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Reverse-learning-enhanced sine cosine algorithm for joint inversion of seismic refraction and Rayleigh-wave dispersion data

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Abstract

Rayleigh-wave dispersion and seismic refraction observations constrain different but coupled aspects of near-surface velocity structure. This study evaluates a reverse-learning-enhanced sine cosine algorithm (ESCA) for jointly estimating layer thickness, shear-wave velocity, and P-wave velocity in a one-dimensional, horizontally layered medium. Reverse learning is added during Tent-map initialization, while the best-solution-centered exponential update follows an earlier sine cosine formulation. The dimensionless joint objective combines root-mean-square relative residuals of fundamental-mode phase velocities and first-arrival times. ESCA, the standard sine cosine algorithm (SCA), and particle swarm optimization (PSO) were evaluated in 30 independent runs using multimodal benchmark functions, synthetic data generated from a model comprising 4 horizontal layers, and previously published field datasets from the Beşirli and MAE sites in Türkiye. The mean final joint-objective values for ESCA, SCA, and PSO, respectively, were 0.0028, 0.0070, and 0.0068 for noise-free synthetic data; 0.0336, 0.0356, and 0.0382 for synthetic data with 10% noise; 0.0343, 0.0393, and 0.0404 for the Beşirli dataset; and 0.0717, 0.0740, and 0.0722 for the MAE dataset. The corresponding runtimes for noise-free synthetic data were 145.32 ± 3.46 , 137.89 ± 5.64 , and 210.33 ± 4.65 s, respectively. ESCA required 0.99% more objective-function evaluations than the baseline algorithms, although its improvement over PSO for the MAE dataset was small. The field-data results demonstrate consistency within the assumed one-dimensional model but do not account for dipping interfaces or lateral variations in velocity.

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Citation: Wang C, Le Z, Huo Z. Reverse-learning-enhanced sine cosine algorithm for joint inversion of seismic refraction and Rayleigh-wave dispersion data. *J Seismic Explor.*
doi: 10.36922/JSE026290140

Received: July 14, 2026**Revised:** October 13, 2026**Accepted:** October 13, 2026**Published online:** October 28, 2026

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Keywords: Rayleigh-wave dispersion; Seismic refraction; Joint inversion; Sine cosine algorithm; Near-surface characterization

1. Introduction

Near-surface seismic surveys commonly use surface waves and refracted body waves to describe complementary parts of the subsurface model. Active-source Rayleigh-wave analysis is particularly sensitive to shear-wave velocity (V_s) and is therefore widely used in geotechnical and engineering characterization.¹⁻⁴ Seismic refraction, by comparison, primarily constrains P-wave velocity (V_p) and refractor geometry. Combining these sensitivities offers a route to a more coherent elastic model than either dataset can generally provide alone.

Dispersion inversion is a nonlinear, high-dimensional problem.⁵⁻⁷ Deterministic algorithms can be efficient, but their outcome can depend on the starting model and local minima. Data-driven studies address model-space design⁸ and staged inversion.⁹ Yang *et al.*¹⁰ selected 500 training examples from an initial pool of 5,000 synthetic layered-earth model–dispersion pairs; each example consists of a candidate model and its forward-calculated dispersion curve. These counts refer to training examples, not receivers or frequency samples. Preprocessing can improve the input observations,¹¹ but model parameterization, weighting, and validation remain necessary.

Refraction data have been inverted by linearized schemes^{12,13} and population-based searches.^{14,15} A specific difficulty is the hidden-layer condition: a thin low-velocity unit, or acquisition geometry that undersamples it, may leave no identifiable first arrival. Refraction picks alone can therefore produce an incomplete or physically implausible velocity sequence.¹⁶

Joint inversion addresses these limitations by requiring a common earth model to explain datasets with different parameter sensitivities. Surface-wave-derived V_s information has been used to constrain refraction tomography,¹⁷ and one-dimensional formulations have combined Rayleigh-wave dispersion with P-wave travel times.¹⁸ Other examples include multi-objective particle swarm optimization (PSO) for simultaneous refraction and surface-wave inversion¹⁹ and iterative integration of first-arrival traveltimes tomography with multichannel surface-wave analysis.²⁰ A recent study applied an improved sparrow search algorithm to the same refraction–dispersion problem.²¹ Collectively, these studies show the value of complementary constraints but also demonstrate that the recovered model depends on the search strategy and on how the two residual terms are balanced.

Population-based searches explore a bounded model space without requiring derivatives. Here, each model parameter has finite lower and upper limits supplied by prior information. Their geophysical applications span several search families.^{14,15,22-30} Performance depends on the bounds, update rules, dimensionality, and information content of the data.³¹ We compare the proposed enhanced sine cosine algorithm (ESCA) with the standard sine cosine algorithm (SCA) and PSO.

The principal algorithmic trade-off is between exploration and exploitation. Broad sampling helps preserve population diversity and reduces the chance of missing distant basins of attraction, whereas local refinement is needed to improve promising models. A search that remains too exploratory converges slowly; one that contracts too early may stagnate around a local

solution. Population initialization and the iteration-dependent update rule are therefore the two elements targeted in this study.

The standard SCA³² updates candidate models by alternating sine and cosine operators. An earlier improved formulation³³ used Tent-map initialization and a best-solution-centered exponential update for joint inversion of Rayleigh-wave dispersion and vertical electrical sounding data. We retain those components and add reverse learning during initialization for the coupled dispersion–refraction problem. The extension changes the observed quantities and parameter sensitivities and is evaluated here against standard SCA and PSO.

