

ARTICLE

Least-squares one-way wave-equation migration with matrix-decomposition propagation and sparse denoising regularization

Guangyin Wang^{1,2}, Pengyuan Sun^{1,2}, Xinyu Zhang^{3*}, Bin Li^{1,2}, Xueying Hu^{1,2}, Heng Zhang^{1,2}, and Jiachun You³ 

¹BGP Inc., China National Petroleum Corporation, Zhuozhou, Hebei, China

²National Engineering Research Center of Oil and Gas Exploration Computer Software, Zhuozhou, Hebei, China

³College of Geophysics, Chengdu University of Technology, Chengdu, Sichuan, China

Abstract

Conventional one-way wave-equation migration methods often suffer from phase errors, amplitude distortions, and low-amplitude artifacts in the presence of strong lateral velocity variations, complex structures, and noisy data. To address these problems, we propose a sparse-denoising-constrained least-squares one-way wave-equation migration method. A one-way propagation operator is first constructed in the space–frequency domain using matrix eigenvalue decomposition to improve the stability and phase accuracy of wavefield depth extrapolation in laterally heterogeneous media. Recursive primary wavefield modeling is then used to establish the forward relationship between the reflectivity model and surface primary reflection data, and the data-misfit gradient is computed through adjoint propagation. Furthermore, a curvelet-transform denoiser regularization term is introduced, so that the data-residual gradient and denoising-residual gradient jointly guide the reflectivity update. Numerical experiments on the Lens, Marmousi, and Overthrust models show that the proposed method produces clearer reflection interfaces, more continuous events, and more stable residual convergence than least-squares migration based on the generalized-screen propagation operator. Noisy-data tests further demonstrate that the proposed sparse denoising constraint improves peak signal-to-noise ratio (SNR) and structural similarity index under 5 dB, 1 dB, and –5 dB noise conditions, especially preserving structural similarity under strong noise. These results demonstrate the improved stability, resolution, and structural fidelity of the proposed method for complex synthetic models and low-SNR synthetic data.

*Corresponding author:

Xinyu Zhang
(202205060204@stu.cdut.edu.cn)

Citation: Wang G, Sun P, Zhang X, *et al.* Least-squares one-way wave-equation migration with matrix-decomposition propagation and sparse denoising regularization. *J Seismic Explor.*
doi: 10.36922/JSE026280132

Received: July 7, 2026

Revised: August 14, 2026

Accepted: August 26, 2026

Published online: September 14, 2026

Copyright: © 2026 Author(s). This is an Open-Access article distributed under the terms of the Creative Commons Attribution License, permitting distribution, and reproduction in any medium, provided the original work is properly cited.

Publisher's Note: AccScience Publishing remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Keywords: One-way wave-equation migration; Least-squares migration; Matrix eigenvalue decomposition; Curvelet-transform denoiser; Sparse denoising constraint

1. Introduction

High-resolution seismic imaging is essential for subsurface structural interpretation, reservoir characterization, and hydrocarbon exploration. Conventional seismic migration methods, including Kirchhoff migration,¹ one-way wave-equation

migration, reverse time migration,² and two-way wave-equation depth-extrapolation methods,³⁻⁷ can generally be interpreted as adjoint operations of linearized seismic modeling operators. Although widely used in seismic data processing, these methods do not provide rigorous inversion solutions of the subsurface reflectivity model. Under non-ideal acquisition and geological conditions, such as irregular acquisition geometry, limited aperture, complex overburden, uneven illumination, and band-limited data, conventional migration images often suffer from amplitude distortion, migration artifacts, reduced spatial resolution, and blurred reflectors. Least-squares migration addresses these limitations by reformulating seismic imaging as a linearized inverse problem, in which the reflectivity model is iteratively updated by minimizing the misfit between observed and simulated data, thereby improving resolution, amplitude fidelity, and illumination compensation.

The theoretical foundation of least-squares seismic imaging traces back to linearized seismic inversion and generalized least-squares inversion. Tarantola established a linearized inversion framework for seismic reflection data under the acoustic approximation, providing an important basis for least-squares migration.^{8,9} Nemeth *et al.* introduced least-squares migration for incomplete reflection data and showed that iterative inversion can reduce imaging errors caused by irregular acquisition and improve migration resolution.¹⁰ Subsequent studies extended this framework to migration deconvolution,^{11,12} illumination compensation,^{13,14} true-amplitude migration,¹⁵ and image-domain correction based on Hessian approximations.¹⁶⁻¹⁹ These studies indicate that least-squares migration relies on a linearized modeling operator and its adjoint operator to compensate, through iterative updating, for the non-unitary nature of the imaging operator, finite-aperture effects, and nonuniform illumination.

With the development of wave-equation imaging, least-squares reverse time migration (LSRTM) has become a major branch of least-squares migration. Based on the two-way wave equation, reverse time migration can accurately describe wave propagation in complex media and is well suited for steeply dipping structures, turning waves, multipathing, and strong lateral velocity variations.^{20,21} Within the Born linearized modeling framework, LSRTM updates the reflectivity model through repeated forward modeling, residual back-propagation, and gradient calculation, thereby improving imaging quality in complex geological settings.²² Considerable efforts have been made to improve its efficiency and stability, including multi-source encoding,^{23,24} plane-wave encoding,²⁵ illumination preconditioning,²⁶ deblurring filtering,²⁷ viscoacoustic

compensation,²⁸⁻³⁰ elastic extensions,^{31,32} anisotropic imaging,³³ and deep-learning-assisted updating.^{34,35} Nevertheless, LSRTM requires repeated solutions of the two-way wave equation and usually involves large-scale wavefield storage or reconstruction, leading to high computational and memory costs. Its imaging quality is also sensitive to the background velocity model, source wavelet, and the accuracy of the linearized modeling operator, which limits its routine application to large-scale three-dimensional industrial data.

One-way wave-equation migration employs recursive depth extrapolation to propagate wavefields and provides a natural framework for iterative least-squares seismic imaging. Since the development of phase-shift migration,³⁶ Fourier migration,³⁷ phase-shift-plus-interpolation migration,³⁸ split-step Fourier migration,³⁹ and Fourier finite-difference migration,⁴⁰ one-way wavefield extrapolation has become an important tool in prestack depth migration. To improve its adaptability to strong lateral velocity variations and wide-angle propagation, generalized-screen,⁴¹ complex-screen,⁴² high-order-screen,⁴³ and true-amplitude one-way wave-equation methods^{44,45} have been developed. These methods separately extrapolate downgoing and upgoing wavefields along depth and construct subsurface images using appropriate imaging conditions. By avoiding full two-way time stepping, conventional one-way propagation methods can reduce computational and wavefield-storage requirements associated with two-way wave-equation imaging. However, their actual computational efficiency depends strongly on the specific propagation operator employed.

Although LSRTM has been extensively investigated, least-squares imaging based on one-way wavefield extrapolation remains relatively limited. From an operator perspective, one-way wave-equation migration can also be formulated as a linearized inverse problem: the one-way propagation operator serves as a demigration or linearized modeling operator that maps the reflectivity model to seismic reflection data, whereas its adjoint corresponds to the migration imaging process. Following this idea, Köhl and Sacchi⁴⁶ incorporated extended double-square-root one-way wave-equation migration and modeling operators into a constrained least-squares inversion framework for AVP/AVA inversion. Zhou *et al.*⁴⁷ constructed linearized modeling and adjoint migration operators using a generalized-screen one-way propagator and introduced seismic illumination as a preconditioning constraint. Stanton and Sacchi⁴⁸ extended this framework to elastic media by constructing multiparameter forward and adjoint operators through elastic wavefield decomposition,

extrapolation, and recombination, while reducing artifacts caused by noise, undersampling, and parameter crosstalk using regularization. More recently, Abolhassani and Verschuur⁴⁹ developed depth-dependent Hessian-approximation preconditioners in a space-frequency-domain one-way wave-equation framework to reduce computational cost and accelerate convergence, while Liu *et al.*⁵⁰ proposed an efficient point-spread-function inversion strategy for image-domain least-squares one-way wave-equation migration using a deep-learning optimizer and a recalibrated loss function. These studies demonstrate the potential of least-squares one-way wave-equation migration to combine one-way extrapolation with the resolution enhancement, amplitude fidelity, and illumination compensation of least-squares inversion.

However, existing least-squares one-way wave-equation migration methods still face several challenges. The accuracy of the one-way propagation operator directly affects the consistency between the linearized modeling operator and the adjoint imaging operator, thereby influencing inversion convergence and imaging fidelity. Conventional generalized-screen, split-step Fourier, and related approximate operators are efficient, but they may introduce phase errors and amplitude distortions with strong lateral velocity variations, complex structural undulations, and steeply dipping reflectors. Compared with conventional approximate one-way propagators, many of which rely on reference-medium approximations or truncated expansions, the matrix-decomposition approach directly constructs the discrete squared depth-wavenumber operator in the space-frequency domain, thereby incorporating lateral velocity variations and wavefield coupling into a unified formulation. This approach can improve phase accuracy and adaptability to laterally heterogeneous media and provides a stable propagation basis for subsequent least-squares inversion. To address these issues, we propose a curvelet-transform (CT)-denoiser-based sparse-denoising-constrained least-squares one-way wave-equation migration method. The proposed method constructs a space-frequency-domain one-way propagation operator using matrix eigenvalue decomposition, establishes a recursive primary-wavefield modeling framework for reflectivity inversion, and incorporates a curvelet-transform denoiser regularization term into the gradient-based updating scheme. However, using matrix eigenvalue decomposition adds computational cost during operator construction, which may become significant for large-scale problems, particularly in three-dimensional applications. Numerical experiments on the Lens, Marmousi, and Overthrust models are conducted to evaluate the imaging performance of the proposed method under local velocity anomalies, complex lateral

heterogeneity, and strongly deformed structures.

2. Theory

2.1. One-way wavefield propagation operator based on matrix eigenvalue decomposition

Under the acoustic-medium assumption and neglecting the source term, the two-dimensional pressure wavefield satisfies the homogeneous acoustic wave equation, as shown in **Equation 1**:

$$\frac{\partial^2 p(x, z, t)}{\partial x^2} + \frac{\partial^2 p(x, z, t)}{\partial z^2} - \frac{1}{v^2} \cdot \frac{\partial^2 p(x, z, t)}{\partial t^2} = 0 \quad (1)$$

where $p(x, z, t)$ denotes the pressure wavefield at spatial position (x, z) and time t , v is the medium velocity, and x and z represent the horizontal and depth coordinates, respectively. **Equation 1** describes acoustic wave propagation in the medium and provides the theoretical basis for constructing the one-way wavefield propagation operator.

By transforming **Equation 1** into the space-frequency domain, the frequency-domain acoustic wave equation can be written as **Equation 2**:

$$\frac{\partial^2 p(x, z, \omega)}{\partial x^2} + \frac{\partial^2 p(x, z, \omega)}{\partial z^2} + \frac{\omega^2}{v^2} p(x, z, \omega) = 0 \quad (2)$$

where ω denotes the angular frequency, and $p(x, z, \omega)$ represents the monochromatic pressure wavefield at frequency ω .

