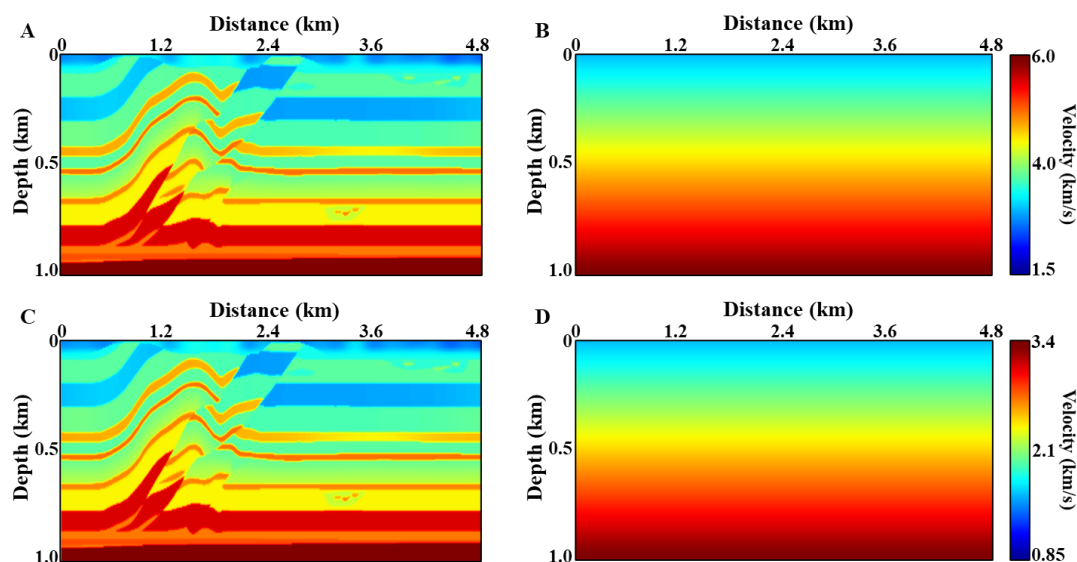


Figure 2. The two-dimensional elastic overthrust model used in the inversion. (A) The true V_p model. (B) The initial V_p model. (C) The true V_s model. (D) The initial V_s model.

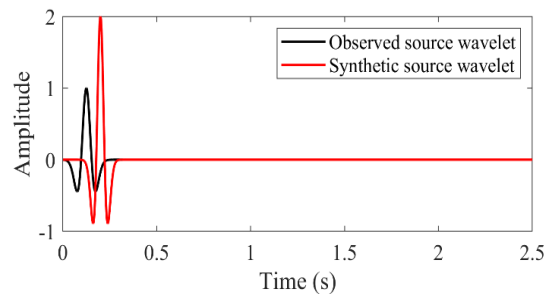


Figure 3. The source wavelets of the observed and synthetic seismic data

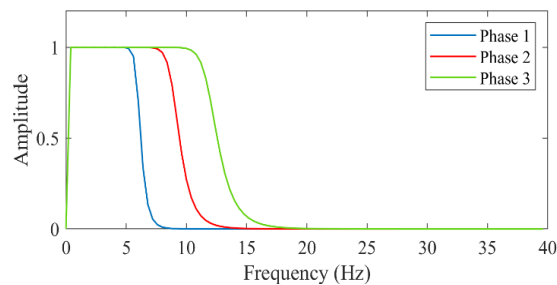


Figure 4. The frequency spectrum of $g(t)$ during the three phases of elastic frequency-controllable envelope inversion

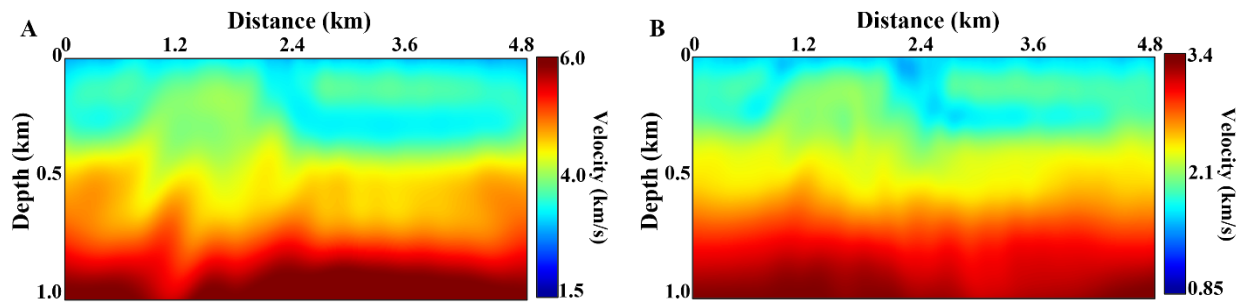


Figure 5. Inverted velocity models of Phase 1 obtained by elastic frequency-controllable envelope inversion. (A) V_p . (B) V_s .

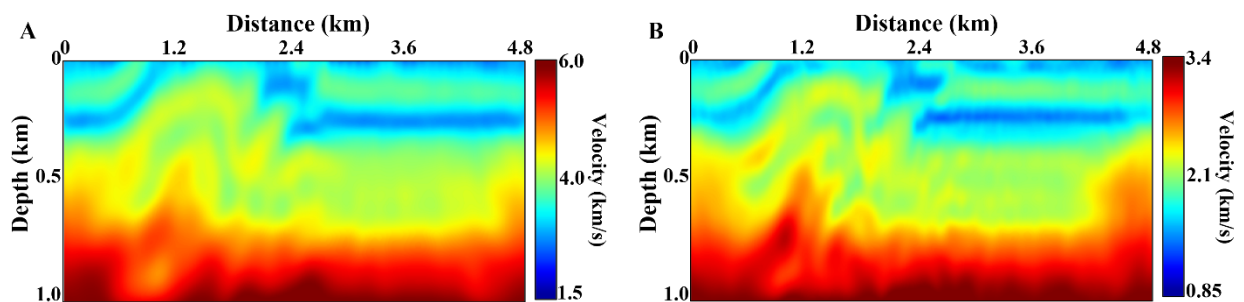


Figure 6. Inverted velocity models of Phase 2 obtained by elastic frequency-controllable envelope inversion. (A) V_p . (B) V_s .

phases, the data misfit decreased significantly as iterations progressed, indicating that the inversion gradually reduced the difference between the observed and synthetic data.

These inverted low-wavenumber velocity models provide a reliable reference and can serve as accurate initial models for subsequent EFWI.

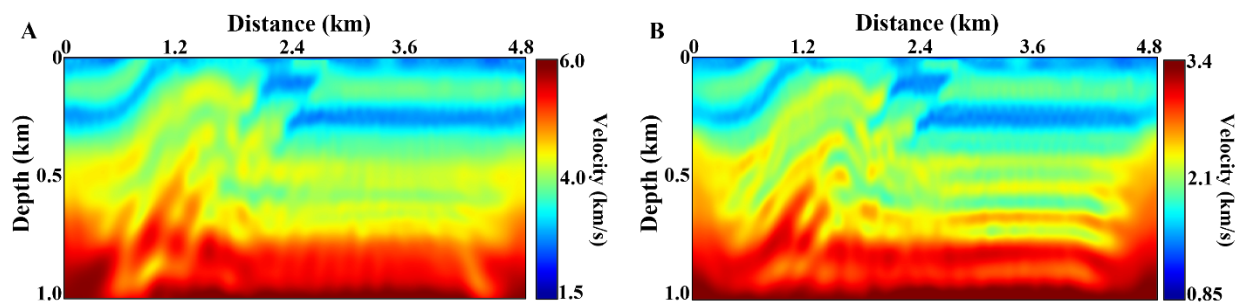


Figure 7. Inverted velocity models of Phase 3 obtained by elastic frequency-controllable envelope inversion. (A) V_p . (B) V_s .

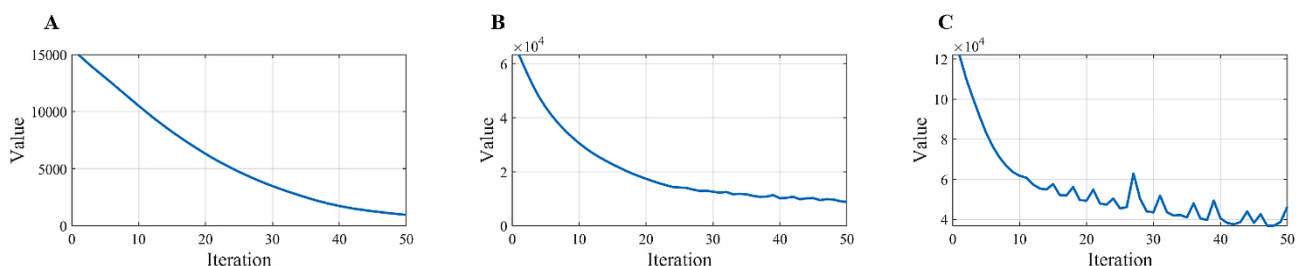


Figure 8. Data misfit curves of elastic frequency-controllable envelope inversion at different inversion phases. Data loss of (A) Phase 1, (B) Phase 2, and (C) Phase 3.

Second, EFCEI was reapplied under a source-wavelet mismatch between the observed and synthetic data, while the same three-phase inversion strategy was maintained. The corresponding inverted low-wavenumber P- and S-wave velocity models at different inversion stages are shown in Figures 9–11, and the data misfit curves are plotted in Figure 12. Only the Phase 1 inversion weakly reconstructed the thrust-fault structure. In addition, the inverted S-wave velocity model delineated some horizontal layered variations in the shallow zone. However, as the inversion progressed, the inverted low-wavenumber P- and S-wave velocity models no longer showed reliable subsurface structural features, in sharp contrast to the EFCEI results obtained using the correct source wavelet. Although the data misfit curves generally decreased, a joint analysis of the misfit curves and inversion results indicates that the inversion was trapped in an undesired local minimum. The final inverted velocity models were dominated by over-smoothed and nearly layered structures. The fault structures were not effectively reconstructed, and the layered variations beyond 2.4 km horizontally were also poorly recovered. In addition, pronounced wave-like artifacts appeared in the shallow zone. These problems were more evident in the inverted S-wave velocity model than in the inverted P-wave velocity model, which may be attributed to the different wavenumber sensitivities of P- and S-wave velocities during the inversion. Overall,

these observations demonstrate that the inaccurate source wavelet drives the inversion toward an undesired local optimum. Although EFCEI uses envelope data with enriched low-frequency information and can partially mitigate the influence of wavelet inaccuracies, such severe source wavelet inconsistency still leads to inversion failure.