The study has three objectives. Component-wise benchmark tests isolate the effects of Tent mapping, reverse learning, and the revised update rule. Synthetic experiments compare ESCA, SCA, and PSO for individual-data and joint inversion under identical noise-free and noisy conditions. Two published Turkish datasets test the consistency and repeatability of the recovered V_s , V_p , and layer-thickness profiles within a one-dimensional parameterization. The Discussion evaluates the implications of that parameterization, including dipping interfaces, lateral heterogeneity, and dispersion-mode selection.

2. Sine cosine algorithm and the proposed enhanced sine cosine algorithm

In the standard SCA,³² each candidate model may be initialized from prior information or, when such information is unavailable, sampled randomly within the prescribed bounds. Subsequent positions are generated with the sine- and cosine-based rule in **Equation 1**:

$$x_i^{t+1} = \begin{cases} x_i^t + r_1 \times \sin(r_2) \times |r_3 p_i^t - x_i^t|, & r_4 < 0.5 \\ x_i^t + r_1 \times \cos(r_2) \times |r_3 p_i^t - x_i^t|, & r_4 > 0.5 \end{cases} \quad (1)$$

In **Equation 1**, x_i^t is the value of the i -th model parameter for the current candidate at iteration t , and p_i^t is the corresponding component of the best model retained at that iteration. The independent random numbers r_2 , r_3 , and r_4 are sampled from $[0, 2\pi]$, $[0, 2]$, and $[0, 1]$, respectively. Parameter r_4 selects the sine or cosine branch, r_2 controls the displacement direction and magnitude, and r_3 randomly scales the attraction towards the current best solution. The amplitude r_1 decreases according to **Equation 2**:

$$r_1 = a - t \frac{a}{T} \quad (2)$$

Here, t and T denote the current and maximum iteration numbers. The constant a sets the initial magnitude of r_1 ; $a = 2$ is used for the standard SCA tests, consistent with Ref.²⁸ At $t = T$, **Equation 2** gives $r_1 = 0$. This creates no singularity in **Equation 1**: the displacement becomes zero, the current candidate is retained, and the run terminates after the final evaluation.

The ESCA implementation follows the improved sine cosine formulation in Ref.³³ Tent-map initialization and the exponential, best-solution-centered update are retained. The added element is reverse learning during initialization, combined here with an objective for coupled refraction and Rayleigh-wave inversion.

Random initialization can leave parts of a bounded search domain poorly represented. ESCA instead generates a Tent chaotic sequence, whose distribution is comparatively uniform and whose deterministic recurrence promotes broader initial coverage.³⁴ To avoid short or unstable cycles, the perturbed recurrence proposed in Ref.³⁵ is used, as given in **Equation 3**:

$$x_{n+1} = \begin{cases} \frac{1}{p}(x_n + \frac{1}{N} \times \text{rand}(0,1)), & 0 \leq x_n \leq p \\ \frac{1}{p}(1 - x_n + \frac{1}{N} \times \text{rand}(0,1)), & p \leq x_n \leq 1 \end{cases} \quad (3)$$

In **Equation 3**, $p = 0.5$ is the Tent-map breakpoint, N is the sequence length, and $\text{rand}(0,1)$ is a uniformly distributed random number. The perturbation prevents repeated short cycles. Each normalized chaotic value is then transformed to the physical bounds of the corresponding model parameter using **Equation 4**.

$$x_{\text{new}} = \min + (\max - \min)x_{\text{old}} \quad (4)$$

In **Equation 4**, $\min$ and $\max$ are the lower and upper bounds of a model parameter. After mapping N chaotic candidates to the search domain, one opposite candidate is constructed for each using the reverse-learning relation in **Equation 5**.³⁵ ESCA therefore evaluates $2N$ candidates during initialization and retains the best N as the starting population. This additional initial set expands the portion of the domain examined before the first iteration and reduces dependence on a single chaotic realization.

$$x_{\text{new}2} = \min + \max - x_{\text{old}} \quad (5)$$

After initialization, ESCA updates each candidate with the best-solution-centered rule in **Equation 6**:

$$x_i^{t+1} = \begin{cases} p_i^t + r_1 \times \sin(r_2) \times |r_3 p_i^t - x_i^t|, & r_4 < 0.5 \\ p_i^t + r_1 \times \cos(r_2) \times |r_3 p_i^t - x_i^t|, & r_4 > 0.5 \end{cases} \quad (6)$$

The search amplitude r_1 follows the exponential schedule in **Equation 7**, adopted from the earlier improved sine cosine formulation:³³

$$r_1 = \exp[-(\frac{t}{T})], \quad t = 1, 2, 3, \dots, T \quad (7)$$

Equations 1 and **6** differ in their reference points. Standard SCA applies its trigonometric displacement to the current candidate x_i^t , whereas ESCA forms the new candidate around the best retained position p_i^t . **Equation 7** then contracts the admissible step magnitude as t/T increases. The resulting search is broad at early iterations and increasingly focused near the best model later in the run. **Algorithm 1** summarizes the standard SCA workflow.

Algorithm 1: Standard SCA workflow

- 01 Specify parameter bounds, population size N , and maximum iteration count T .
- 02 Sample N candidate models within the bounds.
- 03 Evaluate the objective function for every candidate and retain the best-so-far model.
- 04 For $t = 1, \dots, T$:
 - 05 Compute r_1 from Equation 2 and draw r_2, r_3 , and r_4 .
 - 06 Move each candidate according to Equation 1.
 - 07 Enforce the parameter bounds and reevaluate the population.
 - 08 Update the best-so-far model.
- 09 End for.
- 10 Return the retained model and its objective value.