In the one-way wavefield depth-extrapolation framework, the depth direction z is commonly treated as the wavefield marching direction, while the lateral spatial variation is incorporated into the depth-wavenumber operator. From **Equation 2**, the squared depth-wavenumber operator can be expressed as **Equation 3**:

$$k_z^2 = \frac{\omega^2}{v^2} + \frac{\partial^2}{\partial x^2} = \text{diag} \left[\frac{\omega^2}{v^2} \right] + \frac{1}{\Delta x^2} \begin{bmatrix} -2 & 1 & & \\ 1 & -2 & \ddots & \\ & \ddots & \ddots & 1 \\ & & 1 & -2 \end{bmatrix} \quad (3)$$

where k_z^2 denotes the squared depth-wavenumber operator, and Δx is the horizontal sampling interval. The tridiagonal matrix on the right-hand side represents the finite-difference discretization of the lateral second-order derivative operator. **Equation 3** indicates that the squared depth-wavenumber operator in the discrete space domain contains both the velocity-dependent term and the lateral wavefield-coupling term. If a Fourier transform is further applied in the horizontal direction, $\frac{\partial^2}{\partial x^2} \rightarrow -k_x^2$, and

Equation 3 reduces to the conventional dispersion relation $k_z^2 = \frac{\omega^2}{v^2} - k_x^2$. Compared to conventional approximate one-way propagators, this matrix formulation directly incorporates lateral velocity variations and wavefield coupling into the discrete squared depth-wavenumber operator, thereby improving the accuracy of wavefield extrapolation in laterally heterogeneous media.

According to linear algebra theory, the matrix k_z^2 can be decomposed into eigenvalues and eigenvectors using the standard eigenvalue decomposition method (**Equation 4**):

$$k_z^2 = VMV^T \quad (4)$$

where V denotes the matrix composed of eigenvectors and is assumed to be an orthogonal matrix; M denotes the diagonal matrix composed of eigenvalues; the matrix k_z^2 is symmetric; and the superscript T denotes the matrix transpose.

According to **Equation 4**, the matrix expression of k_z can be obtained as **Equation 5**:

$$k_z = VM^{\frac{1}{2}}V^T \quad (5)$$

The eigenvalue spectrum of the squared depth-wavenumber matrix k_z^2 may contain both positive and negative eigenvalues. Positive eigenvalues yield real depth wavenumbers and correspond to propagating modes, whereas negative eigenvalues yield purely imaginary depth wavenumbers and correspond to evanescent modes that decay exponentially with propagation distance in the depth direction. In the numerical implementation, spectral truncation is applied before constructing k_z : only the positive eigenvalues and their associated eigenvectors are retained, while the modes associated with negative eigenvalues are discarded. Consequently, the one-way propagation operators are constructed exclusively within the propagating-wave subspace, thereby suppressing evanescent components and improving the numerical stability of recursive depth extrapolation.

After the matrix of k_z is obtained, the propagation of downgoing and upgoing wavefields between adjacent depth layers can be respectively expressed as a phase-shift process along the depth direction. Therefore, the one-way wavefield propagation operators based on matrix eigenvalue decomposition can be expressed as **Equation 6**:

$$\begin{cases} W^+(z_n; z_{n+1}) = e^{ik_z \Delta z} = e^{\left[VM^{\frac{1}{2}}V^T \right] \Delta z} \\ W^-(z_{n+1}; z_n) = e^{-ik_z \Delta z} = e^{\left[-VM^{\frac{1}{2}}V^T \right] \Delta z} \end{cases} \quad (6)$$

where $W^+(z_n; z_{n+1})$ denotes the propagation operator for the

downgoing wavefield from depth z_n to z_{n+1} , and $W^-(z_{n+1}; z_n)$ denotes the propagation operator for the upgoing wavefield from depth z_n to z_{n+1} .

2.2. Primary wavefield modeling based on recursive propagation

Based on the space-frequency-domain propagation operator, primary wavefield modeling (PWMod) describes the propagation, reflection, and transmission of the source wavefield in the subsurface medium through layer-by-layer recursive extrapolation. The main idea is as follows: the downgoing wavefield generated by the surface source is first extrapolated layer by layer along the depth direction; when the downgoing wavefield reaches a reflection interface, a primary upgoing scattered wavefield is generated; this upgoing wavefield then continues to propagate upward to the surface and forms the receiver record. This process considers only primary reflections and does not explicitly include multiple-scattering terms; therefore, it can serve as the forward-modeling basis for subsequent least-squares wave-equation migration.⁴⁹

Figure 1 illustrates the reflection and transmission relationship of wavefields at a single depth interface. When the downgoing wavefield p^+ is incident on the interface z_n , part of its energy is converted into an upgoing primary reflection by the upward reflection coefficient $R^\cup$, while the remaining energy continues to propagate downward after being modulated by the downgoing transmission coefficient T^+ . Similarly, when the upgoing wavefield p^- passes through the interface, it is jointly affected by the downward reflection coefficient $R^\cap$ and the upgoing transmission coefficient T^- . This local wavefield updating relationship forms the physical basis for defining the reflection and transmission coefficients in PWMod.

At each depth layer z_n , the interaction between the wavefield and the reflection interface can be described by local reflection and transmission coefficients. For an angle-independent acoustic reflection coefficient model, the transmission coefficients during downgoing and upgoing propagation can be defined as **Equation 7**:

$$\begin{cases} T^+(z_n) = I + \text{diag}(R^\cup(z_n)) \\ T^-(z_n) = I - \text{diag}(R^\cap(z_n)) \end{cases} \quad (7)$$

where $T^+(z_n)$ and $T^-(z_n)$ denote the downgoing and upgoing transmission coefficients at depth layer z_n , respectively; I is the identity matrix; $R^\cup(z_n)$ denotes the upward reflection coefficient at depth layer z_n , and $R^\cap(z_n)$ denotes the downward reflection coefficient at depth layer z_n . The two coefficients satisfy $R^\cup(z_n) = -R^\cap(z_n)$. The operator $\text{diag}(\cdot)$ constructs the reflection coefficients into a diagonal matrix.

To describe the propagation of the source wavefield from the surface to an arbitrary depth layer and the propagation of the primary reflected wavefield from the subsurface interface back to the surface, the global downgoing and upgoing propagation operators are introduced as **Equation 8**:

$$\begin{cases} U^+(z_n; z_m) = \left(\prod_{j=n+1}^{m-1} W^+(z_{j-1}; z_j) T^+(z_j) \right) W^+(z_{m-1}; z_m) \\ U^-(z_n; z_m) = \left(\prod_{j=m-1}^{n+1} W^-(z_{j-1}; z_j) T^-(z_j) \right) W^-(z_{m-1}; z_m) \end{cases} \quad (8)$$

where $U^+(z_n; z_m)$ denotes the global downgoing propagation operator that propagates the wavefield from the surface depth layer z_n to the depth layer z_m , and $U^-(z_n; z_m)$ denotes the global upgoing propagation operator that propagates the wavefield from the depth layer z_m to the surface depth layer z_n . **Equation 8** shows that the global propagation operator is formed by layer-by-layer coupling of wavefield extrapolation between adjacent depth layers and interface transmission effects; therefore, it can describe the recursive propagation of wavefields in multilayer media.

Based on the above global propagation operators, the primary-reflection forward-modeling process of PWMod can be uniformly expressed as **Equation 9**:

$$\begin{cases} p_{\text{mod}}^+(z_m) = U^+(z_m; z_0) s^+(z_0) \\ p_{\text{mod}}^-(z_0) = \sum_{j=1}^m U^-(z_0; z_j) [R^\cup(z_j) \cdot p_{\text{mod}}^+(z_j)] \end{cases} \quad (9)$$

where $p_{\text{mod}}^+(z_m, \omega)$ denotes the downgoing modeled wavefield propagated to depth layer z_m , $p_{\text{mod}}^-(z_0, \omega)$ denotes the upgoing modeled wavefield received at the surface, and $R^\cup(z_n)$ denotes the upward reflection coefficient at the depth layer z_n .

The first line of **Equation 9** indicates that the surface source wavefield is extrapolated to the target depth layer through the global downgoing propagation operator, forming the incident downgoing wavefield at that depth. The second line indicates that the reflection coefficients at different depth layers interact with the incident downgoing wavefield to generate primary upgoing scattered wavefields, which are then propagated back to the surface through the global upgoing propagation operator and superposed to form the modeled records. Therefore, PWMod can be summarized as a recursive primary-wave modeling process of “downgoing propagation–interface reflection–upgoing propagation–surface superposition.”

2.3. Least-squares inversion framework with sparse denoising constraint for reflection coefficients

Based on the primary wavefield modeling process

described above, the subsurface reflection coefficient model $R^\cup$ controls the generation of surface primary reflected wavefields through the downgoing wavefield, local reflection interaction, and upgoing propagation operator. The objective of least-squares inversion is to iteratively update the subsurface reflection coefficient model under the constraint of observed data, so that the simulated data gradually approach the observed data. However, under the influence of limited frequency bandwidth, limited aperture, acquisition noise, and velocity errors, conventional least-squares inversion is prone to incoherent noise, low-amplitude artifacts, and nonuniform illumination. To improve inversion stability and enhance the noise robustness of imaging results, a sparse regularization constraint based on a curvelet-transform denoiser is introduced into the data-misfit term, leading to the following objective function (**Equation 10**):^{51,52}

$$\begin{aligned} J = F_1 + F_2 = & \frac{1}{2} \sum_{\omega} \|p_{\text{mod}}^-(z_0) - p_{\text{obs}}(z_0)\|_2^2 \\ & + \frac{1}{2} \lambda (R^\cup)^T [R^\cup - f_{CT}(R^\cup)] \end{aligned} \quad (10)$$

where F_1 is used to measure the error between the surface-modeled wavefield $p_{\text{mod}}^-(z_0)$ and the observed wavefield $p_{\text{obs}}(z_0)$; F_2 is the denoiser regularization term based on the curvelet-transform denoiser, which is used to constrain the sparsity and structural stability of the reflection coefficient model. The parameter λ is the regularization factor, which balances the data-misfit term and the denoising regularization term; $f_{CT}(\cdot)$ denotes the denoising function based on the curvelet transform.

The curvelet transform has multiscale, multidirectional, and anisotropic representation capabilities, and can effectively characterize structural features in seismic imaging results, such as continuous reflection events, curved interfaces, and fault boundaries. In the curvelet domain, effective reflection structures are usually concentrated in a small number of large-amplitude coefficients, whereas random noise and incoherent artifacts are mostly distributed among a large number of small-amplitude coefficients. Therefore, the curvelet transform is used in this study to construct the denoising function $f_{CT}(\cdot)$. Its basic procedure includes the following steps: First, the current reflection coefficient model $R^\cup$ is transformed into the curvelet domain to obtain curvelet coefficients at different scales and directions; second, hard thresholding is applied to the curvelet coefficients, whereby coefficients with amplitudes greater than a prescribed threshold are retained, whereas those below the threshold are set to zero; third, the processed coefficients are reconstructed back to the space domain through the inverse curvelet transform

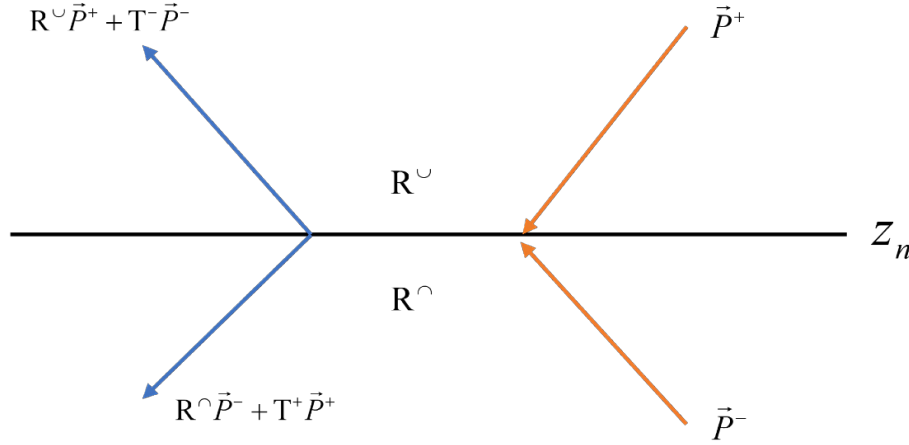


Figure 1. Schematic diagram of the continuous wavefield propagation process

to obtain the denoised reflection coefficient model.