Finally, SIEFCEI was conducted under source-wavelet mismatch while maintaining the same three-phase inversion strategy as before. Figures 13–15 present the inverted low-wavenumber P- and S-wave velocity models at different inversion phases, respectively. The Phase 1 inverted velocity models show thrust-related uplift structures and horizontal layered variations above 0.5 km depth, which are highly consistent with those obtained by EFCEI using the correct source wavelet. After the Phase 2 inversion, these key structural features were more clearly delineated, and the structure located at approximately 0.7 km depth and 1.2 km horizontal distance was also significantly updated. The final inverted velocity models shown in Figure 15 agree well with the results in Figure 7, indicating that SIEFCEI remained effective and produced meaningful large-scale velocity updates despite severe source wavelet mismatch. Figure 16 presents the convergence process of SIEFCEI. Taken together, these results show that SIEFCEI produces substantial updates to the background velocity models and effectively recovers the main geological features, including thrust faults and

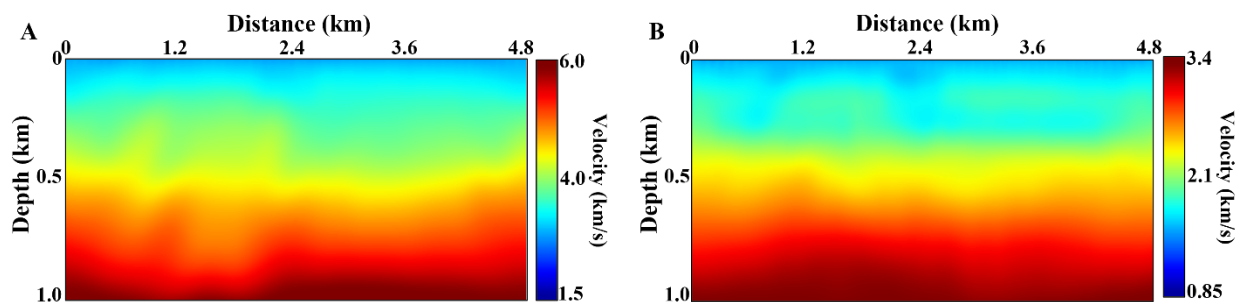


Figure 9. Inverted velocity models of Phase 1 obtained by elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) Vp. (B) Vs.

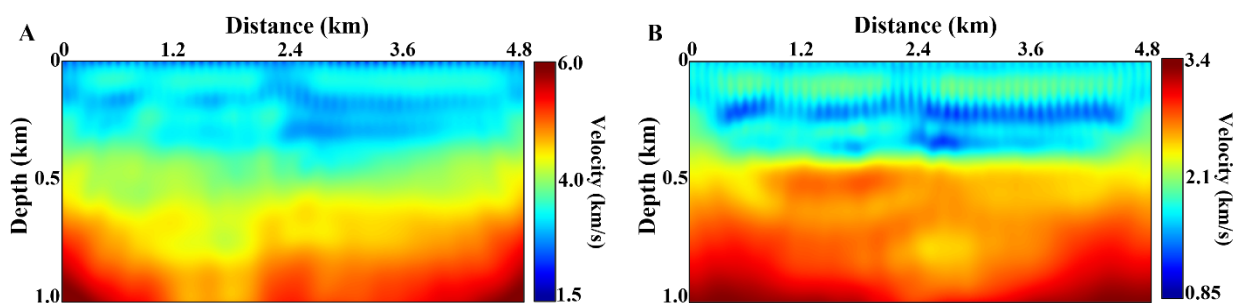


Figure 10. Inverted velocity models of Phase 2 obtained by elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) Vp. (B) Vs.

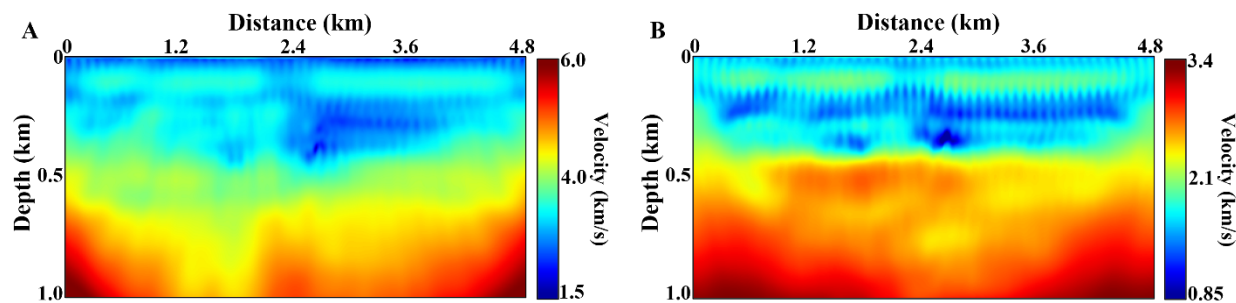


Figure 11. Inverted velocity models obtained by elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) Vp. (B) Vs.

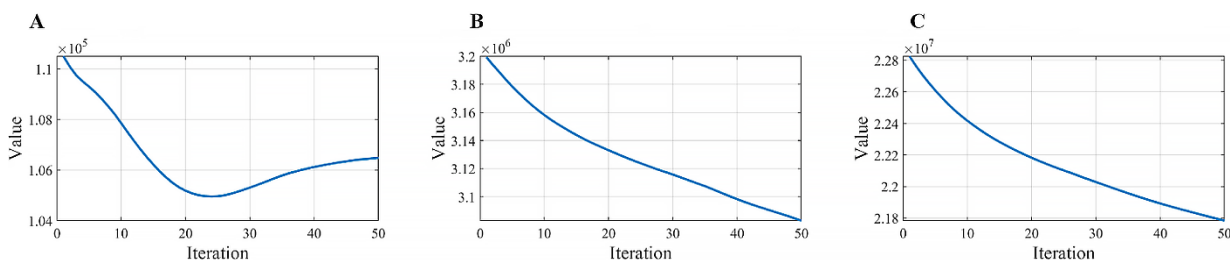


Figure 12. Data misfit curves of elastic frequency-controllable envelope inversion with an inaccurate wavelet at different inversion phases. Data loss of (A) Phase 1, (B) Phase 2, and (C) Phase 3.

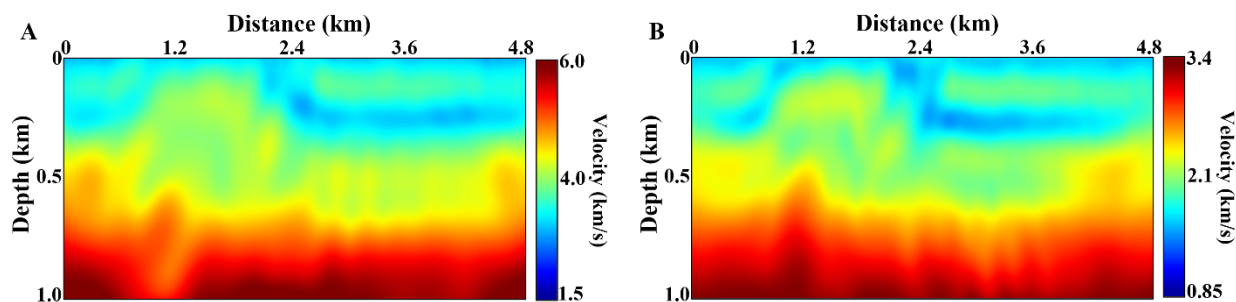


Figure 13. Inverted velocity models of Phase 1 obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) V_p . (B) V_s .

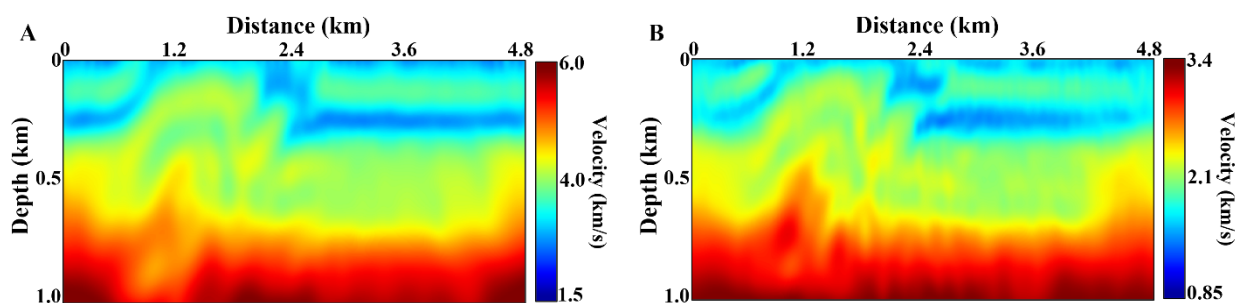


Figure 14. Inverted velocity models of Phase 2 obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) V_p . (B) V_s .

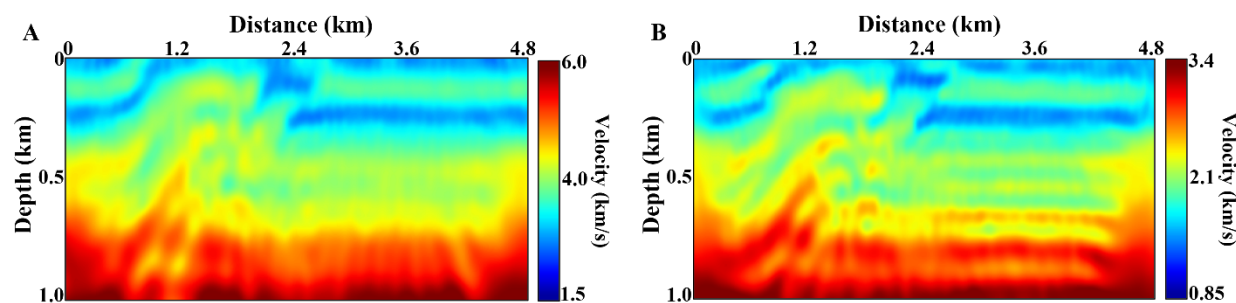


Figure 15. Inverted velocity models obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. (A) V_p . (B) V_s .