For SCA, the initial candidates are evaluated once, the best model is retained, and all candidates are moved with **Equation 1**. The linear schedule in **Equation 2** decreases r_1 until the stopping iteration T is reached. This baseline is kept unchanged in the comparisons. **Algorithm 2** gives the corresponding ESCA sequence.

Algorithm 2: ESCA workflow

- 01 Specify parameter bounds, retained population size N , and maximum iteration count T .
- 02 Generate N candidates with the perturbed Tent map and N opposite candidates by reverse learning.
- 03 Evaluate all $2N$ candidates, retain the best N , and identify the initial best-so-far model.
- 04 For $t = 1, \dots, T$:
 - 05 Draw r_2, r_3 , and r_4 and compute r_1 from **Equation 7**.
 - 06 Update every retained candidate around the best model using **Equation 6**.
 - 07 Regenerate any out-of-bounds component uniformly within its admissible interval.
 - 08 Evaluate N candidates and update the best-so-far model.
 - 09 End for.
 - 10 Return the retained model and its objective value.

The ESCA first evaluates N Tent-map candidates and their N reverse-learning counterparts, then retains the

best N candidates. Each subsequent iteration evaluates N updated candidates. With $N = 300$ and $T = 100$, SCA and PSO each require $N(T + 1) = 30,300$ objective-function evaluations, whereas ESCA requires $N(T + 2) = 30,600$, an increase of 0.99%. The comparison therefore matches retained population size and iteration count, but not exactly the total evaluation budget. The stopping criterion is the fixed limit T ; any out-of-bounds component is regenerated uniformly within its admissible interval. The convergence plots show the arithmetic mean of the best-so-far objective over all 30 runs at each iteration. The 30-run summaries characterize repeatability; they do not establish paired-seed or exactly evaluation-budget-matched performance differences.

3. Benchmark-function assessment

Four two-dimensional multimodal functions provide component-wise ablation tests (Table 1). Standard SCA,

three partially modified variants, and ESCA were compared over 30 runs with common population and iteration limits. ESCA achieved the best or joint-best mean for all functions and generally the smallest standard deviation. Synthetic and field tests provide the application-level assessment.

4. Synthetic experiments

The synthetic tests use the four-layer model in Table 2, comprising three homogeneous, isotropic, horizontal layers over a homogeneous half-space.³⁶ Elastic properties vary only with depth; the free surface and interfaces are planar and horizontal, and no dip or lateral anomaly is included. Rayleigh-wave dispersion primarily constrains V_s and layer thickness, while refraction first arrivals constrain V_p and interface depths. Both forward calculations use this same one-dimensional structure. This controlled model permits comparison of the optimizers and complementary data constraints within the assumed physics. It does not test

Table 1. Benchmark performance of SCA, 3 partially modified SCA variants, and ESCA across 30 independent runs

Name	Expression	Search space	Global optimum	SCA	SCA + Tent map	SCA + reverse learning	SCA + best-centered update	ESCA
Ackley	$-20\exp[-0.2\sqrt{\frac{1}{d}\sum_{i=1}^d x_i^2}]$ $-\exp[\frac{1}{d}\sum_{i=1}^d \cos(2\pi x_i)]$ $+20 + \exp(1)$	[-5, 5]	0	3.05×10^{-7}	13.43×10^{-9}	7.04×10^{-10}	6.23×10^{-13}	8.88×10^{-16}
				1.66×10^{-6}	0.1125	0	0	0
Rastrigin	$10d + \sum_{i=1}^d [x_i^2 - 10\cos(2\pi x_i)]$	[-5.12, 5.12]	0	0.0007	0.5413	0.2779	0	0
				0.0080	0.2297	0	0	0
Eggholder	$-(x_2 + 47)\sin\sqrt{ x_2 + \frac{x_1}{2} + 47 }$ $-x_1\sin\sqrt{ x_1 - (x_2 + 47) }$	[-512, 512]	-959.6407	-873.3654	-636.7226	-624.7731	-827.7267	-957.7331
				47.8321	7.6379	5.7892	3.0600	9.6074
Schwefel	$418.9829d - \sum_{i=1}^d x_i \sin\sqrt{ x_i }$	[-500, 500]	0	68.2483	0.8525	0.0154	0.0086	0.0038
				47.6352	0.3267	0.4569	0.7801	0.0004

Notes: $N = 300$; $T = 100$; $d = 2$. For each method, the upper and lower entries represent the mean and standard deviation, respectively, across 30 independent runs. The search intervals represent conventional benchmark domains; changing these intervals changes the optimization problem. In the expressions, d denotes the number of dimensions, x_i denotes coordinate the i th coordinate, and $\exp$ denotes the natural exponential function, with $\exp(1) = e$ (Euler's number). Coordinates and objective values are dimensionless.