On this basis, the regularization term in **Equation 10** is constructed using the reflection coefficient model $R^\cup$ and its denoising residual $R^\cup - f_{CT}(R^\cup)$. The physical meaning of this term is to constrain the inversion result to gradually approach the structured model denoised in the curvelet domain, so that the model update satisfies data fitting while continuously suppressing low-amplitude incoherent noise, migration artifacts, and local oscillations, and preserving the main reflection structures with geological significance as much as possible.

For the data-misfit term F_1 , its gradient with respect to the reflection coefficient $R^\cup$ can be written as **Equation 11**:

$$\begin{aligned} \nabla R_1^\cup(z_n) &= \frac{\partial F_1}{\partial R^\cup(z_n)} \\ &= \sum_{\omega} U^-(z_0; z_n)^H \cdot (p_{\text{mod}}^+(z_n))^H \cdot (p_{\text{mod}}^-(z_0) - p_{\text{obs}}(z_0)) \end{aligned} \quad (11)$$

where the superscript H denotes the conjugate transpose.

For the hard-thresholding CT-denoiser used in this study, these properties can be examined explicitly. Let C denote the curvelet analysis operator and C^* its adjoint synthesis operator, and let Θ_τ denote the hard-thresholding operator with threshold τ . The denoiser can then be written as $f_{CT}(R^\cup) = C^* \Theta_\tau(CR^\cup)$. For a fixed active set of curvelet coefficients, i.e., when no coefficient crosses the threshold under a sufficiently small perturbation, hard thresholding is locally homogeneous, such that $f_{CT}(CR^\cup) = CF_{CT}(R^\cup)$ is sufficiently close to 1.

Accordingly, away from threshold-transition points, the gradient of the denoiser regularization term with respect to the reflection coefficient can be expressed as **Equation 12**:

$$\nabla R_2^\cup(z_n) = \frac{\partial F_2}{\partial R^\cup(z_n)} = \lambda [R^\cup - f_{CT}(R^\cup)] \quad (12)$$

At threshold-transition points, the hard-thresholding operator is nondifferentiable, and the classical Jacobian is not defined. **Equation 12** should therefore be understood as a piecewise-valid gradient expression within regions where the active set of curvelet coefficients remains unchanged.

Combining the gradient of the data-misfit term and the gradient of the regularization term, the total gradient of the objective function J with respect to the reflection coefficient $R^\cup$ can be obtained as **Equation 13**:

$$\begin{aligned} \nabla R^\cup(z_n) &= \frac{\partial J}{\partial R^\cup(z_n)} = \nabla R_1^\cup(z_n) + \nabla R_2^\cup(z_n) \\ &= \sum_{\omega} U^-(z_0; z_n)^H \cdot (p_{\text{mod}}^+(z_n))^H \cdot (p_{\text{mod}}^-(z_0) - p_{\text{obs}}(z_0)) \\ &\quad + \lambda [R^\cup - f_{CT}(R^\cup)] \end{aligned} \quad (13)$$

After obtaining the gradient of the data-misfit term, the current reflection coefficient model can be updated along the negative gradient direction to obtain the updated reflection coefficient model (**Equation 14**):

$$r_{\text{new}}(z_n) = r_{\text{old}}(z_n) - \alpha \cdot \nabla R^\cup(z_n) \quad (14)$$

where $r_{\text{old}}(z_n)$ denotes the reflection coefficient model in the current iteration, $r_{\text{new}}(z_n)$ denotes the updated reflection coefficient model, and α is the iteration step length.^{53,54}

As shown in **Figure 2**, the sparse-denoising-constrained least-squares inversion workflow adopted in this study can be summarized as an iterative updating process. First, the initial reflection coefficient model, velocity model, and source wavelet are input, and the modeled upgoing

wavefield corresponding to the current model is computed using the PWMod forward modeling operator. Then, the simulated data are subtracted from the observed data to obtain the data residual, and the gradient of the data-misfit term is computed through adjoint imaging. Meanwhile, the curvelet-transform denoising function is applied to the current reflection coefficient model for structure-preserving denoising, and the denoiser regularization gradient is constructed. Finally, the data-misfit gradient and the denoiser regularization gradient are combined to form the total gradient, and the reflection coefficient model is updated along its negative direction. While ensuring that the simulated data gradually approach the observed data, this workflow introduces a curvelet-domain denoising constraint to suppress random noise, incoherent artifacts, and low-amplitude perturbations, thereby improving the stability and imaging quality of the inversion results.

3. Numerical examples

To validate the effectiveness of the matrix-eigenvalue-decomposition-based one-way propagation operator and the corresponding least-squares imaging framework, the numerical experiments were organized progressively from simple to complex geological models and from noise-free to noisy data conditions. Three representative models, namely the Lens, Marmousi, and Overthrust models, were first used to evaluate the imaging performance of the proposed method under local velocity anomalies, complex lateral heterogeneity, and strong structural deformation. Subsequently, using the Lens model as an example, random noise with different intensities was added to the shot gathers to further assess the stabilizing effect and noise-suppression capability of the CT-denoiser sparse denoising constraint for low signal-to-noise-ratio (SNR) imaging. For clarity,

generalized-screen-propagator-based least-squares one-way wave-equation migration and matrix-decomposition-based least-squares one-way wave-equation migration are hereafter denoted as LS-GSP and LS-MD, respectively. For all regularized numerical experiments, λ was increased stage-wise from 0.012 (iterations 1–10) to 0.024 (iterations 11–20) and 0.036 (iterations 21–30). This schedule was empirically determined through preliminary tests to balance data fitting and regularization strength.

3.1. Lens model

A relatively simple Lens model with a pronounced local velocity anomaly was first used to evaluate the basic imaging capability of the LS-MD method for strong velocity anomalies and their reflection boundaries. Figure 3 shows the Lens velocity model. The shallow part of the model exhibits evident lateral velocity variations and contains a high-velocity lens body, whereas the middle and lower parts are dominated by relatively smooth layered structures. This high-velocity anomaly induced wavefield focusing and phase distortion and was therefore suitable for testing the accuracy of different propagation operators in laterally heterogeneous media. The Lens model spanned 4,720 m horizontally and 500 m in depth, with horizontal and vertical grid spacings of 20 m and 10 m, respectively. A Ricker wavelet with a dominant frequency of 15 Hz was used as the source wavelet. The recording duration was 2 s, with a temporal sampling interval of 0.001 s. Receivers were deployed along the entire surface at 20-m intervals. A total of 118 shots were acquired with a shot spacing of 40 m, and the receiver array remained fixed for all shots.

Figure 4 compares the imaging results of the LS-GSP and LS-MD methods for the Lens model. The red arrows

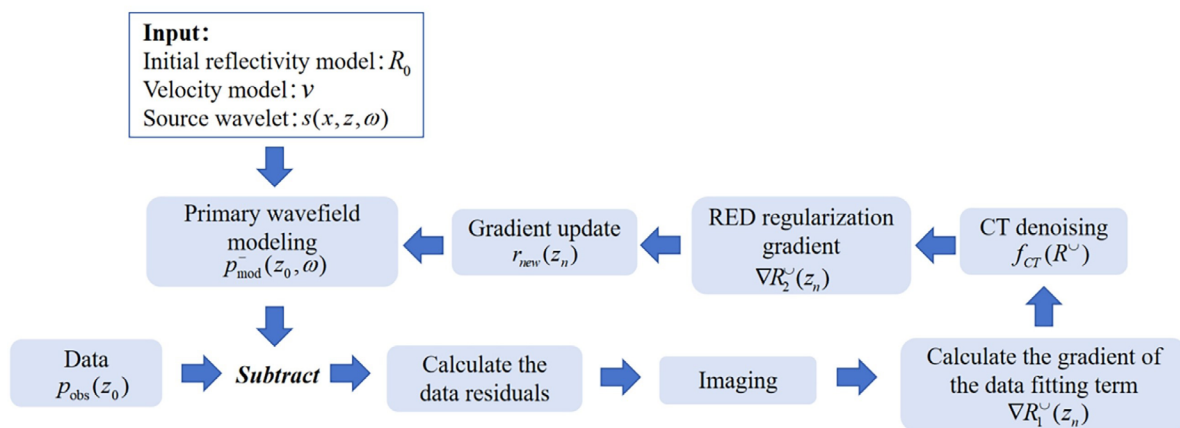


Figure 2. Overall iterative workflow of least-squares inversion with sparse denoising constraint
Abbreviations: CT: Curvelet transform; RED: Regularization by denoising.