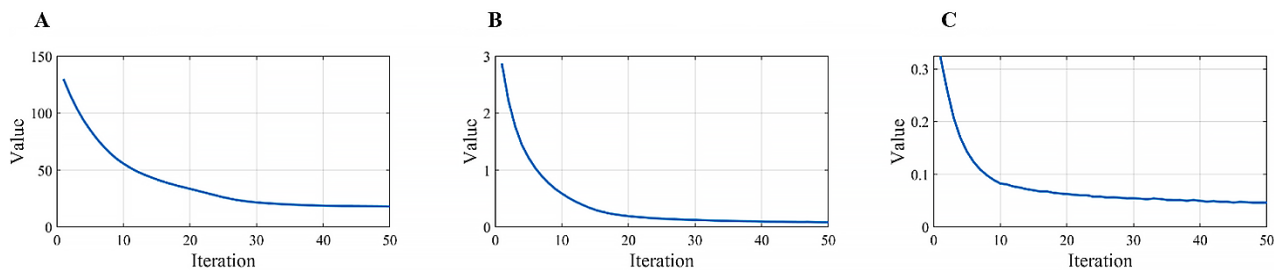


Figure 16. Data misfit curves of source-independent elastic frequency-controllable envelope inversion with an inaccurate wavelet at different inversion phases. Data loss of (A) Phase 1, (B) Phase 2, and (C) Phase 3.

layered structures, with reasonable velocity-variation trends and overall geometries. Only a few local structures differed from the EFCEI-inverted results obtained under the assumption of source-wavelet consistency, and these differences were mainly confined to localized regions and did not alter the large-scale background trends. More importantly, both EFCEI and SIEFCEI were designed to construct low-wavenumber velocity models, which were then used as the initial models for EFWI. From this perspective, small local differences are acceptable, because the primary requirement at this stage is to construct reliable initial P- and S-wave velocity models that guide the subsequent EFWI toward the correct solution and mitigate cycle skipping.

To further verify the effectiveness of SIEFCEI, we used the velocity models recovered by EFCEI and SIEFCEI under inconsistent source wavelets as the respective initial models for subsequent EFWI. The number of iterations was set to 70. It is worth noting that the SIEFWI,²⁴ rather than traditional EFWI, was adopted because the inversion is conducted under source inconsistency. Figure 17 presents the SIEFWI results obtained when the velocity models inverted by EFCEI under source inconsistency were used as the initial models. The inversion failed to converge to the true solution and instead became trapped in an incorrect one, mainly because of the inaccurate low-wavenumber starting models. Consequently, the recovered structures were blurred, and key geological features remained poorly resolved. In contrast, when the SIEFCEI inversion result was used as the initial model for SIEFWI, the corresponding inversion results in Figure 18 show an improved reconstruction of subsurface structures. Both the P- and S-wave velocity models were updated toward the true models, and the subsurface structural details, including fault geometry and stratigraphic variations, were recovered more clearly. Although a perfect match was still difficult to achieve because of the limited dominant frequency and bandwidth of the source wavelet, the

substantial improvement from Figure 17 to Figure 18 demonstrates that SIEFCEI can provide reliable initial models for EFWI even under significant source-wavelet inaccuracies.

3.2. Volve field data

We selected a two-dimensional line from the three-dimensional OBC survey that Statoil acquired in 2010 over the Volve field. The dataset consists of pressure records. Multidimensional deconvolution was applied to suppress free-surface multiples and transform the original acquisition geometry into a virtual configuration with co-located sources and receivers at the seafloor. The resulting dataset contained 180 virtual sources and 180 receivers with a nominal horizontal spacing of 25 m. Source and receiver depths varied from approximately 86.8 to 99.2 m, following the seafloor topography. Because all were positioned on the seafloor, the water-layer effect need not be taken into account during the inversion. In addition, the field data underwent a series of preprocessing operations, including denoising, vector-fidelity correction, anti-aliasing filtering, and a $\sqrt{t}$ scaling to compensate for the difference in geometrical spreading between three-dimensional field propagation and two-dimensional numerical modeling.

Because the field-data experiment focused on reconstructing the long-wavelength components of the V_p and V_s models, the pressure data were low-pass filtered to retain only the frequency components between 0 and 10 Hz. Figure 19 shows the initial velocity models used for the inversion. The initial P-wave velocity model was obtained by smoothing the tomographic P-wave velocity model of the Volve field, while the initial S-wave velocity model was derived from the P-wave model using the empirical relation $V_s = V_p/\sqrt{3}$. Both EFCEI and SIEFCEI were applied to this dataset using a three-phase inversion strategy. The density was fixed at 1,000 kg/m³ and was not treated as an inversion parameter. The maximum

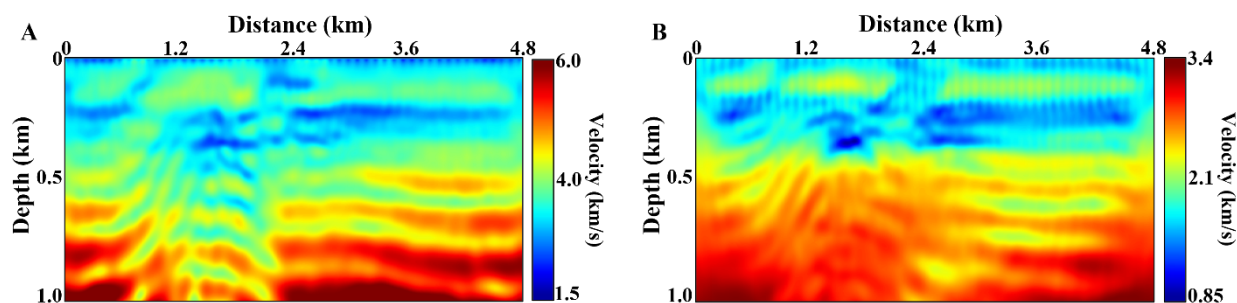


Figure 17. Inverted velocity models obtained by source-independent elastic frequency-controllable envelope inversion + elastic frequency-controllable envelope inversion (under source wavelet mismatch). (A) V_p . (B) V_s .

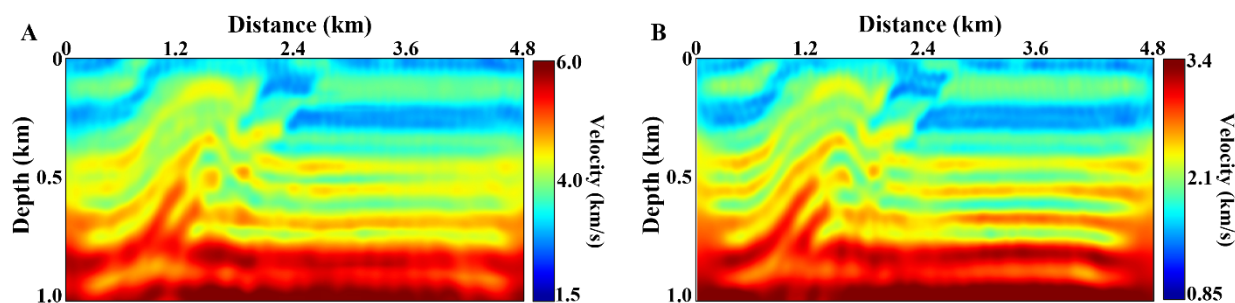


Figure 18. Inverted velocity models obtained by source-independent elastic frequency-controllable envelope inversion + elastic frequency-controllable envelope inversion (under source wavelet mismatch). (A) Vp. (B) Vs.

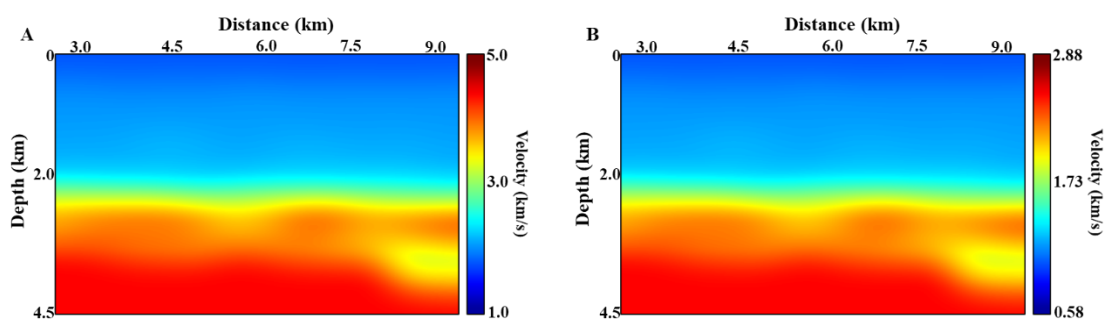


Figure 19. The initial models used in the inversion. (A) The initial Vp model. (B) The initial Vs model.

frequencies of the three phases were 4 Hz, 7 Hz, and 10 Hz, respectively, and 15 iterations were performed in each phase. A fixed step size was used for the model updates. A Ricker wavelet with a dominant frequency of 4 Hz, which was deliberately used as an inaccurate source wavelet, was adopted in both inversion experiments. Notably, although pressure records can be used for EFWI, they are generally more sensitive to variations in Vp than to those in Vs. Therefore, the inverted Vp model is generally more reliable than the inverted Vs model.