Abbreviations: ESCA: Enhanced sine cosine algorithm; SCA: Sine cosine algorithm.

recovery of a dipping or laterally heterogeneous structure. The normalized joint objective (*Obj*) combines the residual terms as follows:

$$\text{Obj}(h, V_s, V_p) = w_1 \sqrt{\sum_{i=1}^{M_1} \frac{[V_{\text{obs}}^r(f_i) - V_{\text{cal}}^r(f_i, h, V_s, V_p)]^2}{V_{\text{obs}}^r(f_i)^2}} + w_2 \sqrt{\sum_{i=1}^{M_2} \frac{[t_{\text{obs}}(x_i) - t_{\text{cal}}(x_i, h, V_p)]^2}{t_{\text{obs}}(x_i)^2}} \quad (8)$$

The symbols in **Equation 8** are defined below.

- V_{obs}^r and V_{cal}^r are the observed and calculated Rayleigh wave phase velocities.
- M_1 represents the total number of frequency samples.
- t_{obs} and t_{cal} are the observed and calculated first-arrival times, respectively.
- M_2 is the number of refraction picks.
- w_1 and w_2 are the weighting factors for the dispersion and first-arrival residual terms, respectively.

Only the fundamental Rayleigh-wave mode was considered; consequently, the dispersion term in **Equation 8** contains one residual per frequency sample, without summation over multiple modes.

To assign equal weight to the two datasets while accounting for their different numbers of observations, w_1 is set to one half divided by the square root of M_1 , and w_2 is set to one half divided by the square root of M_2 . Accordingly, **Equation 8** represents an equally weighted sum of the root-mean-square relative residuals of the two datasets. Each residual divides a difference by an observation with the same physical unit, so both terms and the final joint objective are dimensionless; the numerical values in the Abstract and objective-history figures are not errors in m/s or s. Observation-specific uncertainty weights were not used because pointwise standard deviations were unavailable. Here we report the combined normalized objective, not separate physical-unit RMSEs or individual residual-component histories.

Density is assigned from the empirical relations in

Refs.^{37,38} Rayleigh-wave phase velocities are evaluated for a horizontally layered medium using the dispersion calculation described in Ref.³⁹ and first-arrival times are calculated by layered-earth ray tracing.¹⁵ Neither forward calculation includes dip or lateral velocity variations. The corresponding applicability limits are discussed in Section 6.

4.1. Noise-free synthetic experiment

Sine cosine algorithm, ESCA, and PSO were first compared using exact synthetic observations. The dispersion calculation is restricted to the fundamental Rayleigh mode. PSO uses inertia $w = 0.729$, individual attraction $c_1 = 2.041$, and social attraction $c_2 = 0.948$.⁵

Phase velocities were sampled from 5 to 80 Hz at 1 Hz intervals ($M_1 = 76$). Refraction travel times were generated at offsets 5, 7, ..., 49 m, giving $M_2 = 23$ picks at 2 m spacing. All optimizers used a retained population of 300 candidates and 100 iterations; the resulting evaluation counts are stated in Section 2. Thirty independent inversions were performed for each stochastic method, and parameter means and standard deviations summarize central tendency and run-to-run variation.

For each optimizer, the same bounds, retained population size, and iteration limit were also used in two individual-data inversions, one based only on the fundamental-mode dispersion curve and the other only on refraction arrivals. This design isolates the effect of coupling the datasets. **Figure 1** shows that all three methods reproduce the exact observations closely, so data fit alone does not reveal the differences among the recovered models.

The recovered velocity-depth models in **Figure 2** reveal clearer differences among the optimization algorithms. ESCA produces V_s and V_p profiles closest to the prescribed

Table 2. True model parameters and inversion bounds for the four-layer synthetic model

Layers	Model parameters				Model space		
	V_s (m/s)	V_p (m/s)	ρ (g/cm ³)	h (m)	V_s (m/s)	h (m)	V_p (m/s)
1	220	350	1.8	3	100~350	1~5	100~600
2	420	700	1.7	12	200~650	5~20	300~1,100
3	260	450	1.8	5	100~400	2~8	200~700
Half-space	710	1,200	1.9	∞	300~1,100	∞	600~2,000

Notes: The one-dimensional model consists of three finite, horizontal layers overlying a half-space. Density values were estimated using the empirical relationships described by Xia *et al.*³⁷ and Xia *et al.*³⁸ and held constant during inversion. The half-space is assumed to have infinite thickness and therefore has no thickness bounds.

four-layer structure for both inversion strategies. The SCA and PSO estimates show larger departures at several interfaces and velocity levels despite their acceptable response fits.

For ESCA, coupling dispersion and refraction data reduces the deviations observed in the individual-data inversions. The improvement is less consistent for SCA and PSO. This contrast illustrates the non-uniqueness of the inverse problem: similar response curves can be produced by models that differ appreciably, and complementary observations are most useful when the optimizer explores the common model space effectively.

Figure 3 compares the 30-run mean best-so-far objective histories. After 100 iterations, the final mean objective is 0.0028 for ESCA, 0.0070 for SCA, and 0.0068 for PSO. The early decreases are similar, but ESCA continues to improve and terminates at the smallest value. The higher SCA and PSO plateaus are consistent with the larger model errors in Figure 2 and suggest stagnation in less favorable regions of the search space.

Computational time was measured on the theoretical noise-free four-layer model. Across 30 runs, mean $\pm$

standard deviation per inversion was 145.32 ± 3.46 s for ESCA, 137.89 ± 5.64 s for SCA, and 210.33 ± 4.65 s for PSO. ESCA was 7.43 s (5.39%) slower than SCA but 65.01 s (30.91%) faster than PSO in this setting; these timings should not be generalized across implementations.

The 30-run parameter statistics in Table 3 provide the quantitative comparison. Relative errors and standard deviations are generally smaller for ESCA, indicating that its improved model recovery is repeatable rather than the result of a single favorable realization.