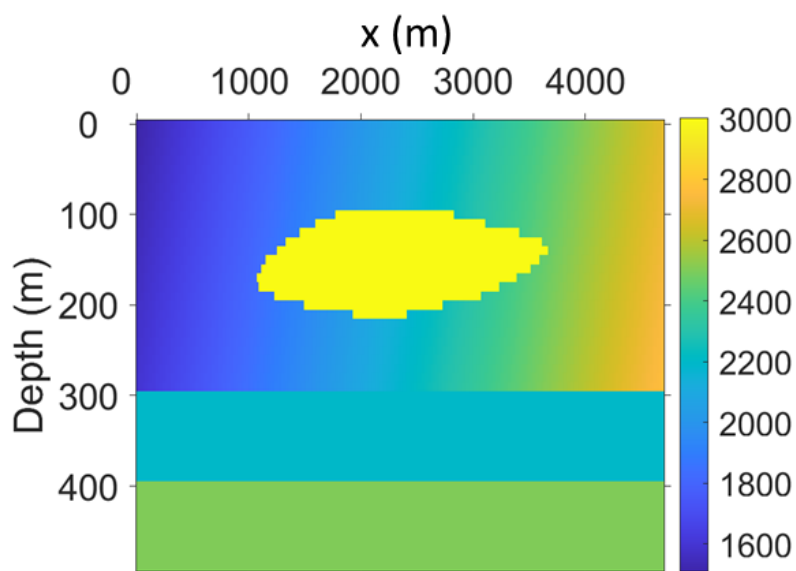


Figure 3. Lens velocity model

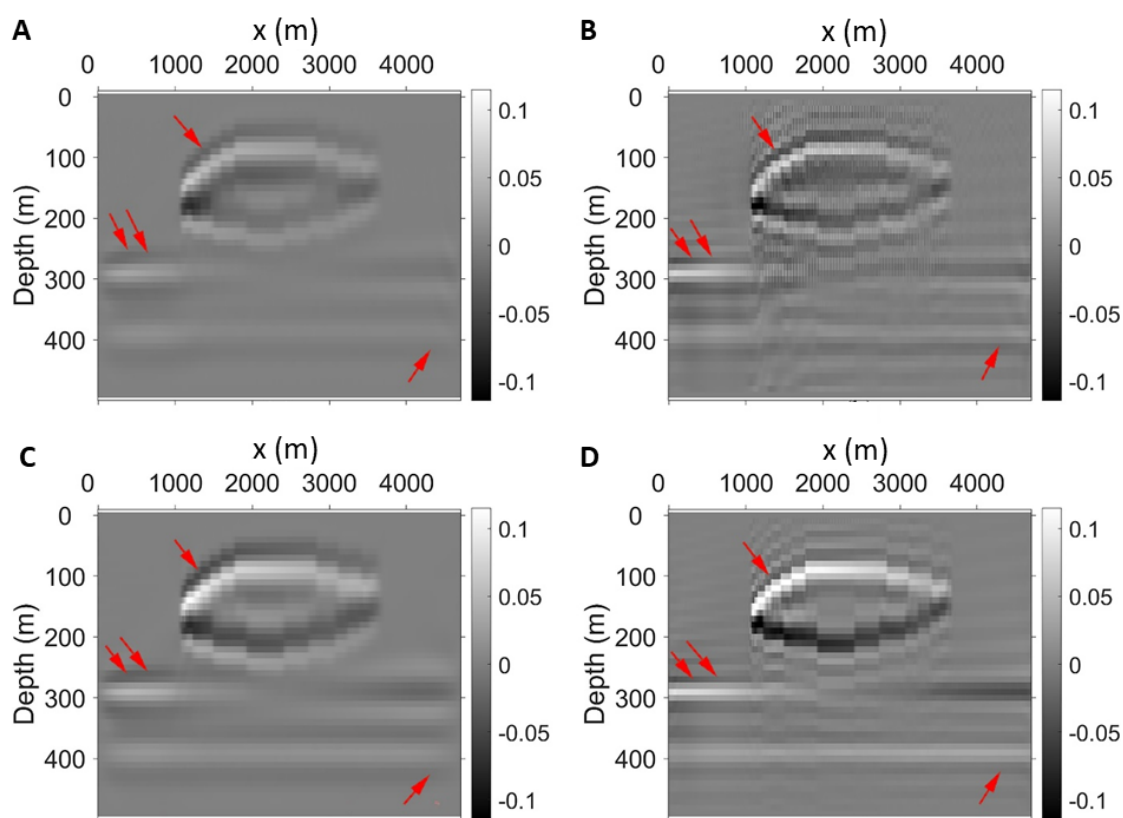


Figure 4. Comparison of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP) and matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD) imaging results for the Lens model. (A) Imaging result of the LS-GSP method after the 1st iteration. (B) Imaging result of the LS-GSP method after the 30th iteration; (C) Imaging result of the LS-MD method after the 1st iteration. (D) Imaging result of the LS-MD method after the 30th iteration. The red arrows indicate the imaging differences.

indicate representative regions for highlighting differences in reflector continuity and boundary focusing. Figure 4A shows the result of the LS-GSP method after the first iteration. The outline of the lens body could be identified, but the overall image was relatively blurred, and the reflection boundaries lacked continuity. Figure 4B shows the LS-GSP result after 30 iterations. Compared to the initial iteration, the lens boundaries were enhanced to some extent; however, reflection energy remained diffused, low-amplitude artifacts were evident, and local interfaces were not sharply delineated. Figure 4C shows the LS-MD result after the first iteration, where the main boundaries of the lens body were more clearly recovered, and the reflection energy was better focused than in the LS-GSP result. Figure 4D shows the LS-MD result after 30 iterations. The upper and lower boundaries of the lens body became more continuous, the reflection energy was more concentrated, and migration artifacts were effectively suppressed. As highlighted by the red arrows, the LS-MD result exhibited sharper lens boundaries and better continuity of the deeper horizontal reflectors than the LS-GSP result. These results indicate that the LS-MD method can more accurately describe one-way wave propagation in laterally heterogeneous media, thereby improving the imaging quality of local velocity anomalies.

Figure 5 presents the standardized data-residual curves

for the Lens model, where the blue curve denotes the LS-GSP method, and the red curve denotes the LS-MD method. The LS-GSP residual decreased rapidly during the early iterations but gradually flattened, indicating limited improvement in data fitting during subsequent iterations. In contrast, the LS-MD method exhibited a more stable residual decay and reached a lower residual level within fewer iterations. Combined with the imaging results in Figure 4, these observations suggest that the LS-MD method not only improves the imaging clarity of the lens reflection boundaries but also achieves a more stable residual reduction during the least-squares inversion.

3.2. Marmousi model

The Marmousi model, which has a higher degree of structural complexity, was adopted to evaluate the applicability of the LS-MD method in media with strong lateral velocity variations and complex layered structures. Figure 6 shows the Marmousi velocity model. It can be observed that the model contained multiple sets of curved reflectors, local high-velocity anomalies, and complex fault structures, with significant lateral velocity heterogeneity. These features place higher demands on the propagation operator to describe wavefield propagation paths and preserve phase accuracy. The Marmousi model spanned 3,500 m horizontally and 3,750 m in depth and

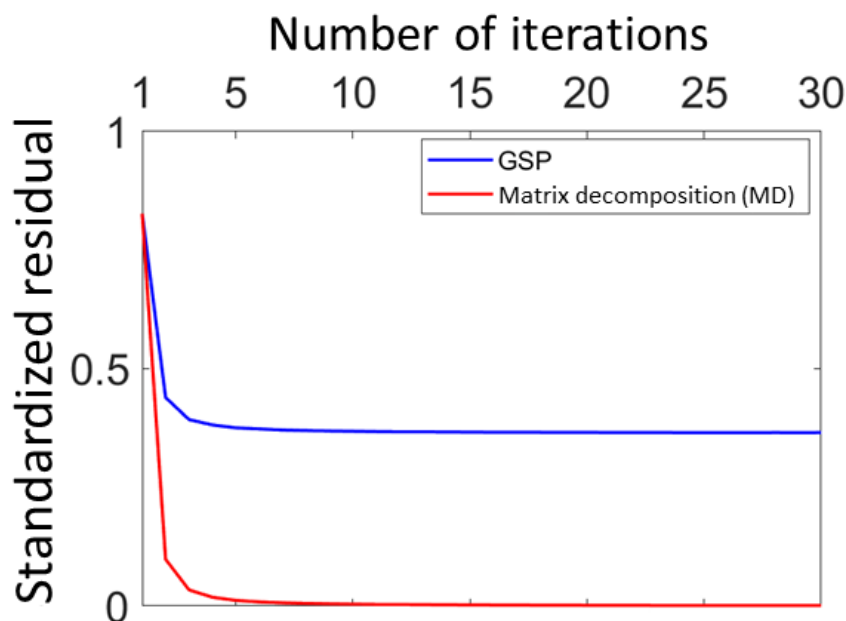


Figure 5. Comparison of standardized data-residual curves. The blue curve denotes the standardized residual curve of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP), and the red curve denotes the standardized residual curve of matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD).

was discretized with a uniform grid spacing of 10 m in both directions. Seismic data were generated using a Ricker wavelet with a dominant frequency of 20 Hz. The recording duration and temporal sampling interval were 4 s and 0.001 s, respectively. Receivers were distributed along the entire surface at 10-m intervals. The survey consisted of 116 shots with a shot spacing of 30 m, while the receiver array remained fixed for all shots.

Figure 7 presents the imaging results of the LS-GSP and LS-MD methods for the Marmousi model. The red arrows mark representative deep reflectors and structurally complex regions where differences in reflector delineation can be readily observed. Figure 7A shows the imaging result of the LS-GSP method after the first iteration, where the responses of the main reflection interfaces were weak, the energy of deep structures was insufficient, and some dipping interfaces and fault features were difficult to identify clearly. Figure 7B shows the LS-GSP result after 30 iterations. The shallow and middle reflection layers were enhanced to some extent, but problems such as insufficient reflection continuity, local energy divergence, and migration artifacts remained in complex structural regions. Figure 7C shows the LS-MD result after the first iteration. Compared to Figure 7A, the main structural outlines and reflection layers were more effectively recovered, and the imaging responses in complex structural regions were more pronounced. Figure 7D shows the LS-MD result after 30 iterations. It can be seen that the shallow layered reflections, middle fault structures, and deep dipping interfaces were more clearly delineated, with better continuity of reflection events, more accurate positioning of structural boundaries, and a more balanced distribution of imaging energy. The arrowed regions further show that the LS-MD method provided a more complete delineation of deep-dipping reflectors and complex reflection events, with reduced energy smearing compared with LS-GSP. These results indicate that the LS-MD method can effectively improve wavefield imaging quality in complex laterally heterogeneous media and enhance the resolution capability for complex structures.

Figure 8 shows the standardized data-residual curves for the Marmousi model. It can be observed that the residual of the LS-GSP method decreased during the initial iterations and then quickly stabilized, indicating that its data-fitting capability in complex velocity backgrounds is limited by the approximation accuracy of the propagation operator. In contrast, the LS-MD method maintained a continuous residual-decreasing trend during the iteration process and eventually reached a lower standardized residual level. This indicates that the matrix-eigenvalue-decomposition-based propagation operator can more accurately describe

wavefield propagation in complex heterogeneous media, thereby improving the data-fitting accuracy and convergence stability of least-squares inversion.

3.3. Overthrust model

On the basis of the preceding experiment with complex layered media, the Overthrust model with typical thrust structures was introduced to further increase structural complexity and evaluate the applicability of the proposed method under strong structural deformation. Figure 9 shows the Overthrust velocity model. Multiple thrust faults, strong velocity anomalies, and steeply dipping structural interfaces were developed in the middle and deep parts of the model, and the velocity field exhibited strong lateral variations. As a result, the wavefield propagation paths and phase characteristics were more complex, imposing higher requirements on the phase-preserving capability of the propagation operator and the imaging accuracy of deep structures. For the Overthrust experiment, the model spanned 10,000 m horizontally and 2,800 m in depth, with a spatial grid spacing of 20 m in both directions. A Ricker wavelet with a dominant frequency of 20 Hz was adopted as the source wavelet. Each shot record had a duration of 2 s and was sampled at a temporal interval of 0.001 s. Receivers were deployed over the entire surface with a spacing of 20 m. A total of 100 shots were distributed at 100-m intervals, and the receiver array remained fixed for all shots.

Figure 10 compares the imaging results of the LS-GSP and LS-MD methods for the Overthrust model. The red arrows identify representative thrust-related and steeply dipping structures, while the red rectangle encloses a middle-to-deep region with pronounced structural deformation for detailed comparison. Figure 10A shows the imaging result of the LS-GSP method after the first iteration. The main structural framework was initially imaged, but the reflection interfaces are generally blurred, and the spatial positions of steeply dipping interfaces were not clearly delineated. Figure 10B shows the LS-GSP result after 30 iterations. The energy of the main thrust structures was enhanced to some extent, but energy dispersion still occurred in local regions, and the focusing of deep reflections remained limited. Figure 10C shows the LS-MD result after the first iteration. The spatial distribution of the imbricated thrust structures was clearer, and the reflection energy in the middle and deep parts was more concentrated, indicating that the LS-MD method can improve the imaging response of complex structures even at the initial iteration stage. Figure 10D shows the LS-MD result after 30 iterations. The imaging positions of steeply dipping interfaces were more accurate, the energy focusing at structural turning points was more pronounced, and