The final inverted P- and S-wave long-wavelength velocity models of SIEFCEI and EFCEI are shown in Figures 20 and 21, respectively. The EFCEI result exhibited nearly horizontal and regularly spaced oscillatory structures, whereas the SIEFCEI result showed clear lateral variations in the amplitude and distribution of the velocity structures. In addition, distinct velocity updates were observed in the lower-middle part of the model, as indicated by the black arrows in the SIEFCEI result. Figure 22 further illustrates these differences. The Vp updates of EFCEI were characterized by regularly alternating oscillatory bands and were mainly concentrated above approximately 2.5 km, with only weak updates at greater depths. In contrast, although the SIEFCEI updates also exhibited a banded pattern, they showed stronger spatial

heterogeneity, and clear structural variations were also observed in the lower-middle part of the model. The regularly oscillatory structures in the EFCEI results likely stem from the inaccurate source wavelet. The inaccurate source wavelet introduced coherent and periodically oscillatory waveform mismatches between the observed and calculated data. Because these mismatches exhibited similar characteristics across different shots, they may generate similar oscillatory components in the velocity gradients. After summing gradients from different shots, these similar oscillatory components accumulated and were enhanced, producing long, regularly spaced oscillatory bands. In contrast, by reducing the influence of the source wavelet, SIEFCEI makes the model updates less affected by coherent oscillatory waveform mismatches. Consequently, the recovered velocity structures show clear lateral heterogeneity and localized variations, which are more geologically plausible than the nearly horizontal, regularly spaced oscillatory structures observed in the EFCEI result. These results and analysis suggest that EFCEI is strongly affected by the inaccurate source wavelet, producing pronounced oscillatory artifacts, whereas SIEFCEI substantially reduces these effects and yields a more geologically reasonable long-wavelength velocity reconstruction.

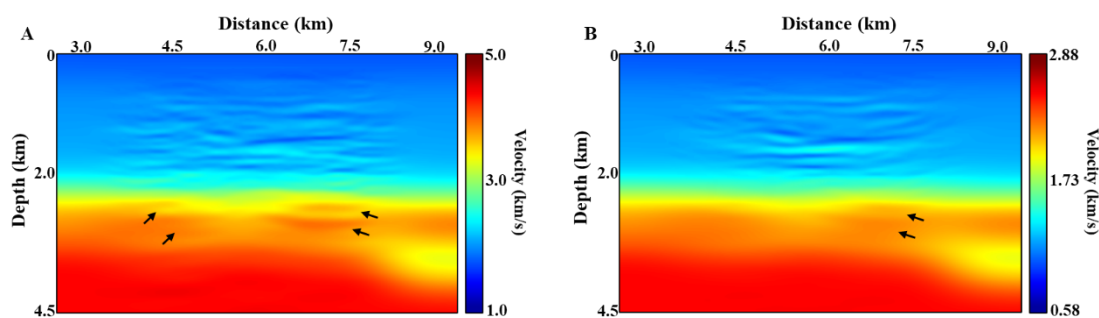


Figure 20. The inverted models using source-independent elastic frequency-controllable envelope inversion. (A) V_p model. (B) V_s model.

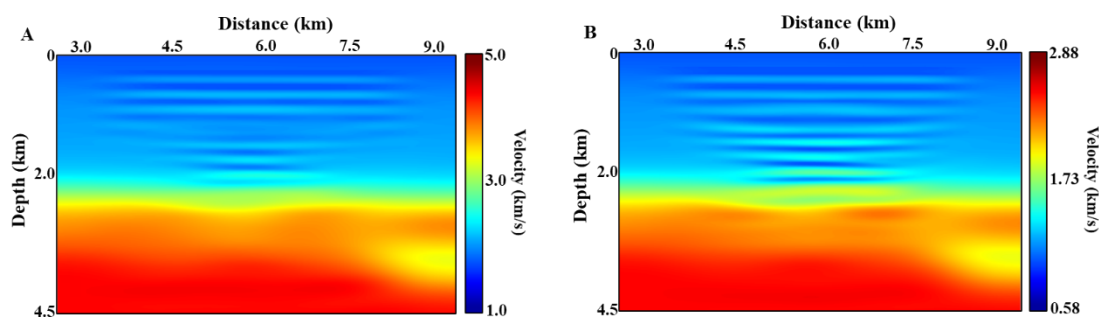


Figure 21. The inverted models using elastic frequency-controllable envelope inversion. (A) V_p model. (B) V_s model.

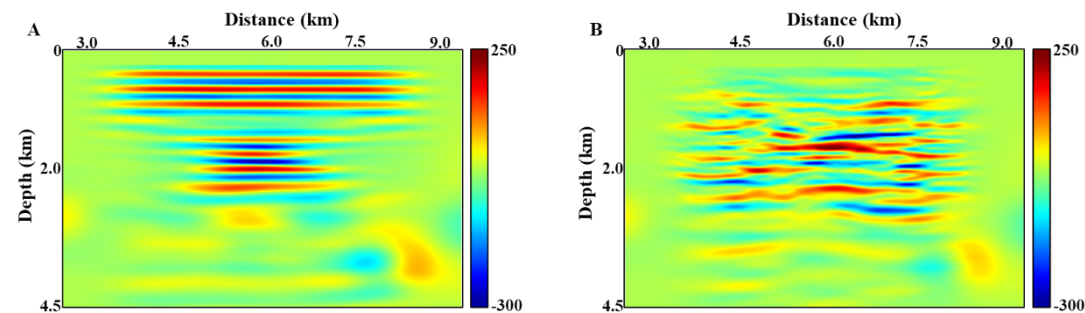


Figure 22. Comparison of the V_p updates from the initial model for (A) elastic frequency-controllable envelope inversion. (B) Source-independent elastic frequency-controllable envelope inversion.

Figures 23 and 24 show the normalized data loss for SIEFCEI and EFCEI during the three inversion phases, respectively. For SIEFCEI, the data loss decreased continuously in all three phases. In contrast, when the inaccurate source wavelet was used, EFCEI data loss increased throughout the three phases, but the overall change was relatively small. This indicates that the EFCEI model updates failed to reduce the data misfit. Together with the highly regular oscillatory structures observed in Figures 21 and 22, this behavior suggests that a substantial portion of the EFCEI updates is likely associated with the source-wavelet-induced oscillatory components

rather than with effective structural reconstruction. Consequently, the inversion tends to stagnate around these oscillatory updates rather than converging toward lower data misfit.

4. Discussion

Source-independent elastic frequency-controllable envelope inversion is a novel method for constructing low-wavenumber P- and S-wave velocity models, and it offers two key innovations. First, it is a source-independent inversion method that removes the adverse effects of an unknown source wavelet. Unlike widely used source-independent

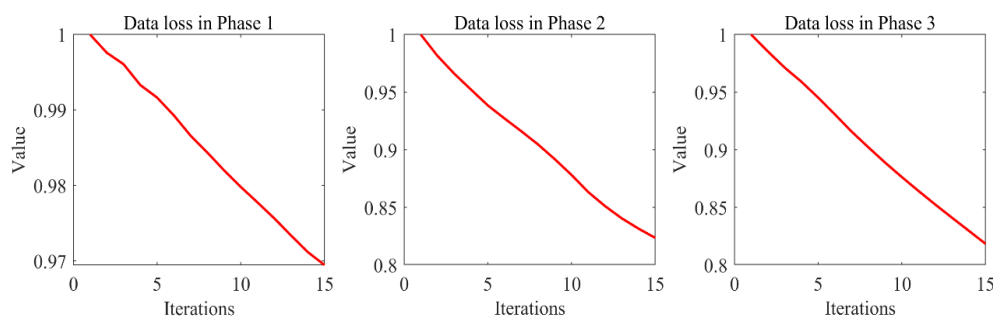


Figure 23. Normalized data loss during the three inversion phases of source-independent elastic frequency-controllable envelope inversion using the Volve dataset

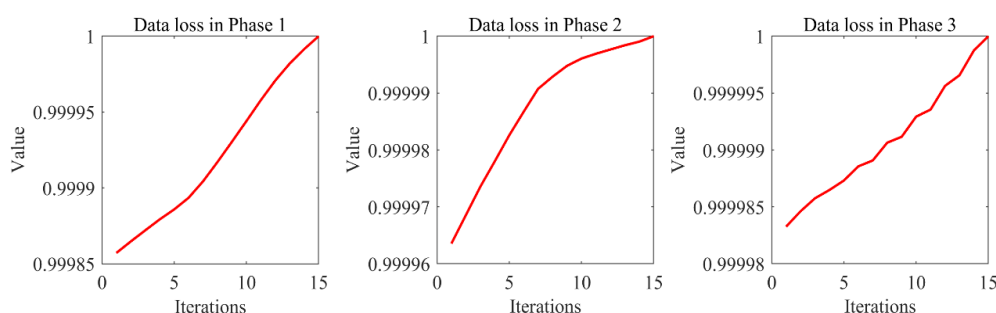


Figure 24. Normalized data loss during the three inversion phases of elastic frequency-controllable envelope inversion using the Volve dataset

strategies that require a suitably chosen reference trace for data matching, SIEFCEI employs an amplitude-semblance objective function to match the data without selecting a reference trace. As discussed above, an inappropriately selected reference trace can increase the risk of inversion failure. By eliminating the need for reference-trace selection, SIEFCEI streamlines the workflow and provides a more convenient and robust inversion approach. Second, the inversion uses elastic frequency-controllable envelope data for data matching. Because of its frequency-control mechanism, this data type provides richer, more focused low-frequency information than conventional elastic envelope data, which is more beneficial for recovering large-scale P- and S-wave velocity models. Additionally, the inversion can be readily performed in a multi-scale manner, progressing from low- to high-frequency bands. This multi-scale envelope strategy stabilizes the inversion and yields more reliable low-wavenumber P- and S-wave velocity models.

To further illustrate these advantages, additional experiments were performed, accounting for source-wavelet phase mismatch, noisy observed data, and comparison with the accumulated energy norm method. Practical limitations and future challenges are also addressed, as described below.

4.1. Inversion under source-wavelet phase mismatch

In the phase-mismatch test, which was carried out on the elastic overthrust model, the observed data were generated using a Ricker wavelet with a dominant frequency of 8 Hz, whereas the synthetic data were generated using the first derivative of a Gaussian function. Details of the wavelets are presented in Figure 25. The inversion was also conducted in three phases, and the frequency spectra of $g(t)$ for the three phases were kept the same as those shown in Figure 4. The inverted P- and S-wave velocity models at different inversion phases are presented in Figures 26–28. The inversion results at different phases indicate that the large-scale structures become progressively clearer as the envelope-data frequency band is gradually broadened. The final inverted models reasonably delineate the main structural features of the overthrust model.