Thus, in the absence of observational noise, the main advantage of ESCA is not merely a closer curve fit but a more accurate and less variable reconstruction of the shared model of V_s , V_p , and layer thickness. The comparison also confirms that the joint data constraint is more informative than either dataset used separately.

4.2. Noise-contaminated synthetic experiment

A second experiment perturbs both synthetic datasets at a nominal 10% level to test sensitivity to measurement error. The random-noise model follows Equation 9:^{5,6}

$$d_i^{\text{noisy}} = d_i + \eta \bar{d}(u_i - v_i), \quad \eta = 0.10 \quad (9)$$

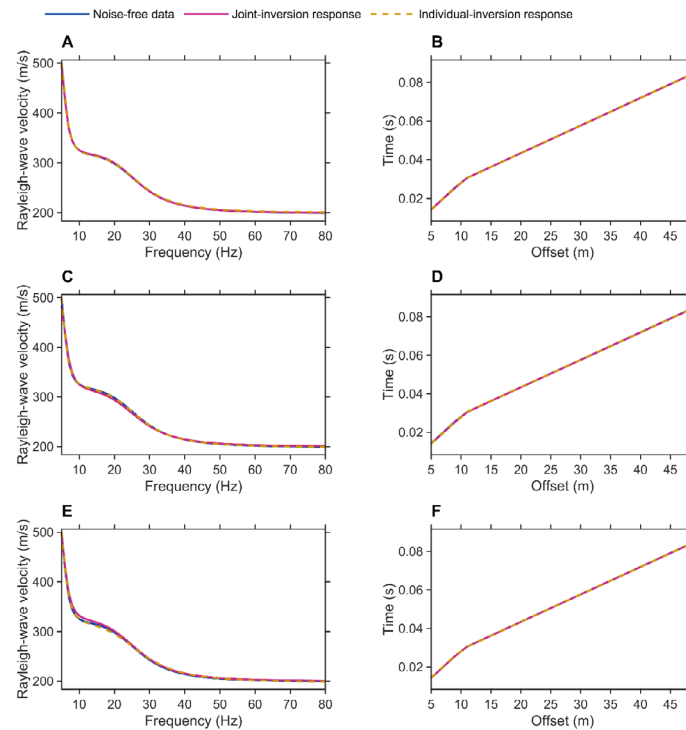


Figure 1. Fits to the noise-free synthetic observations obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Blue solid curves denote the noise-free input data, pink solid curves the joint-inversion responses, and gold dashed curves the corresponding individual-data inversion responses. (A) Rayleigh-wave dispersion fit obtained with ESCA. (B) Refraction first-arrival time fit obtained with ESCA. (C) Rayleigh-wave dispersion fit obtained with SCA. (D) Refraction first-arrival time fit obtained with SCA. (E) Rayleigh-wave dispersion fit obtained with PSO. (F) Refraction first-arrival time fit obtained with PSO.

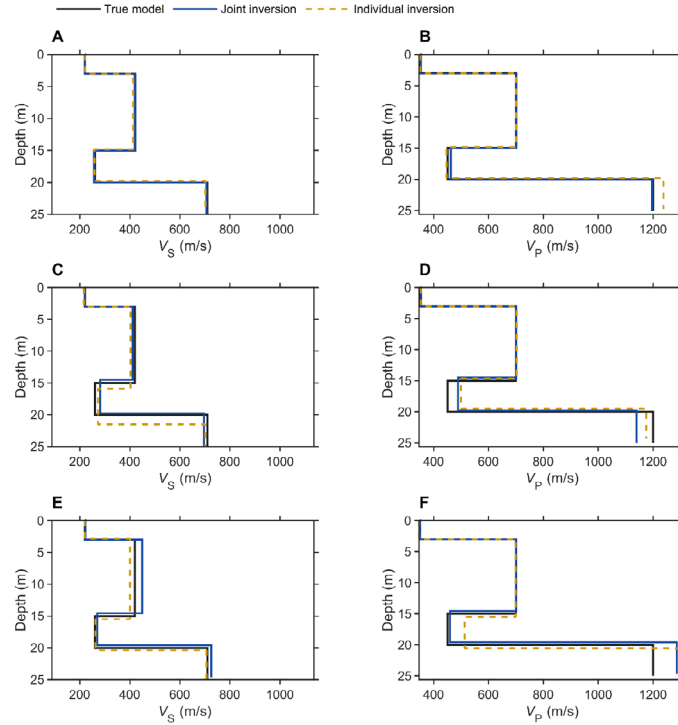


Figure 2. Velocity-depth models for the noise-free synthetic test obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Black solid curves indicate the true model, blue solid curves the joint-inversion solutions, and gold dashed curves the individual-inversion solutions. (A) Shear-wave velocity (V_s) profile obtained with ESCA. (B) P-wave velocity (V_p) profile obtained with ESCA. (C) V_s profile obtained with SCA. (D) V_p profile obtained with SCA. (E) V_s profile obtained with PSO. (F) V_p profile obtained with PSO. These are depth-dependent profiles for horizontally layered models; no lateral coordinate or interface dip is inverted.