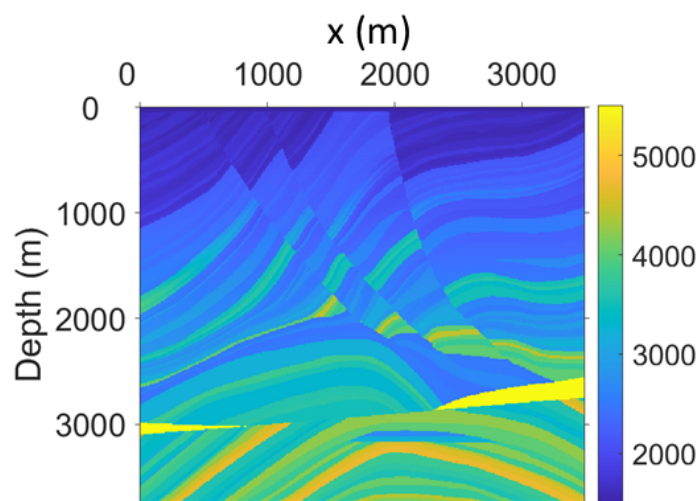


Figure 6. Marmousi velocity model

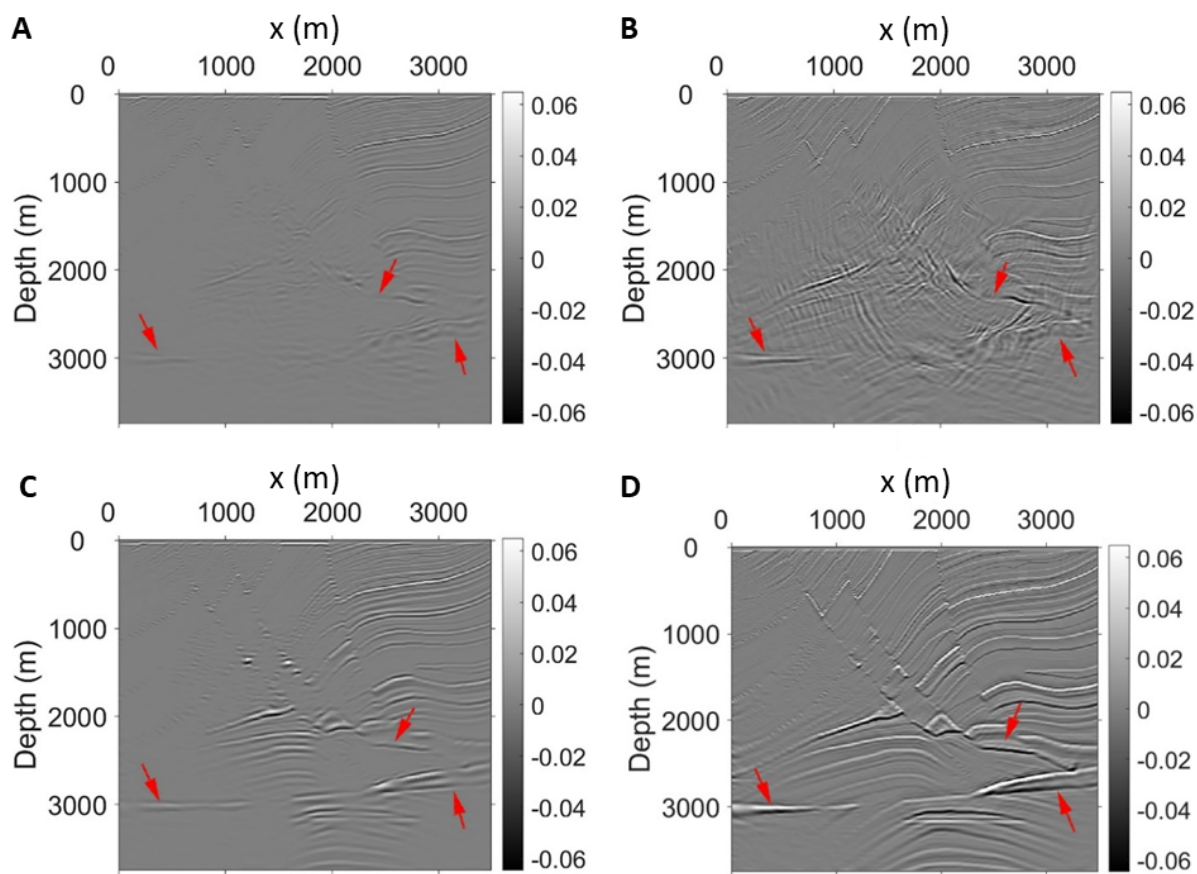


Figure 7. Comparison of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP) and matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD) imaging results for the Marmousi model. (A) Imaging result of the LS-GSP method after the 1st iteration. (B) Imaging result of the LS-GSP method after the 30th iteration. (C) Imaging result of the LS-MD method after the 1st iteration. (D) Imaging result of the LS-MD method after the 30th iteration. The red arrows indicate the imaging differences.

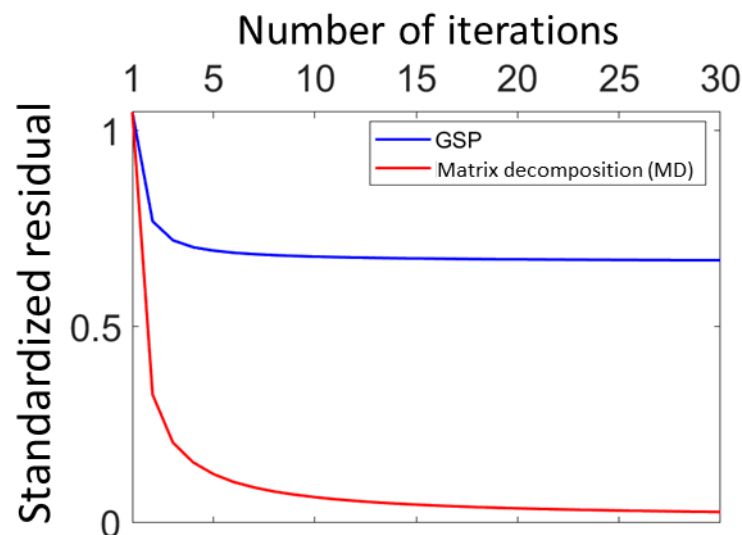


Figure 8. Comparison of standardized data-residual curves. The blue curve denotes the standardized residual curve of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP), and the red curve denotes the standardized residual curve of matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD).

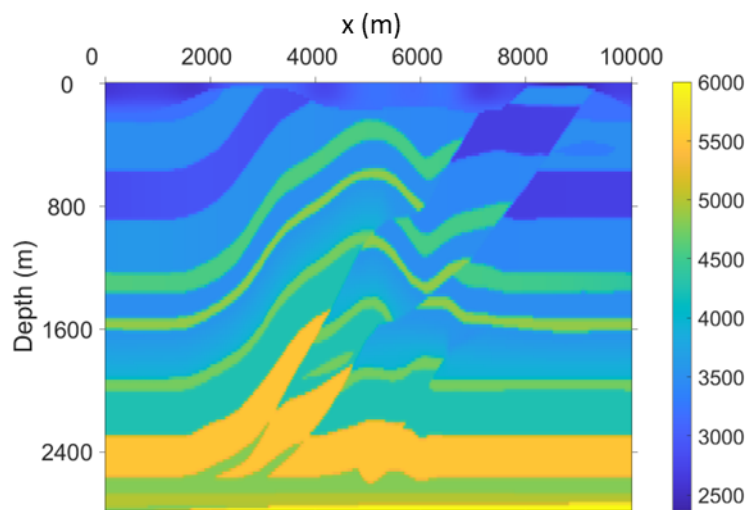


Figure 9. Overthrust velocity model

deep complex reflection events were recovered more stably. In particular, the annotated regions show that LS-MD better resolved the geometry of steeply dipping interfaces and structural turning points and yielded clearer reflection events within the strongly deformed middle-to-deep region. These results indicate that the LS-MD method can improve wavefield propagation accuracy and inversion stability under complex thrust-structure conditions, thereby producing deep imaging results with higher structural fidelity.

Figure 11 shows the standardized data-residual curves for the Overthrust model. It can be observed that, under complex thrust-structure conditions, the residual reduction of the LS-GSP method gradually slowed down with increasing iterations, indicating that certain data-matching errors remained under strong lateral velocity variations and steeply dipping interfaces, limiting further improvement in imaging quality during subsequent iterations. In contrast, the LS-MD method exhibited a smoother and more continuous data-error decay during

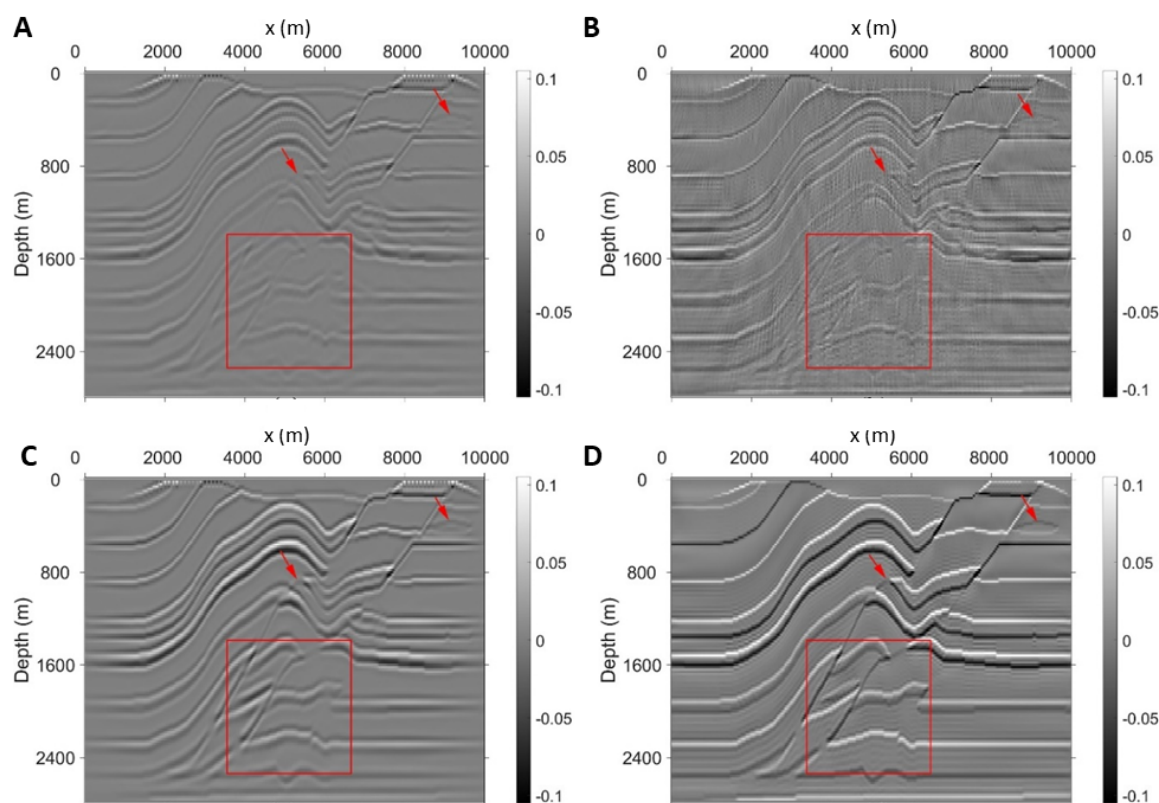


Figure 10. Comparison of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP) and matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD) imaging results for the Overthrust model. (A) Imaging result of the LS-GSP method after the 1st iteration. (B) Imaging result of the LS-GSP method after the 30th iteration. (C) Imaging result of the LS-MD method after the 1st iteration. (D) Imaging result of the LS-MD method after the 30th iteration. The red arrows and red rectangle indicate the imaging differences.

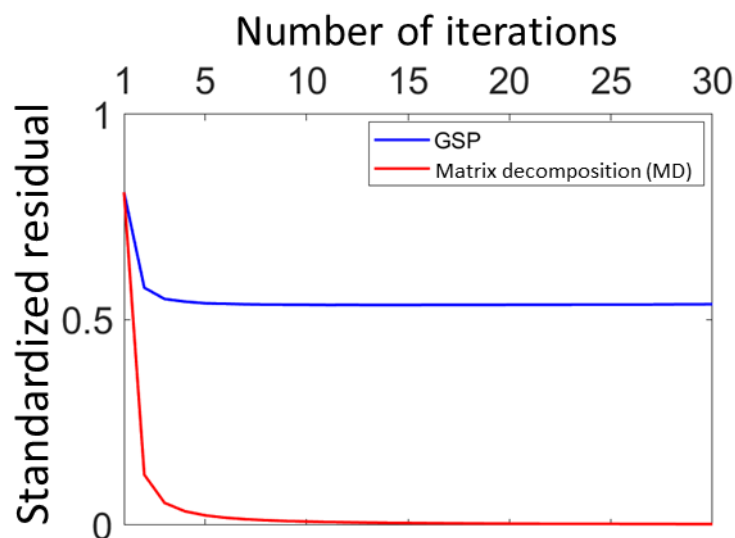


Figure 11. Comparison of standardized data-residual curves. The blue curve denotes the standardized residual curve of generalized-screen-propagator-based least-squares one-way wave-equation migration (LS-GSP), and the red curve denotes the standardized residual curve of matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD).

the iteration process. This indicates that the matrix-eigenvalue-decomposition-based propagation operator can more accurately describe wavefield phase evolution and energy propagation characteristics in complex velocity fields, thereby improving the imaging reliability of deep structures and steeply dipping interfaces.