4.2. Noise robustness test

In the noisy-data experiment, the observed data were contaminated with Gaussian noise at a signal-to-noise ratio of 10 dB. Figure 29 compares the observed shot records of different components with their corresponding noisy shot records. The observed data were generated using a Ricker wavelet, whereas the synthetic data were

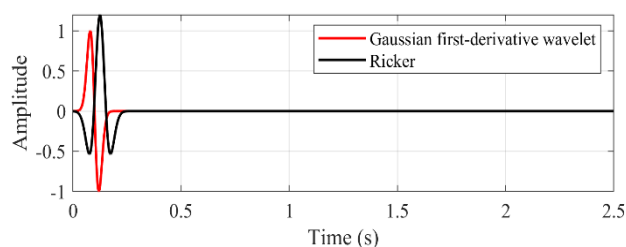


Figure 25. The wavelets used in the phase-mismatch test

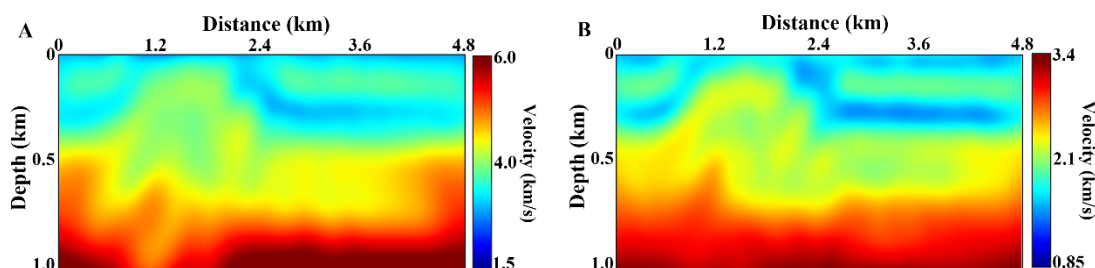


Figure 26. The inverted velocity models of Phase 1 obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. The observed data are generated using a Ricker wavelet, whereas the synthetic data are generated using the first derivative of a Gaussian function. (A) Vp. (B) Vs.

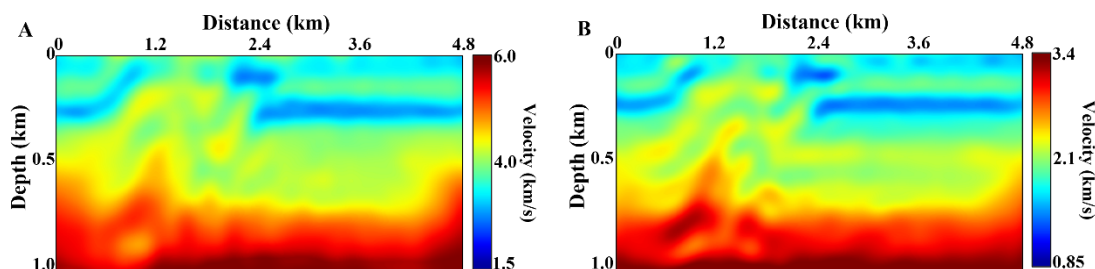


Figure 27. The inverted velocity models of Phase 2 obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. The observed data are generated using a Ricker wavelet, whereas the synthetic data are generated using the first derivative of a Gaussian function. (A) Vp. (B) Vs.

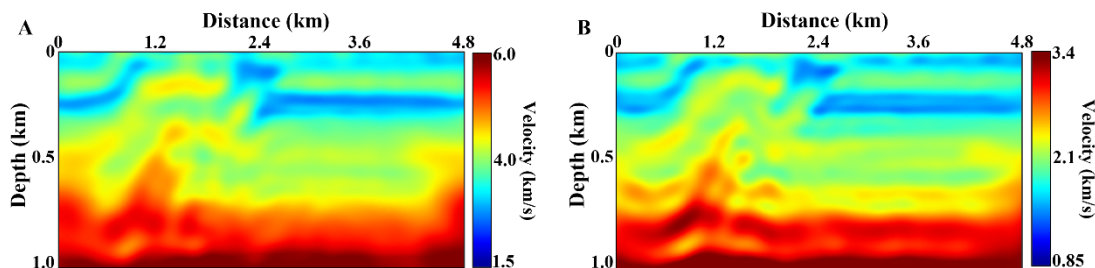


Figure 28. The inverted velocity models of Phase 3 obtained by source-independent elastic frequency-controllable envelope inversion under source wavelet mismatch. The observed data are generated using a Ricker wavelet, whereas the synthetic data are generated using the first derivative of a Gaussian function. (A) Vp. (B) Vs.

generated using the first derivative of a Gaussian function. The inversion was also conducted in three phases, and the frequency spectra of $g(t)$ for the three phases were kept the same as those shown in Figure 4. Figure 30 presents the final inverted velocity models after three inversion phases; the thrust-fault structures and the right-side horizontal layered variations are clearly delineated. The experimental results demonstrate that, even under noisy-data conditions, the proposed SIEFCEI can still effectively recover the low-wavenumber components of the subsurface structures.

4.3. Comparison with the accumulated energy norm method

The accumulated energy norm method has been shown

to be effective for recovering the long-wavelength velocity model in acoustic inversion.⁶ This method does not directly match the original seismic waveforms; instead, it constructs accumulated-energy data from seismic records to extract low-frequency information for macro-velocity model building. In this sense, it shares a similar motivation with SIEFCEI, since both methods aim to enhance the low-wavenumber information in the inversion by extracting useful information from the original seismic data through nonlinear transformations. Therefore, as a comparison, we further tested the dependence of the accumulated energy norm method on source-wavelet accuracy in the elastic inversion case.

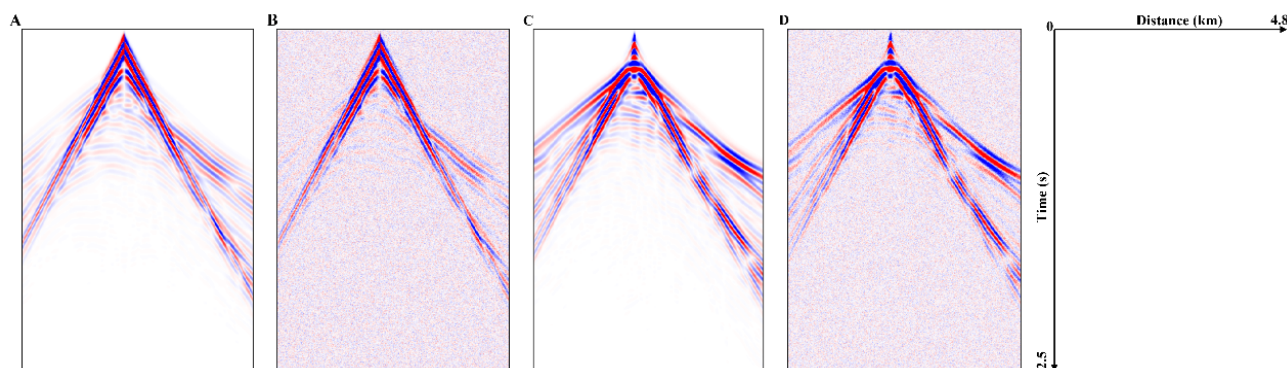


Figure 29. Illustration of observed shot records for different components and their corresponding noisy shot records. (A) x -component shot gather. (B) Noisy x -component shot gather. (C) z -component shot gather. (D) Noisy z -component shot gather.

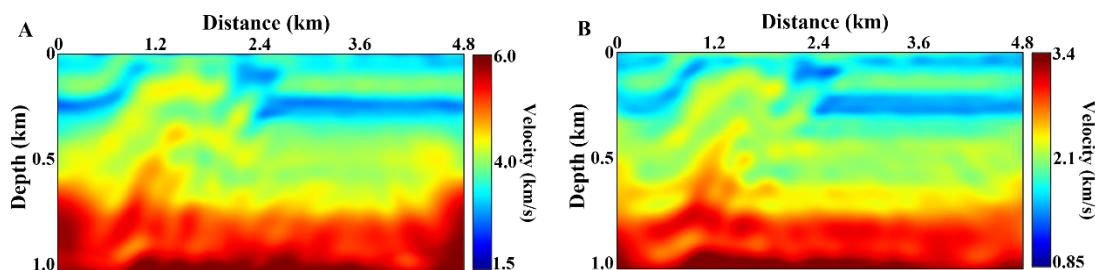


Figure 30. Final inverted velocity models obtained by source-independent elastic frequency-controllable envelope inversion under noisy-data conditions and source wavelet mismatch. (A) V_p . (B) V_s .