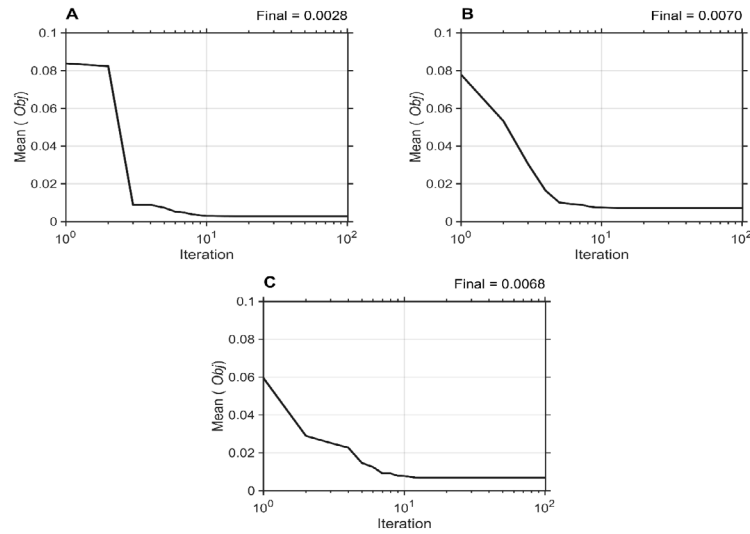


Figure 3. Mean best-so-far objective histories over 30 independent noise-free joint-inversion runs: (A) enhanced sine cosine algorithm, (B) sine cosine algorithm, and (C) particle swarm optimization. The plotted joint objective is dimensionless. The final mean objective is indicated above each panel.

In **Equation 9**, d_i is an unperturbed observation (phase velocity in m/s or first-arrival time in s); d_i^{noisy} is its perturbed counterpart in the same unit. The overbar

is the mean of the corresponding unperturbed dataset; η is the dimensionless noise fraction 0.10; and u_i and v_i are independent uniform draws on $[0, 1]$ for sample index i .

Table 3. Parameter-recovery statistics from 30 independent joint inversions using noise-free and 10% noisy synthetic data.

Algorithm	Parameters	True value	Noise-free data			10% noisy data		
			Mean	RE	SD	Mean	RE	SD
ESCA	$V_{s1}/(\text{m}\cdot\text{s}^{-1})$	220	220.1	0.0032	1.92	220.7	0.0033	0.20
	$V_{s2}/(\text{m}\cdot\text{s}^{-1})$	420	421.1	0.0049	2.91	438.7	0.0445	0.66
	$V_{s3}/(\text{m}\cdot\text{s}^{-1})$	260	256.3	0.0140	2.60	248.1	0.0457	6.79
	$V_{s4}/(\text{m}\cdot\text{s}^{-1})$	710	710	0.0314	9.22	700.6	0.0131	11.49
	$h_1/(\text{m})$	3	3.0	0.0162	0.05	2.9	0.0199	0.08
	$h_2/(\text{m})$	12	11.9	0.0080	0.13	11.8	0.0388	0.66
	$h_3/(\text{m})$	5	5.0	0.0075	0.01	4.9	0.0104	0.07
	$V_{p1}/(\text{m}\cdot\text{s}^{-1})$	350	352.9	0.0084	1.92	342.5	0.0212	7.74
	$V_{p2}/(\text{m}\cdot\text{s}^{-1})$	700	701.7	0.0025	1.21	706.3	0.0090	0.39
	$V_{p3}/(\text{m}\cdot\text{s}^{-1})$	450	462.3	0.0289	18.41	452.6	0.0348	22.15
	$V_{p4}/(\text{m}\cdot\text{s}^{-1})$	1,200	1,196.6	0.0214	36.39	1,212.2	0.0102	10.80
SCA	$V_{s1}/(\text{m}\cdot\text{s}^{-1})$	220	221.2	0.0055	0.63	219.0	0.0043	0.46
	$V_{s2}/(\text{m}\cdot\text{s}^{-1})$	420	410.8	0.0218	5.15	428.4	0.0202	4.17
	$V_{s3}/(\text{m}\cdot\text{s}^{-1})$	260	280.6	0.0792	23.45	266.2	0.0407	14.99
	$V_{s4}/(\text{m}\cdot\text{s}^{-1})$	710	696.2	0.0194	3.55	642.6	0.0948	18.06
	$h_1/(\text{m})$	3	3.0	0.0165	0.05	2.9	0.0262	0.02
	$h_2/(\text{m})$	12	11.4	0.0437	0.62	11.2	0.0662	1.12
	$h_3/(\text{m})$	5	5.2	0.0506	0.02	5.4	0.0822	0.11
	$V_{p1}/(\text{m}\cdot\text{s}^{-1})$	350	354.5	0.0128	0.80	342.0	0.0228	4.76
	$V_{p2}/(\text{m}\cdot\text{s}^{-1})$	700	701.2	0.0038	3.85	705.4	0.0077	1.65
	$V_{p3}/(\text{m}\cdot\text{s}^{-1})$	450	488.3	0.0852	42.92	494.6	0.0992	6.70
	$V_{p4}/(\text{m}\cdot\text{s}^{-1})$	1,200	1,140.1	0.0678	115.21	1,275.5	0.0629	62.38
PSO	$V_{s1}/(\text{m}\cdot\text{s}^{-1})$	220	221.5	0.0070	0.71	218.3	0.0122	3.80
	$V_{s2}/(\text{m}\cdot\text{s}^{-1})$	420	448.9	0.0690	16.73	425.4	0.0130	3.73
	$V_{s3}/(\text{m}\cdot\text{s}^{-1})$	260	269.2	0.0970	35.67	263.8	0.0585	21.53
	$V_{s4}/(\text{m}\cdot\text{s}^{-1})$	710	725.4	0.1036	104.08	724.2	0.0200	10.38
	$h_1/(\text{m})$	3	2.9	0.0020	0.01	2.9	0.0941	0.39
	$h_2/(\text{m})$	12	11.5	0.0613	1.04	11.1	0.0714	1.12
	$h_3/(\text{m})$	5	5.0	0.0348	0.24	4.4	0.1002	0.32
	$V_{p1}/(\text{m}\cdot\text{s}^{-1})$	350	350.0	0.0041	2.06	347.0	0.0523	25.92
	$V_{p2}/(\text{m}\cdot\text{s}^{-1})$	700	700.2	0.0009	0.97	694.0	0.0126	12.54
	$V_{p3}/(\text{m}\cdot\text{s}^{-1})$	450	459.0	0.1320	84.06	401.5	0.1077	51.78
	$V_{p4}/(\text{m}\cdot\text{s}^{-1})$	1,200	1,286.9	0.0724	3.3	1,255.7	0.0652	110.73