3.4. Noise suppression with sparse denoising constraint

Field seismic data are commonly affected by various types of noise, and low-SNR data may lead to noise amplification and low-amplitude artifacts during least-squares imaging. As a controlled synthetic test of noise sensitivity, random noise with different intensities was added to the shot gathers of the Lens model to evaluate the noise robustness of the CT-denoiser sparse denoising constraint.

Figure 12 shows the shot gathers of the Lens model under different noise levels. Figure 12A shows the noise-free shot gather, in which the reflection patterns are clear, and the events are continuous. Figure 12B shows the shot gather under the 5 dB noise condition. The random noise was mainly superimposed on the background region and slightly affected the continuity of effective reflections, while the main events remained distinguishable. Figure 12C shows the shot gather under the 1 dB noise condition, where the noise energy was further increased; the continuity of reflection events was significantly affected, and weak reflections became more difficult to identify. Figure 12D shows the shot gather under the -5 dB noise condition. In this case, the effective reflection signals were strongly disturbed by random noise; the continuity of reflection events was obviously degraded, and the data quality deteriorated significantly. These results indicate that, as the signal-to-noise ratio decreases, effective wavefield information in the shot gathers is gradually obscured by random noise, posing greater challenges to subsequent imaging stability.

Figure 13 presents the true reflectivity model and the LS-MD imaging results after 30 iterations under different noise levels. Figure 13A shows the true reflectivity model, which is used as the reference for the subsequent quantitative peak SNR (PSNR) and structural similarity index (SSIM) evaluations. Figure 13B shows the reference imaging result under the noise-free condition, where the upper and lower boundaries of the lens body were clear, the reflection energy was concentrated, and migration artifacts were relatively weak. Figure 13C shows the imaging result under the 5 dB noise condition without the CT-denoiser constraint. The main structure of the lens body was still recognizable, but random noise and strip-like artifacts appeared in the background. Figure 13D shows the result

under the 5 dB noise condition with the CT-denoiser constraint. Compared to Figure 13C, the background noise was effectively suppressed, the lens boundaries became more continuous, and the imaging result was closer to the noise-free reference. Figure 13E shows the result under the 1 dB noise condition without the CT-denoiser constraint. As the noise level increased, random interference in the image became more pronounced, and weak reflections around the lens body were partially obscured. Figure 13F shows the result under the 1 dB noise condition with the CT-denoiser constraint, where the main boundaries of the lens body were well preserved, and background noise and low-amplitude artifacts were significantly reduced. Figure 13G shows the result under the -5 dB noise condition without the CT-denoiser constraint. The image was severely disturbed, with substantial random noise in the background, and the lens boundaries became blurred and discontinuous. Figure 13H shows the result under the -5 dB strong-noise condition with the CT-denoiser constraint. Although some details were still affected by strong noise, the main lens structure was effectively recovered, and random noise was significantly reduced, indicating that the CT-denoiser sparse denoising constraint can improve the stability and interpretability of LS-MD imaging under low-SNR conditions.

To quantitatively evaluate the noise-suppression performance of the CT-denoiser sparse denoising constraint, the PSNR and SSIM were calculated between the imaging results and the true reflectivity model under different noise levels. Unlike the visual comparisons above, for which the noise-free LS-MD result was used as the baseline, the true reflectivity model in Figure 13A was used as the reference for the quantitative evaluation. Let the true reflectivity model and the evaluated imaging result be denoted as a and b , respectively, with a model size of $M \times N$. The mean

squared error is defined as $MSE(a, b) = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (a_{ij} - b_{ij})^2$, and

PSNR can be expressed as $PSNR(a, b) = 10 \log_{10}(\frac{MAX_I^2}{MSE})$, where

the maximum possible amplitude $MAX_I = 0.2$. The SSIM is

defined as $SSIM(a, b) = \frac{(2\mu_a\mu_b + C_1)(2\sigma_a\sigma_b + C_2)}{(\mu_a^2 + \mu_b^2 + C_1)(\sigma_a^2 + \sigma_b^2 + C_2)}$, where μ_a

and μ_b denote the mean values of a and b , respectively; σ_a and σ_b denote the variance of a and b ; C_1 and C_2 are stability constants introduced to avoid division by zero. A higher PSNR indicates a smaller overall reconstruction error relative to the true reflectivity model, whereas an SSIM value closer to 1 indicates higher structural similarity to the true model.⁵⁵ The results are listed in Table 1. Under the 5 dB noise condition, the unregularized LS-MD result yielded a PSNR of 16.3667 dB and an SSIM of 0.6435. After introducing the CT-denoiser constraint, the PSNR

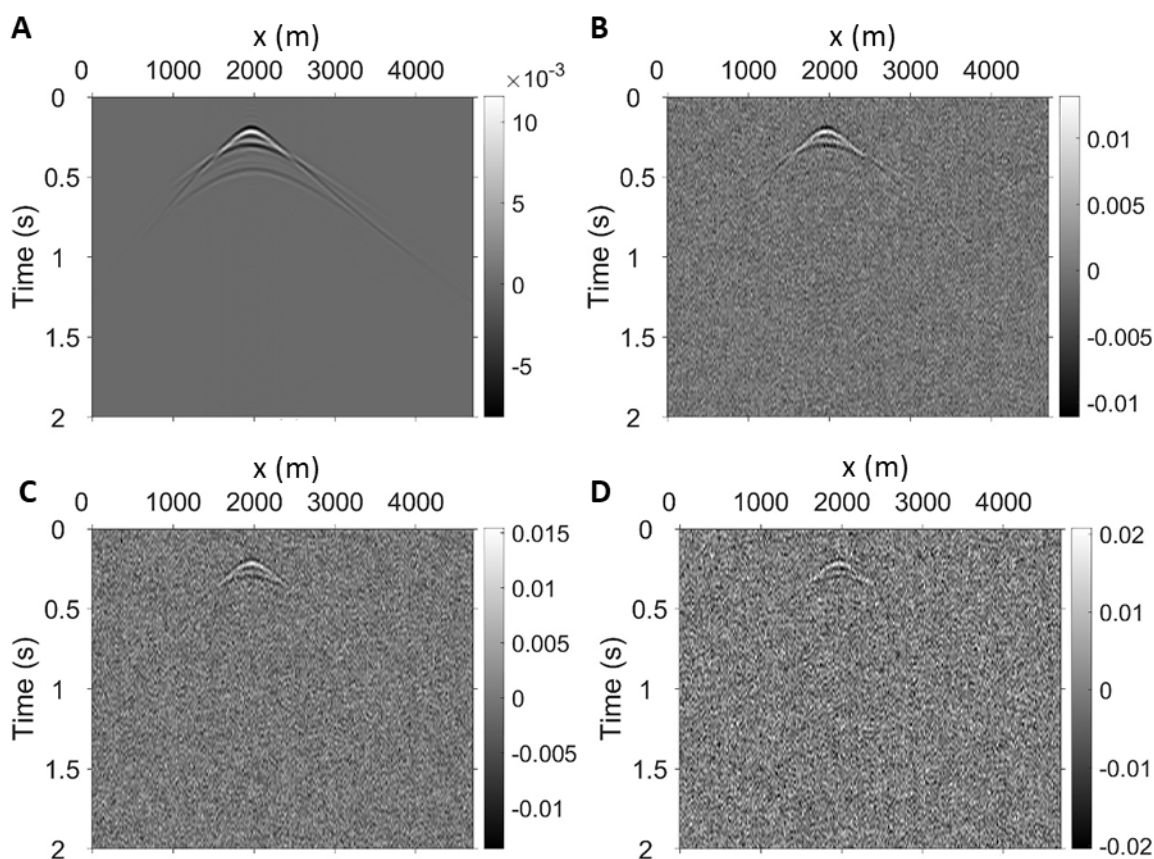


Figure 12. Shot gathers of the Lens model under different noise levels. (A) Noise-free. (B) 5 dB. (C) 1 dB. (D) -5 dB.

increased to 17.3225 dB, and the SSIM increased to 0.7266. Under the 1 dB noise condition, the PSNR increased from 16.2974 dB to 17.2291 dB, while the SSIM increased from 0.6139 to 0.7109. Under the -5 dB strong-noise condition, the improvement became more pronounced, with PSNR increasing from 12.9809 dB to 16.8805 dB and the SSIM increasing from 0.3115 to 0.6258. Because all metrics were calculated relative to the true reflectivity model, these results quantitatively demonstrate that the CT-denoiser constraint reduces the reconstruction error and improves the structural similarity of the inverted reflectivity model under different noise levels.

4. Discussion

Although the proposed LS-MD method improves the accuracy and stability of least-squares one-way wave-equation migration, several limitations remain. First, the method is based on one-way depth extrapolation and therefore remains subject to the intrinsic assumptions of depth-marching wavefield propagation. Its applicability

may be limited for wave phenomena involving turning waves, strong backscattering, or propagation paths that cannot be adequately represented by monotonic depth extrapolation. Second, the current PWMod framework considers only primary reflections and does not explicitly account for multiple scattering. Consequently, information carried by multiples is not exploited in the present inversion framework.

The improved wavefield representation of the matrix-decomposition propagator is accompanied by additional computational and memory costs associated with eigenvalue decomposition and operator construction. Although the propagation operators can be reused during inversion when the background velocity model is fixed, these costs increase with model size and may become significant for large-scale and three-dimensional applications. The present study therefore focuses on imaging performance rather than establishing a computational-efficiency advantage over LS-GSP. Future work will investigate efficient eigensolvers, low-rank or