We first evaluated the performance of the accumulated energy norm method in elastic inversion for building low-wavenumber P- and S-wave velocity models under a consistent source-wavelet condition. In this test, the non-logarithmic form of the objective function was adopted, and an 8 Hz Ricker wavelet was used for both the observed and synthetic data. The overthrust model was used for the inversion experiments, which were conducted within three

different frequency bands, i.e., 0–10 Hz, 0–12 Hz, and 0–14 Hz. Figure 31 shows three loss curves of the accumulated energy norm method corresponding to three frequency bands. The loss curves all decreased and gradually leveled off as iterations proceeded. Furthermore, the convergence rate tends to increase as the inversion frequency band expands. When a wider frequency band was used, more wavefield information contributed to the accumulated

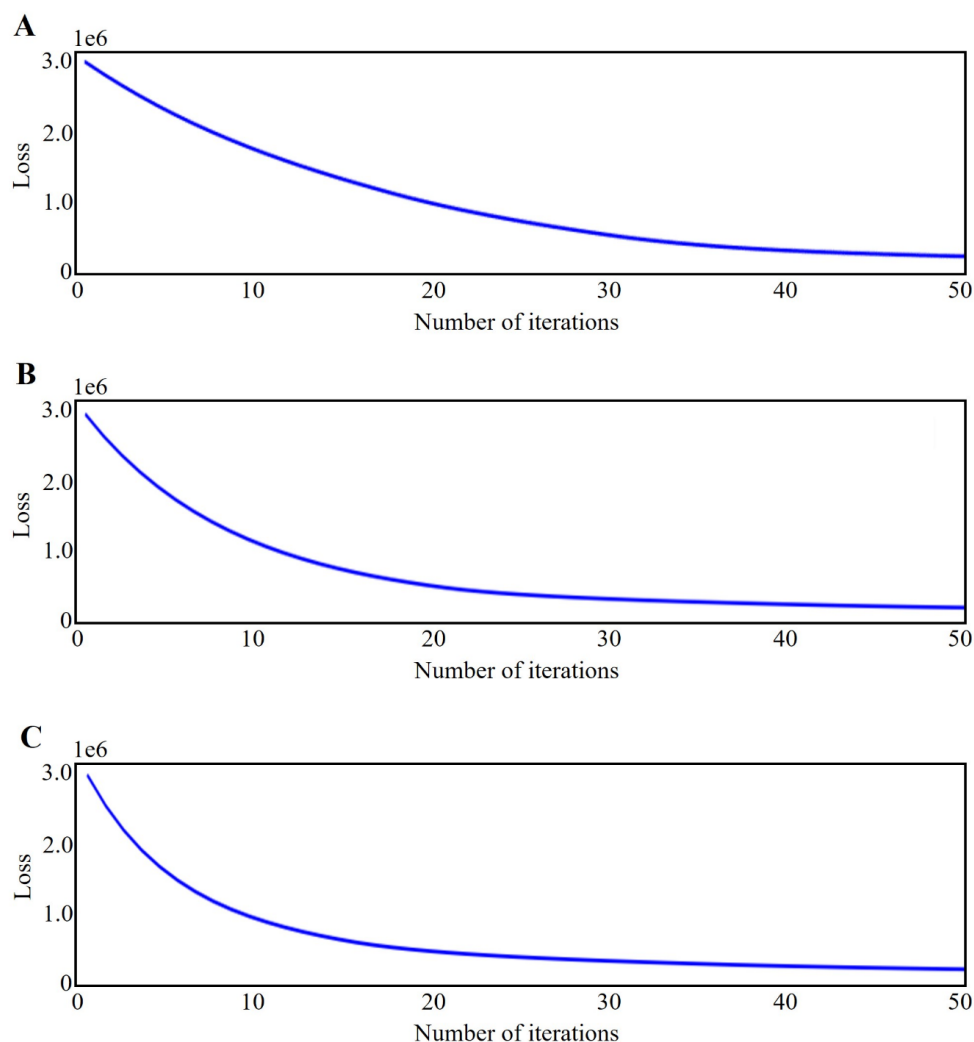


Figure 31. Misfit loss curve for the accumulated energy norm method within different frequency bands under a consistent source-wavelet condition. (A) 0–10 Hz. (B) 0–12 Hz. (C) 0–14 Hz.

energy objective function, which may impose stronger constraints on the model update and lead to a faster reduction in misfit. Although the loss curves exhibited different convergence rates, they all showed a similar tendency toward stabilization in later iterations, implying that the inversions conducted within different frequency bands essentially reached convergence. In addition, the accumulated energy of the elastic seismic records from the 0–14 Hz inversion experiment was compared in Figure 32. As shown, the accumulated energy of the observed and synthetic data remained highly similar, with only minor differences in some weak-energy regions. This close match in accumulated energy further suggests, from another perspective, that the inversion has essentially converged.

Figures 33–35 show the P- and S-wave velocity models obtained from inversions performed within the 0–10 Hz, 0–12 Hz, and 0–14 Hz frequency bands, respectively. The results of the three experiments consistently show that the accumulated energy norm method can significantly improve the long-wavelength velocity structures of the initial model through iterative updating, and the recovered large-scale structures are generally consistent with the true model. These results indicate that, when an accurate source wavelet is available, this method is applicable to long-wavelength velocity model building in elastic media.

Next, we investigated the performance of the accumulated-energy norm method under wavelet mismatch. The observed data were generated using a

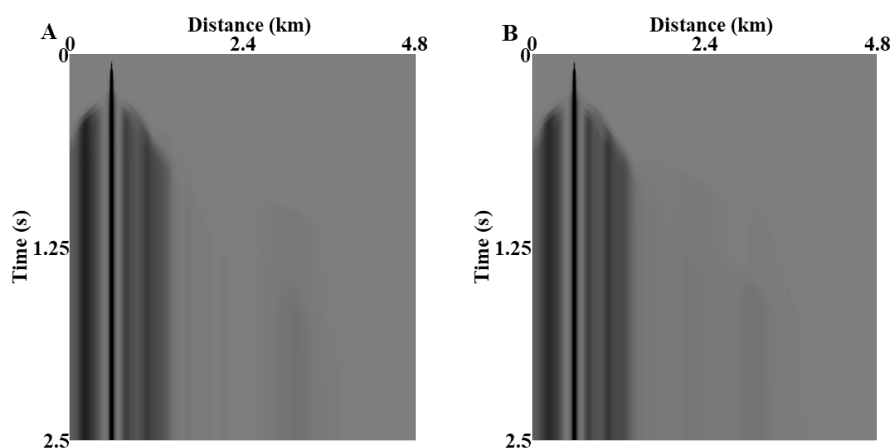


Figure 32. Comparison of accumulated energy of elastic seismic records. (A) Accumulated energy of the observed data. (B) Accumulated energy of the synthetic data generated by forward modeling using the final inverted model within 0–14 Hz.

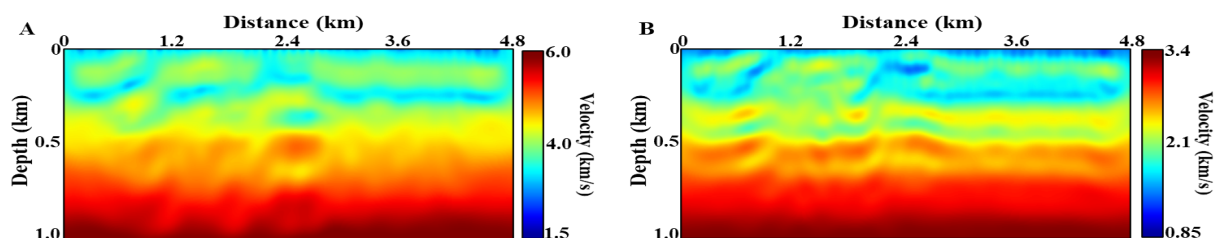


Figure 33. Elastic inversion results obtained using the accumulated energy norm within the 0–10 Hz frequency band under a consistent source-wavelet condition. (A) V_p . (B) V_s .

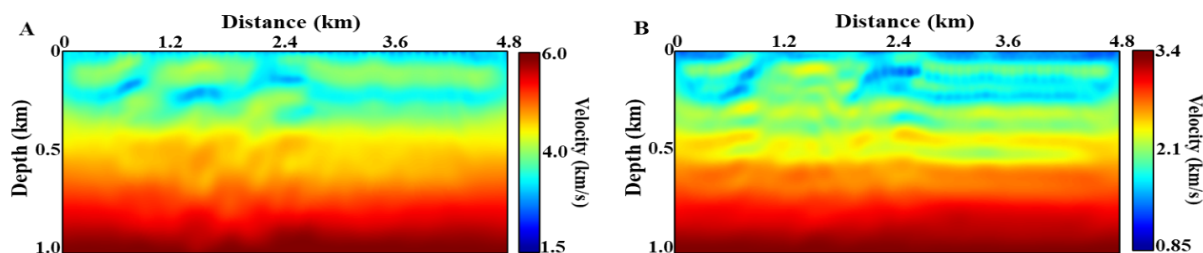


Figure 34. Elastic inversion results obtained using the accumulated energy norm within the 0–12 Hz frequency band under a consistent source-wavelet condition. (A) V_p . (B) V_s .

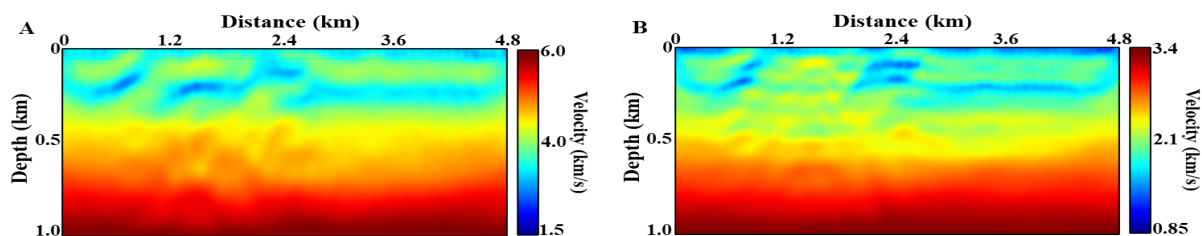


Figure 35. Elastic inversion results obtained using the accumulated energy norm within the 0–14 Hz frequency band under a consistent source-wavelet condition. (A) V_p . (B) V_s .

Ricker wavelet, whereas the synthetic data were generated using the first derivative of a Gaussian function, which is the same as the source-wavelet mismatch shown in Figure 25.

We performed experiments within two frequency bands, 0–10 Hz and 0–14 Hz, respectively. Figures 36 and 37 show the velocity models obtained from inversions performed within the 0–10 Hz and 0–14 Hz frequency bands, respectively. Both results corresponded to the models obtained after 25 iterations. The inverted models did not show effective updates to meaningful structural information. To further illustrate this issue, we present the first-iteration P- and S-wave velocity gradients for the two experiments in Figures 38 and 39. These gradient results clearly show that no geologically reasonable structural information is introduced into the model updates; instead, the gradients are mainly dominated by horizontal stripe-like artifacts. These observations indicate that, in the elastic inversion case, the accumulated energy norm method is sensitive to source-wavelet accuracy, and source-wavelet mismatch can significantly limit its capability to recover meaningful low-wavenumber P- and S-wave velocity structures. In contrast, SIEFCEI can still recover reliable low-wavenumber P- and S-wave velocity models under the same source-wavelet mismatch condition.