Notes: Values are based on 30 independent inversion runs for each optimization method and noise condition. Mean represents the mean estimated parameter value across runs; RE represents the mean absolute relative error calculated for individual runs; and SD represents the standard deviation of the estimated parameter values across runs. The half-space is designated as Layer 4. RE is dimensionless, whereas SD is expressed in the units of the corresponding parameter.

Abbreviations: ESCA: Enhanced sine cosine algorithm; PSO: Particle swarm optimization; SCA: Sine cosine algorithm.

The optimizer settings, bounds, and individual-versus-joint test design are identical to those of Section 4.1.

All three optimizers reproduce the broad trends of the perturbed dispersion and refraction observations (Figure 4). The corresponding velocity models, however, are more strongly affected than the response curves. Figure 5 shows that SCA and PSO depart substantially from the prescribed structure in some layers, demonstrating again that an acceptable fit does not guarantee a reliable parameter estimate.

The ESCA provides the closest overall reconstruction of the synthetic V_s and V_p profiles. Noise also makes the interaction between the two residual terms more visible: the joint and separate solutions no longer coincide as closely as in the exact-data experiment because perturbations in one dataset can influence the common layer parameters.

The 30-run mean histories in Figure 6 terminate at 0.0336 for ESCA, 0.0356 for SCA, and 0.0382 for PSO, all higher than their noise-free counterparts. Their overall shapes remain comparable, and ESCA again reaches the smallest final mean objective.

Noise generally increases model-parameter error, but its effect on run-to-run dispersion is parameter dependent: several standard deviations decrease rather than increase (Table 3). For each parameter, RE is the mean over 30 runs of the absolute run-wise deviation from the true value divided by that true value. This definition explains why a rounded mean can equal the true value while RE remains nonzero, as for ESCA V_{s4} in the noise-free case. ESCA nevertheless shows smaller errors and variability for most, but not all, parameters.

5. Field-data applications

The first field example reanalyzes the published Beşirli dataset of Senkaya *et al.*⁴⁰ rather than presenting a new acquisition. The site lies near the Black Sea coast in Trabzon (Ortahisar), Türkiye; Figure 7 provides the updated geographic context for the Beşirli and MAE field examples. The original survey used a 24-channel PASI 16S24-U system and 4.5 Hz vertical geophones, with a 1 ms sampling interval, a 1,024 ms record, 2 m receiver spacing, and a 46 m spread. A 10 kg hammer struck a hard plastic plate five times at each shot point, and the 8 m

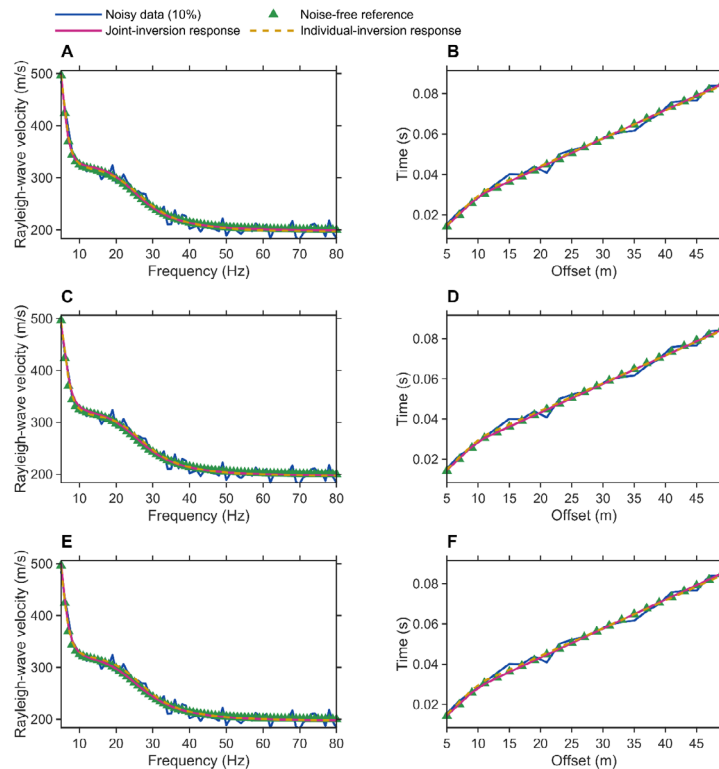


Figure 4. Response fits for the synthetic test with 10% noise obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Blue solid curves denote perturbed observations, green triangles denote the noise-free reference data, pink solid curves denote the joint-inversion responses, and gold dashed curves denote the corresponding individual-data inversion responses. (A) Rayleigh-wave dispersion fit obtained with ESCA. (B) Refraction first-arrival time fit obtained with ESCA. (C) Rayleigh-wave dispersion fit obtained with SCA. (D) Refraction first-arrival time fit obtained with SCA. (E) Rayleigh-wave dispersion fit obtained with PSO. (F) Refraction first-arrival time fit obtained with PSO.