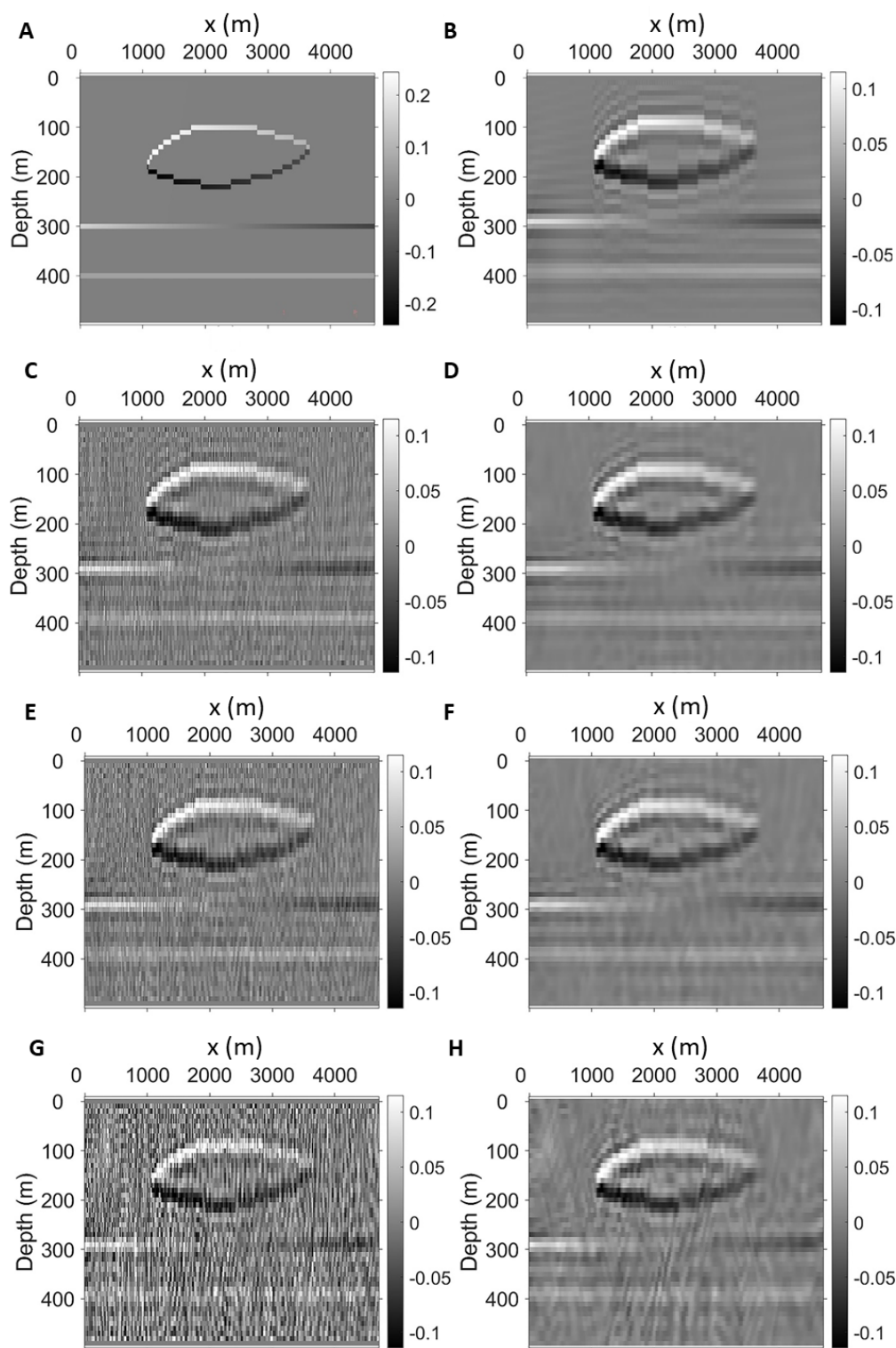


Figure 13. True reflectivity model and matrix-decomposition-based least-squares one-way wave-equation migration (LS-MD) imaging results after 30 iterations under different noise conditions. (A) True reflectivity model. (B) Noise-free result. (C) Result under 5 dB noise. (D) Denoised imaging result under 5 dB noise using curvelet transform (CT)-denoiser regularization. (E) Result under 1 dB noise. (F) Denoised imaging result under 1 dB noise using CT-denoiser regularization. (G) Result under -5 dB noise. (H) Denoised imaging result under -5 dB noise using CT-denoiser regularization.

sparse approximations, parallel computing, and GPU acceleration to improve computational scalability.

Table 1. Quantitative comparison of PSNR and SSIM relative to the true reflectivity model before and after CT-denoiser regularization under different noise levels

Noise level	Method	PSNR (dB)	SSIM
5 dB	Unregularized	16.3667	0.6435
5 dB	Regularized	17.3225	0.7266
1 dB	Unregularized	16.2974	0.6139
1 dB	Regularized	17.2291	0.7109
−5 dB	Unregularized	12.9809	0.3115
−5 dB	Regularized	16.8805	0.6258

Abbreviations: CT: Curvelet transform; PSNR: Peak signal-to-noise ratio; SSIM: Structural similarity index.

The hard-thresholding CT-denoiser used in this study is piecewise differentiable, and the gradient expression in **Equation 12** is valid within regions where the active set of curvelet coefficients remains unchanged. At threshold-transition points, however, the hard-thresholding operator is nondifferentiable, and a global smooth-gradient convergence guarantee therefore does not directly apply. Nevertheless, the stable residual reduction observed in the numerical experiments indicates that these local nonsmooth transitions do not cause noticeable instability under the tested parameter settings.

Finally, the present study is limited to synthetic examples, including controlled tests with additive random noise. Therefore, the robustness of the proposed method to field-data effects, such as irregular acquisition footprints, source and receiver statics, background-velocity uncertainty, and non-Gaussian or coherent noise, has not yet been systematically evaluated. Future work will focus on applications to field seismic data and three-dimensional settings to further assess the practical robustness and scalability of the method under realistic acquisition and noise conditions.

5. Conclusion

In this study, a least-squares one-way wave-equation migration method with a CT-denoiser sparse denoising constraint is proposed. The method constructs a space-frequency-domain one-way propagation operator using matrix eigenvalue decomposition, and combines recursive

primary wavefield modeling with adjoint propagation to update the reflection coefficient model. A CT-denoiser regularization term is further introduced, allowing the data-residual gradient and denoising-residual gradient to jointly guide the inversion and suppress random noise and incoherent artifacts. Numerical experiments on the Lens, Marmousi, and Overthrust models show that the proposed method produces clearer reflection interfaces, more continuous reflection events, and more stable residual convergence under local velocity anomalies, complex lateral heterogeneity, and strong structural variations. Noisy-data tests further demonstrate that the CT-denoiser constraint improves PSNR and SSIM under low-SNR conditions while suppressing low-amplitude artifacts and preserving the main reflection structures. These results demonstrate that the proposed method can improve imaging stability, resolution, and structural fidelity for complex synthetic models and controlled low-SNR data.

Acknowledgments

The authors sincerely thank the anonymous reviewers and the handling editor for their constructive comments and valuable suggestions, which have significantly improved the clarity and quality of this manuscript.

Funding

This work was supported by the open research project of the National Engineering Research Center for Oil and Gas Exploration Computer Software (DFWT-ZYRJ-2025-JS-56) and the Sichuan Science and Technology Program (2026NSFSC0234).

Conflict of interest

The authors declare that they have no competing interests.

Author contributions

Conceptualization: Guangyin Wang, Pengyuan Sun, Xinyu Zhang

Data curation: Guangyin Wang, Heng Zhang

Formal analysis: Guangyin Wang, Pengyuan Sun, Xueying Hu

Investigation: Guangyin Wang, Pengyuan Sun, Bin Li, Xueying Hu

Methodology: Guangyin Wang, Pengyuan Sun

Project administration: Xinyu Zhang

Software: Guangyin Wang, Pengyuan Sun

Supervision: Xinyu Zhang, Jiachun You

Validation: Guangyin Wang, Pengyuan Sun, Bin Li

Visualization: Guangyin Wang, Pengyuan Sun

Writing-original draft: Guangyin Wang

Writing-review & editing: Pengyuan Sun, Xinyu Zhang,

Bin Li, Xueying Hu, Heng Zhang, Jiachun You

Availability of data

Data are available upon reasonable request from the corresponding author.

References

- Gray SH. Efficient traveltimes calculations for Kirchhoff migration. *Geophysics*. 1986;51(8):1685-1688.
doi: 10.1190/1.1442217
- Baysal E, Kosloff DD, Sherwood JWC. Reverse time migration. *Geophysics*. 1983;48(11):1514-1524.
doi: 10.1190/1.1441434
- Wu B, Wu RS, Gao J. Preliminary investigation of wavefield depth extrapolation by two-way wave equations. *Int J Geophys*. 2012;2012(1):968090.
doi: 10.1155/2012/968090
- You J, Wu RS, Liu X, *et al*. Two-way wave equation-based depth migration using one-way propagators on a bilayer sensor seismic acquisition system. *Geophysics*. 2018;83(3):S271-S278.
doi: 10.1190/geo2017-0172.1
- You J, Pan N, Liu W, *et al*. Efficient wavefield separation by reformulation of two-way wave-equation depth-extrapolation scheme. *Geophysics*. 2022;87(4):S209-S222.
doi: 10.1190/geo2021-0629.1
- You JC, Zhang GL, Huang XG, *et al*. Internal-multiple-elimination with application to migration using two-way wave equation depth-extrapolation scheme. *Pet Sci*. 2025;22(1):178-192.
doi: 10.1016/j.petsci.2024.06.021
- Li A, Zhu X, Liu T, *et al*. One-step sparse-matrix method for full wave equation depth migration. *Geophysics*. 2024;89(6):S405-S413.
doi: 10.1190/geo2023-0372.1
- Tarantola A. Inversion of seismic reflection data in the acoustic approximation. *Geophysics*. 1984;49(8):1259-1266.
doi: 10.1190/1.1441754
- Tarantola A. Linearized inversion of seismic reflection data. *Geophys Prospect*. 1984;32(6):998-1015.
doi: 10.1111/j.1365-2478.1984.tb00751.x
- Nemeth T, Wu C, Schuster GT. Least-squares migration of incomplete reflection data. *Geophysics*. 1999;64(1):208-221.
doi: 10.1190/1.1444517
- Hu J, Schuster GT, Valasek PA. Poststack migration deconvolution. *Geophysics*. 2001;66(3):939-952.
doi: 10.1190/1.1444984
- Yu J, Hu J, Schuster GT, Estill R. Prestack migration deconvolution. *Geophysics*. 2006;71(2):S53-S62.
doi: 10.1190/1.2187783
- Duquet B, Marfurt KJ, Dellinger JA. Kirchhoff modeling, inversion for reflectivity, and subsurface illumination. *Geophysics*. 2000;65(4):1195-1209.
doi: 10.1190/1.1444812
- Rickett JE. Illumination-based normalization for wave-equation depth migration. *Geophysics*. 2003;68(4):1371-1379.
doi: 10.1190/1.1598130
- Plessix RE, Mulder WA. Frequency-domain finite-difference amplitude-preserving migration. *Geophys J Int*. 2004;157(3):975-987.
doi: 10.1111/j.1365-246X.2004.02282.x
- Guitton A. Amplitude and kinematic corrections of migrated images for non-unitary imaging operators. *Geophysics*. 2004;69(4):1017-1024.
doi: 10.1190/1.1778244
- Valenciano AA, Biondi B, Guitton A. Target-oriented wave-equation inversion. *Geophysics*. 2006;71(4):A35-A38.
doi: 10.1190/1.2213359
- Valenciano AA, Biondi BL, Clapp RG. Imaging by target-oriented wave-equation inversion. *Geophysics*. 2009;74(6):WCA109-WCA120.
doi: 10.1190/1.3250267
- Aoki N, Schuster GT. Fast least-squares migration with a deblurring filter. *Geophysics*. 2009;74(6):WCA83-WCA93.
doi: 10.1190/1.3155162
- McMechan GA. Migration by extrapolation of time-dependent boundary values. *Geophys Prospect*. 1983;31(3):413-420.
doi: 10.1111/j.1365-2478.1983.tb01060.x
- Zhou HW, Hu H, Zou Z, Wo Y, Youn O. Reverse time migration: A prospect of seismic imaging methodology. *Earth Sci Rev*. 2018;179:207-227.
doi: 10.1016/j.earscirev.2018.02.008
- Zhang Y, Duan L, Xie Y. A stable and practical implementation of least-squares reverse time migration. *Geophysics*. 2014;80(1):V23-V31.
doi: 10.1190/geo2013-0461.1
- Huang Y, Schuster GT. Multi-source least-squares migration of marine streamer and land data with frequency-division encoding. *Geophys Prospect*. 2012;60(4):663-680.
doi: 10.1111/j.1365-2478.2012.01086.x