Because the original implementation code is not publicly available, the reproduction results reported here may not reflect the optimal performance achievable with the accumulated energy norm method. Improved results may be obtained through further parameter tuning, alternative inversion strategies, or implementations in different computational domains. Nevertheless, the comparison between the consistent and inconsistent source-wavelet cases indicates that the accumulated energy norm method is relatively sensitive to source-wavelet accuracy in elastic inversion.

After presenting the above supplementary experiments, we further discuss the parameter selection in SIEFCEI and its possible future extensions. In the SIEFCEI experiments presented above, the filter parameters used to construct the envelope data at different inversion phases were selected empirically. A relatively low cutoff frequency is usually adopted in the first phase to reduce the risk of convergence to an undesired local minimum. In the subsequent phases, the cutoff frequency is typically increased by about 2–3 Hz at each phase. This is a relatively conservative strategy, which helps maintain inversion stability while gradually introducing higher-wavenumber information. Although envelope data can alleviate cycle skipping to some extent, we do not recommend excessively large changes in the

cutoff frequency between adjacent phases, because such a setting may reduce inversion stability. Future work should continue to investigate how to select envelope-related parameters more systematically and in a theoretically grounded manner during inversion.

4.4. Real-world limitations and future challenges

This section discusses the limitations of the current SIEFCEI framework in the presence of anisotropy, rugged topography, and multiparameter crosstalk between density and attenuation, along with possible directions for future research.

4.4.1. Anisotropy

Source-independent EFCEI is derived under the assumption of isotropic elastic wave propagation and therefore does not account for anisotropy. Its source-independent amplitude-semblance objective function is designed primarily to reduce the inversion's dependence on an accurately estimated source wavelet. However, source independence does not make the method insensitive to errors in the assumed wave-propagation physics. In strongly anisotropic formations, the isotropic assumption may introduce mismatches in traveltime, phase, and amplitude. When SIEFCEI is applied directly to data acquired in anisotropic media, these discrepancies may map into the V_p and V_s updates because the inversion does not include the relevant anisotropic parameters. This may lead to inaccurate velocity estimates, misplaced interfaces, and spurious structural features. Although the amplitude-semblance objective function can reduce the influence of source-wavelet errors, it cannot remove residuals caused by anisotropic propagation. Extending SIEFCEI to anisotropic elastic media is therefore an important direction for future work.

4.4.2. Rugged topography

The current numerical implementation and experimental setup assumed a flat acquisition surface, and rugged topography was not considered in this study. Although the amplitude-semblance objective function reduces the influence of source-wavelet errors, it cannot compensate for modeling errors caused by an inaccurate representation of the acquisition surface. Therefore, the application of SIEFCEI to rugged-topography conditions requires forward and adjoint modeling that accurately represents the actual surface topography. Investigating SIEFCEI under rugged-topography conditions and systematically assessing the stability of the amplitude-semblance objective function in such settings will be an important direction for future research.

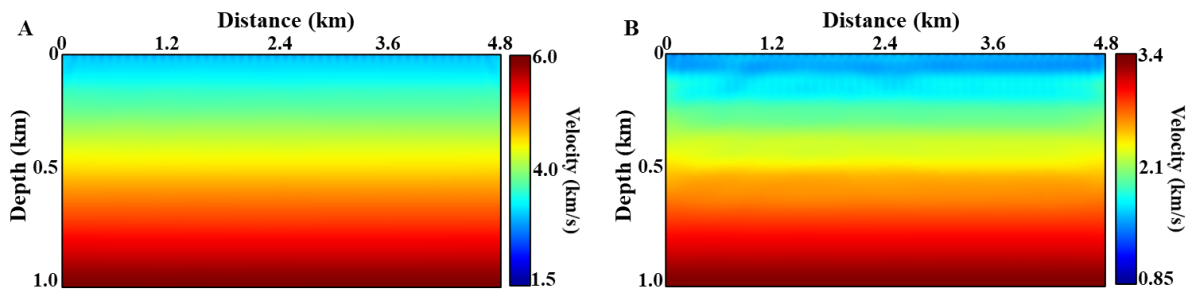


Figure 36. The elastic inversion result obtained from the accumulated energy norm in 0–10 Hz under source wavelet mismatch. (A) Vp. (B) Vs.

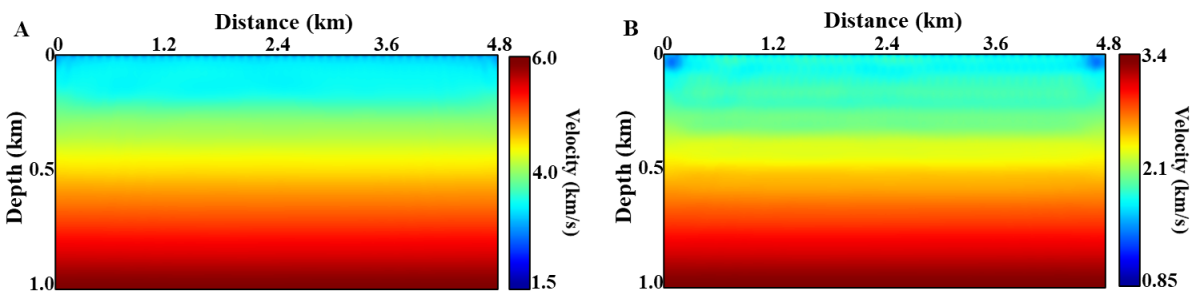


Figure 37. The elastic inversion result obtained from the accumulated energy norm in 0–14 Hz under source-wavelet mismatch. (A) Vp. (B) Vs.

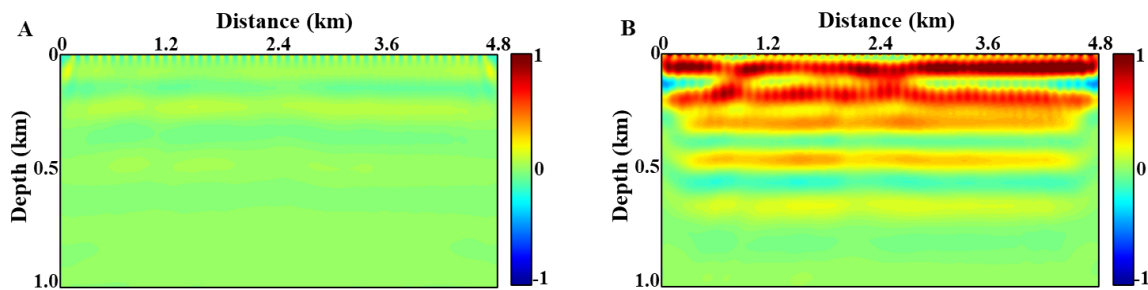


Figure 38. First-iteration gradients obtained using the accumulated energy norm within the 0–10 Hz frequency band under a source-wavelet mismatch condition. (A) P-wave velocity gradient. (B) S-wave velocity gradient.

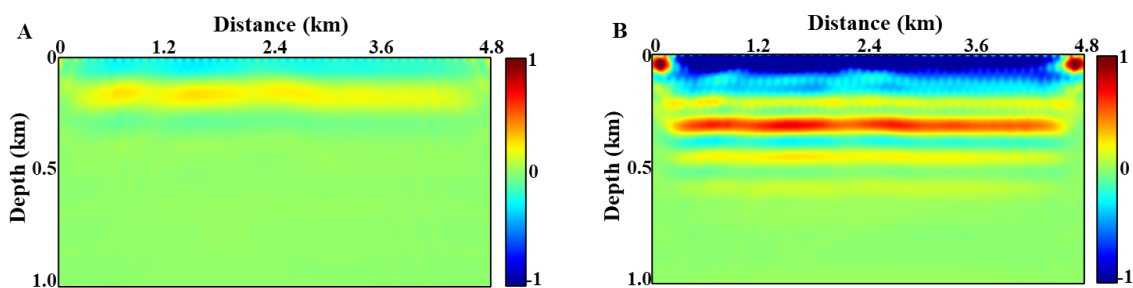


Figure 39. First-iteration gradients obtained using the accumulated energy norm within the 0–14 Hz frequency band under a source-wavelet mismatch condition. (A) P-wave velocity gradient. (B) S-wave velocity gradient.

4.4.3. Multiparameter crosstalk

Multiparameter crosstalk remains a potential limitation of SIEFCEI. The frequency-controllable envelope operator does not provide an explicit mechanism for decoupling the V_p and V_s updates from density-related or attenuation-induced effects. Because SIEFCEI retains the elastic forward operator, model parameterization, and parameter-gradient kernels of conventional EFWI, it inherits the same fundamental parameter-coupling mechanisms. The modified adjoint source may change the strength and spatial distribution of the crosstalk, but it cannot eliminate it. Regarding parameter trade-offs between velocity updates (V_p , V_s) and density/attenuation, we note that seismic data are generally less sensitive to long-wavelength density variations than to velocity variations. Therefore, we gave priority to the reconstruction of V_p and V_s during the initial model-building stage. If long-wavelength density reconstruction is considered in future work, seismic data alone may not provide sufficient information. Complementary information, such as gravity data, density logs, geological facies, or rock-physics relationships, could be incorporated through joint inversion. Such information may help to construct the background density. As for attenuation, it was not included in the current formulation because wave propagation was modeled using the elastic rather than the viscoelastic wave equation. When elastic inversion is applied to attenuative data, attenuation effects may be partially mapped into the velocity updates. Addressing this issue would require a viscoelastic extension of SIEFCEI and a separate investigation of the trade-offs between velocity and attenuation parameters. These issues will require careful consideration in future work and are important for the reliable application of SIEFCEI to practical seismic exploration.

Future work should also extend SIEFCEI to three-dimensional velocity-model building. Moreover, the model parameters were currently updated using a conventional gradient-descent scheme. It is therefore promising to integrate the proposed framework with neural-network parameterizations,³²⁻³⁴ such as implicit neural representations, to enhance optimization efficiency and mitigate parameter crosstalk in multiparameter inversion.