24. Dai W, Fowler P, Schuster GT. Multi-source least-squares reverse time migration. *Geophys Prospect*. 2012;60(4):681-695.
doi: 10.1111/j.1365-2478.2012.01092.x
25. Dai W, Schuster GT. Plane-wave least-squares reverse-time migration. *Geophysics*. 2013;78(4):S165-S177.
doi: 10.1190/geo2012-0377.1
26. Zhang X, Li R, Cui L, Liu D, Liu G, Zhang Z. Least squares reverse time migration imaging with illumination preconditioned based on improved PRP conjugate gradients. *Sci Rep*. 2023;13:13623.
doi: 10.1038/s41598-023-40578-8
27. Dai W, Wang X, Schuster GT. Least-squares migration of multi-source data with a deblurring filter. *Geophysics*. 2011;76(5):R135-R146.
doi: 10.1190/geo2010-0159.1
28. Dutta G, Schuster GT. Attenuation compensation for least-squares reverse time migration using the viscoacoustic-wave equation. *Geophysics*. 2014;79(6):S251-S262.
doi: 10.1190/geo2013-0414.1
29. Yang J, Zhu H. Viscoacoustic least-squares reverse time migration using a time-domain complex-valued wave equation. *Geophysics*. 2019;84(5):S479-S495.
doi: 10.1190/geo2018-0804.1
30. Qu Y, Wang Y, Li Z, Liu C. Q least-squares reverse time migration based on the first-order viscoacoustic quasidifferential equations. *Geophysics*. 2021;86(4):S283-S299.
doi: 10.1190/geo2020-0712.1
31. Feng Z, Schuster GT. Elastic least-squares reverse time migration. *Geophysics*. 2017;82(2):S143-S157.
doi: 10.1190/geo2016-0254.1
32. Ren Z, Liu Y, Sen MK. Least-squares reverse time migration in elastic media. *Geophys J Int*. 2017;208(2):1103-1125.
doi: 10.1093/gji/ggw443
33. Mu X, Huang J, Yang J, Guo X, Guo Y. Least-squares reverse time migration in TTI media using a pure qP-wave equation. *Geophysics*. 2020;85(4):S251-S262.
doi: 10.1190/geo2019-0320.1
34. Torres K, Sacchi M. Least-squares reverse time migration via deep learning-based updating operators. *Geophysics*. 2022;87(6):S315-S333.
doi: 10.1190/geo2021-0491.1
35. Wu B, Yao G, Ma X, Chen H, Wu D, Cao J. Least-squares reverse time migration of simultaneous sources with deep-learning-based denoising. *Geophysics*. 2024;89(3):S289-S304.
doi: 10.1190/geo2023-0320.1
36. Gazdag J. Wave equation migration with the phase-shift method. *Geophysics*. 1978;43(7):1342-1351.
doi: 10.1190/1.1440899
37. Stolt RH. Migration by Fourier transform. *Geophysics*. 1978;43(1):23-48.
doi: 10.1190/1.1440826
38. Gazdag J, Sguazzero P. Migration of seismic data by phase shift plus interpolation. *Geophysics*. 1984;49(2):124-131.
doi: 10.1190/1.1441643
39. Stoffa PL, Fokkema JT, de Luna Freire RM, Kessinger WP. Split-step Fourier migration. *Geophysics*. 1990;55(4):410-421.
doi: 10.1190/1.1442850
40. Ristow D, Rühl T. Fourier finite-difference migration. *Geophysics*. 1994;59(12):1882-1893.
doi: 10.1190/1.1443575
41. Zhao H, Gelius LJ, Tygel M. A new generalized screen propagator for wave equation depth migration. In: *77th EAGE Conference and Exhibition 2015*. Houten, Netherlands: EAGE Publications BV; 2015.
doi: 10.3997/2214-4609.201412786
42. Wu RS. Wide-angle elastic wave one-way propagation in heterogeneous media and an elastic wave complex-screen method. *J Geophys Res Solid Earth*. 1994;99(B1):751-766.
doi: 10.1029/93JB02518
43. Chen SC, Ma ZT. High order generalized screen propagator for wave equation prestack depth migration. *Chin J Geophys*. 2006;49(5):1455-1465.
doi: 10.1002/cjg2.952
44. Zhang Y, Zhang G, Bleistein N. Theory of true-amplitude one-way wave equations and true-amplitude common-shot migration. *Geophysics*. 2005;70(4):E1-E10.
doi: 10.1190/1.1988182
45. Zhang Y, Xu S, Bleistein N. True-amplitude angle-domain common-image gathers from one-way wave-equation migrations. *Geophysics*. 2007;72(1):S49-S58.
doi: 10.1190/1.2399371
46. Kühl H, Sacchi MD. Least-squares wave-equation migration for AVP/AVA inversion. *Geophysics*. 2003;68(1):262-273.
doi: 10.1190/1.1543212
47. Zhou HM, Chen SC, Ren HR, Wang HC, Chen GX. One-way wave equation least-squares migration based on illumination compensation. *Chin J Geophys*. 2014;57(5):726-738.
doi: 10.1002/cjg2.20136
48. Stanton A, Sacchi MD. Elastic least-squares one-way wave-

- equation migration. *Geophysics*. 2017;82(4):S293-S305.
doi: 10.1190/geo2016-0391.1
49. Abolhassani S, Verschuur DJ. Efficient preconditioned least-squares wave-equation migration. *Geophysics*. 2024;89(3):S275-S288.
doi: 10.1190/geo2023-0048.1
50. Liu C, Sun M, Wu W, Dai N, Guo M, Fu H. An efficient point spread function inversion method for image-domain one-way wave-equation least-squares migration. *IEEE Trans Geosci Remote Sens*. 2025;63:1-16.
doi: 10.1109/TGRS.2025.3538530
51. Romano Y, Elad M, Milanfar P. The little engine that could: regularization by denoising (RED). *SIAM J Imaging Sci*. 2017;10(4):1804-1844.
doi: 10.1137/16m1102884
52. Peng G, Gong X, Hu B, Xu Z. Least-squares reverse time migration with regularization by denoising scheme. *IEEE Trans Geosci Remote Sens*. 2024;62:1-14.
doi: 10.1109/TGRS.2024.3439370
53. Nocedal J, Wright SJ. *Numerical Optimization*. New York, NY: Springer; 2006.
doi: 10.1007/978-0-387-40065-5
54. Sun R, You J, Li Z, *et al*. Viscoacoustic full wavefield migration and its application. *IEEE Trans Geosci Remote Sens*. 2025;63:1-14.
doi: 10.1109/TGRS.2025.3574991
55. Hore A, Ziou D. Image Quality Metrics: PSNR vs. SSIM. In: *2010 20th International Conference on Pattern Recognition*. New York, NY: IEEE; 2010:2366-2369.
doi: 10.1109/ICPR.2010.579

Appendix

Appendix A. Wavefield propagation accuracy in a laterally varying velocity model

To further evaluate the propagation accuracy of the generalized-screen propagator (GSP) and matrix-decomposition (MD) operators in laterally heterogeneous media, an additional wavefield comparison is conducted using the velocity model shown in Figure A1A. The velocity varied continuously in both the horizontal and vertical directions, ranging approximately from 2,000 to 6,000 m/s. The model spanned 2,000 m horizontally and 2,000 m in depth, with a spatial grid spacing of 10 m in both directions. The temporal sampling interval was 0.001 s, and the total propagation time was 1.0 s. A 30-Hz Ricker wavelet was used, and the shot point was located at $x = 1,000$ m on the model surface. Wavefield snapshots calculated using the finite-difference (FD), GSP, and MD methods were compared at the same propagation time.

Figure A1B shows the FD wavefield snapshot, which serves as the numerical reference. The red dashed curve traces the principal positive-amplitude wavefront and is superimposed on the GSP and MD results for direct comparison. As shown in Figure A1C, the GSP method reproduced the overall wavefront geometry but exhibited noticeable deviations along the lateral portions, together with local wavefield distortions and weak spurious artifacts. In contrast, the MD result in Figure A1D agrees more closely with the FD reference, showing smaller wavefront-position discrepancies and fewer artifacts. These results indicate that the MD operator provides more accurate wavefield propagation in laterally varying media and further supports its use in the proposed LS-MD imaging framework.

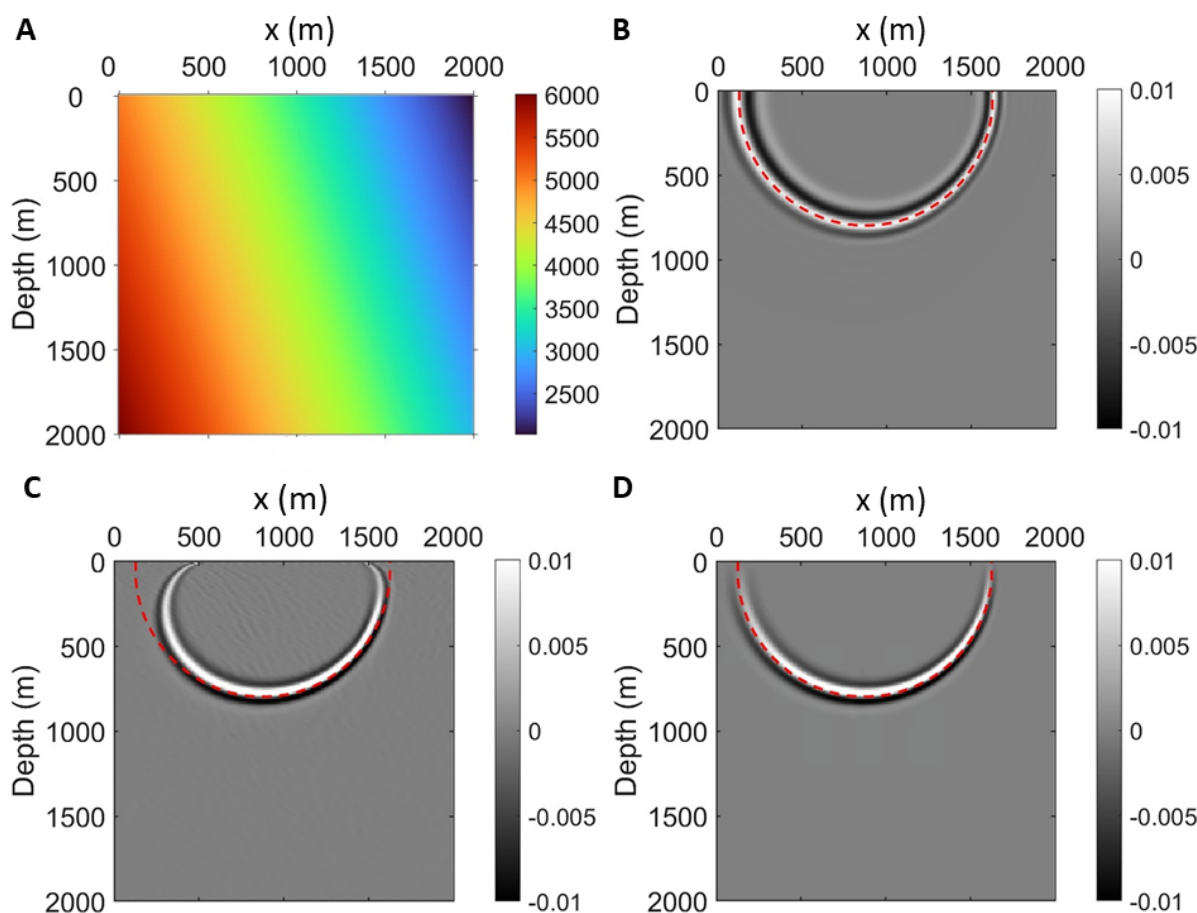


Figure A1. Comparison of wavefield propagation in a laterally varying velocity model. (A) Velocity model. (B) Wavefield snapshot calculated using the finite-difference (FD) method. (C) Wavefield snapshot calculated using the generalized-screen propagator (GSP). (D) Wavefield snapshot calculated using the matrix-decomposition (MD) propagator. The red dashed curve denotes the reference wavefront identified from the FD result and superimposed on the wavefield snapshots to facilitate comparison of the wavefront positions.