5. Conclusion

In this study, we propose SIEFCEI, a source-independent initial-model building method for EFWI. By combining the elastic frequency-controllable envelope operator with an amplitude-semblance-based objective function, SIEFCEI reconstructs low-wavenumber P- and S-wave velocity models without requiring an accurate source wavelet. Unlike conventional source-independent strategies, it eliminates the need for reference-trace selection, thereby

improving practicality and robustness. We also provide a theoretical derivation demonstrating the source-wavelet independence of the proposed misfit function. Numerical experiments on both synthetic and field data indicate that SIEFCEI produces reliable large-scale background velocity models when starting from poor initial models and under significant source-wavelet inconsistency, where conventional EFCEI may fail and become trapped in local minima. The noisy-data experiment further demonstrates that SIEFCEI can still recover reliable low-wavenumber structural information in the presence of noise contamination. Further tests show that, when the recovered models are used as the initial models for source-independent EFWI, the inversion yields more accurate results and a clearer reconstruction of key subsurface structures. These results confirm that SIEFCEI provides reliable initial models for elastic waveform inversion under realistic source-uncertain conditions.

Acknowledgments

None.

Funding

The work was supported by the National Natural Science Foundation of China (Grant No. 42422404).

Conflict of interest

The authors declare no competing interests.

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Conceptualization: Zhaoqi Gao

Data curation: Qiu Du

Funding acquisition: Zhaoqi Gao

Investigation: Qiu Du

Methodology: All authors

Resources: Zhaoqi Gao

Supervision: Zhaoqi Gao

Writing—original draft: Qiu Du

Writing—review & editing: All authors

Availability of data

Data are available from the corresponding author upon reasonable request.

References

1. Tarantola A. Inversion of seismic reflection data in the acoustic approximation. *Geophysics*. 1984;49(8):1259-1266. doi: 10.1190/1.1441754
2. Virieux J, Operto S. An overview of full-waveform inversion in exploration geophysics. *Geophysics*. 2009;74(6):WCC1-WCC26.

- doi: 10.1190/1.3238367
3. Hu Y, Huang X, Fu L, *et al.* Jiyu tan xing bo dong fang cheng de ju bu chi du zou shi fan yan [Elastic wave-equation based local-scale traveltime inversion]. *Chinese Journal of Geophysics*. 2024;67(10):3874–3890. [In Chinese]
doi: 10.6038/cjg2023R0466
 4. Liang Z, Zheng Y, He C, Wu G, Zhang X, Gai Z. Elastic reflection traveltime inversion with pure-wave propagators. *Geophysics*. 2021;86(4):R639–R658.
doi: 10.1190/geo2020-0733.1
 5. Wang G, Wang S, Song J, Dong C, Zhang M. Elastic reflection traveltime inversion with decoupled wave equation. *Geophysics*. 2018;83(5):R463–R474.
doi: 10.1190/geo2017-0631.1
 6. Shin C, Ha W. Accumulated energy norm for full waveform inversion of marine data. *Journal of Applied Geophysics*. 2017;147:91–101.
doi: 10.1016/j.jappgeo.2017.10.005
 7. Hu Y, Wu RS, Han LG, Zhang P. Joint Multiscale Direct Envelope Inversion of Phase and Amplitude in the Time–Frequency Domain. *IEEE Trans Geosci Remote Sensing*. 2019;57(7):5108–5120.
doi: 10.1109/tgrs.2019.2896936
 8. Hu Y, Wu RS, Huang X, Long Y, Xu Y, Han LG. Phase-amplitude-based polarized direct envelope inversion in the time-frequency domain. *Geophysics*. 2022;87(3):R245–R260.
doi: 10.1190/geo2020-0956.1
 9. Luo J, Wu RS. Initial model construction for elastic full waveform inversion using envelope inversion method. In: *International Meeting for Applied Geoscience & Energy*. Vol 1. Society of Exploration Geophysicists; 2015:1420–1424.
doi: 10.1190/segam2015-5820719.1
 10. Hu Y, Wu RS, Luo J, Chen G. A Review of Envelope Inversion for Large-Scale Background Model Building. *Surv Geophys*. 2025;46(6):1217–1246.
doi: 10.1007/s10712-025-09913-8
 11. Guo Q, Alkhalifah T. Elastic reflection-based waveform inversion with a nonlinear approach. *Geophysics*. 2017;82(6):R309–R321.
doi: 10.1190/geo2016-0407.1
 12. Sun H, Demanet L. Deep Learning for Low-Frequency Extrapolation of Multicomponent Data in Elastic FWI. *IEEE Trans Geosci Remote Sensing*. 2022;60:1–11.
doi: 10.1109/tgrs.2021.3135790
 13. Gao Z, Pan Z, Gao J, Wu RS. Frequency Controllable Envelope Operator and Its Application in Multiscale Full-Waveform Inversion. *IEEE Trans Geosci Remote Sensing*. 2019;57(2):683–699.
doi: 10.1109/tgrs.2018.2859219
 14. Du Q, Gao Z, Gao J. Elastic frequency controllable envelope inversion. *J Geophys Eng*. 2024;22(1):265–282.
doi: 10.1093/jge/gxae122
 15. Du Q, Gao Z, Zhang W, Gao J. Structurally Consistent Elastic Frequency- Controllable Envelope Inversion for P- and S-Wave Velocity Model Building. *IEEE Trans Geosci Remote Sensing*. 2025;63:1–21.
doi: 10.1109/tgrs.2025.3631856
 16. Skopintseva LV, Maaø FA, Pedersen Ø. Importance of the Source Estimation in FWI - Sensitivity and Examples. In: *Proceedings of the 78th EAGE Conference and Exhibition 2016*. May 30, 2016–June 2, 2016; Vienna, Austria. European Association of Geoscientists & Engineers; 2016:1–5.
doi: 10.3997/2214-4609.201601015
 17. Fang Z, Wang H, Li M, Gu X, Li P. A Source-Independent TV-Regularized Full Waveform Inversion Method for Peripheral Imaging Around a Borehole. *IEEE Trans Geosci Remote Sensing*. 2024;62:1–9.
doi: 10.1109/tgrs.2024.3401034
 18. Choi Y, Alkhalifah T, DeVault B. Application of the unwrapped phase inversion to land data without source estimation. In: *International Meeting for Applied Geoscience & Energy*. Vol 1. Society of Exploration Geophysicists; 2015:1225–1229.
doi: 10.1190/segam2015-5873514.1
 19. Choi Y, Shin C, Min DJ, Ha T. Efficient calculation of the steepest descent direction for source-independent seismic waveform inversion: An amplitude approach. *J Comput Phys*. 2005;208(2):455–468.
doi: 10.1016/j.jcp.2004.09.019
 20. Choi Y, Min DJ. Source-independent elastic waveform inversion using a logarithmic wavefield. *J Appl Geophys*. 2012;76:13–22.
doi: 10.1016/j.jappgeo.2011.10.013
 21. Seo T, Shin C, Min D. Efficient calculation of steepest descent direction of source signature independent waveform inversion of logarithmic wavefield. In: *SEG Technical Program Expanded Abstracts 2004*. Society of Exploration Geophysicists; 2004;23(1):1838–1841.
doi: 10.1190/1.1845176
 22. Cheong S, Pyun S, Shin C. Two efficient steepest-descent algorithms for source signature-free waveform inversion. *J Seism Explor*. 2006;14(4):335–348.
 23. Choi Y, Alkhalifah T. Source-independent time-domain waveform inversion using convolved wavefields: Application to the encoded multisource waveform inversion. *Geophysics*. 2011;76(5):R125–R134.

- doi: 10.1190/geo2010-0210.1
24. Zhang Q, Zhou H, Li Q, Chen H, Wang J. Robust source-independent elastic full-waveform inversion in the time domain. *Geophysics*. 2016;81(2):R29-R44.
doi: 10.1190/geo2015-0073.1
 25. Ao R, Dong L, Chi B. Buyi lai zi boji yu bao luo de FWI chu shi mo xing jian li fang fa yan jiu [Source-independent envelope-based FWI to build an initial model]. *Chin J Geophys*. 2015;58(6):1998-2010. [In Chinese]
doi: 10.6038/cjg20150615
 26. Zhang P, Wu RS, Han L. Source-independent seismic envelope inversion based on the direct envelope Fréchet derivative. *Geophysics*. 2018;83(6):R581-R595.
doi: 10.1190/geo2017-0360.1
 27. Chi B, Huang L. Source-Independent Amplitude-Semblance Full-Waveform Inversion Using a Hybrid Time- and Frequency-Domain Approach. *CiCP*. 2020;28(1):328-341.
doi: 10.4208/cicp.oa-2018-0199
 28. Wu RS, Luo J, Wu B. Seismic envelope inversion and modulation signal model. *Geophysics*. 2014;79(3):WA13-WA24.
doi: 10.1190/geo2013-0294.1
 29. Bedrosian E. A product theorem for Hilbert transforms. *Proc IEEE*. 1963;51(5):868-869.
doi: 10.1109/proc.1963.2308
 30. Brown JL. A Hilbert transform product theorem. *Proc IEEE*. 1986;74(3):520-521.
doi: 10.1109/proc.1986.13495
 31. Zhang P, Han L, Zhang F, Feng Q, Chen X. Wavefield Decomposition-Based Direct Envelope Inversion and Structure-Guided Perturbation Decomposition for Salt Building. *Minerals*. 2021;11(9):919.
doi: 10.3390/min11090919
 32. Sun J, Innanen K, Zhang T, Trad D. Implicit Seismic Full Waveform Inversion With Deep Neural Representation. *JGR Solid Earth*. 2023;128(3).
doi: 10.1029/2022jb025964
 33. Zhao P, Fang J, Jie C, Zhang J, Wang E, Zhang S. Multiscale Deep Learning Reparameterized Full Waveform Inversion With the Adjoint Method. *IEEE Trans Geosci Remote Sensing*. 2025;63:1-12.
doi: 10.1109/tgrs.2025.3553053
 34. Zhu W, Xu K, Darve E, Biondi B, Beroza GC. Integrating deep neural networks with full-waveform inversion: Reparameterization, regularization, and uncertainty quantification. *Geophysics*. 2021;87(1):R93-R109.
doi: 10.1190/geo2020-0933.1