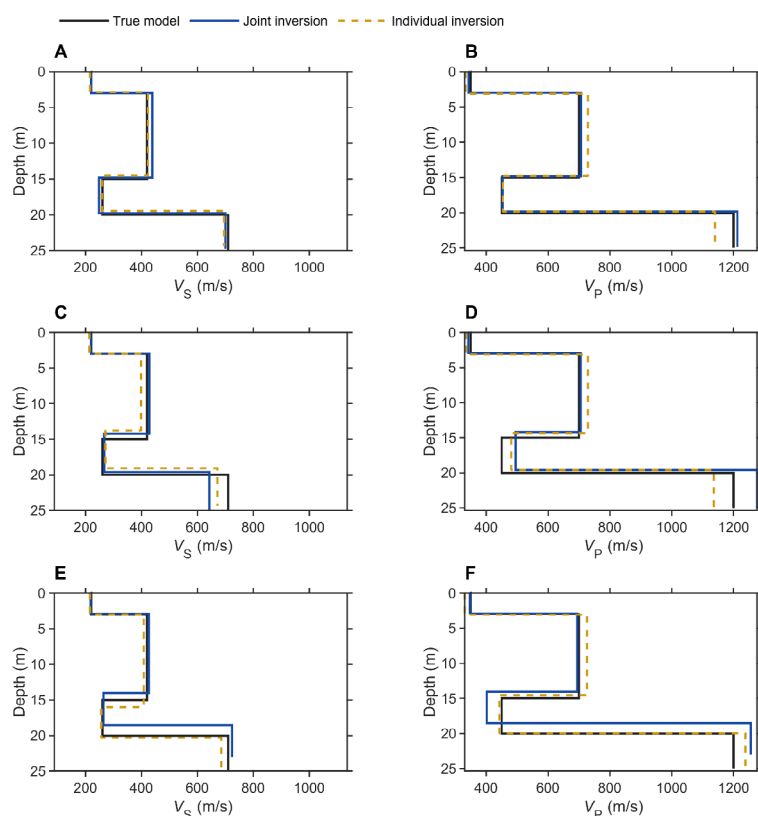


Figure 5. Velocity-depth models recovered from the synthetic data with 10% noise using the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Black solid curves show the true structure, blue solid curves the joint-inversion solutions, and gold dashed curves the corresponding individual-inversion solutions. (A) Shear-wave velocity (V_s) profile obtained with ESCA. (B) P-wave velocity (V_p) profile obtained with ESCA. (C) V_s profile obtained with SCA. (D) V_p profile obtained with SCA. (E) V_s profile obtained with PSO. (F) V_p profile obtained with PSO. These are depth-dependent profiles for horizontally layered models; no lateral coordinate or interface dip is inverted.

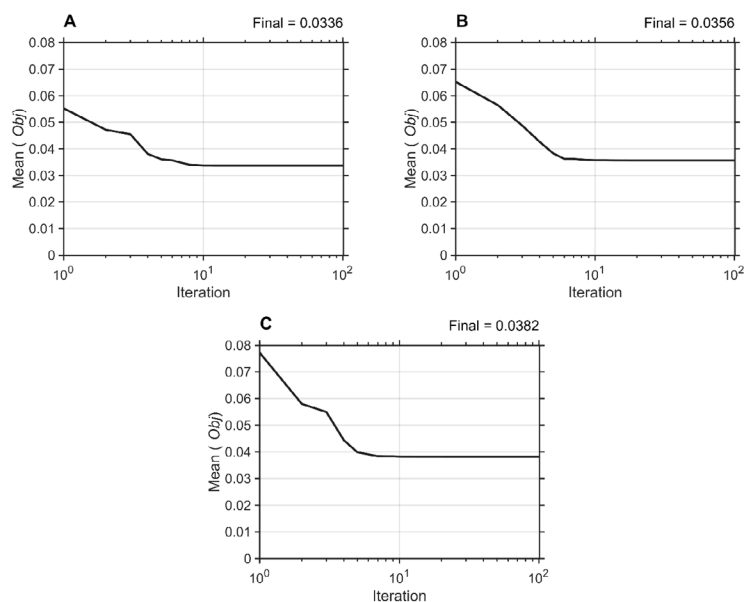


Figure 6. Mean best-so-far objective histories over 30 independent joint-inversion runs for the synthetic data with 10% noise: (A) enhanced sine cosine algorithm, (B) sine cosine algorithm, and (C) particle swarm optimization. The plotted joint objective is dimensionless. The final mean objective is indicated above each panel.

initial offset was selected to limit near-field effects. Senkaya *et al.*⁴⁰ interpreted the observations with a gradient-based joint inversion; a recent improved sparrow-search study also analyzed this refraction–dispersion case.²¹

For each site, we digitized one published dispersion curve and one first-arrival-time curve at their plotted sampling coordinates, obtaining approximately 50 phase-velocity samples and 24 first-arrival picks. No additional smoothing was applied; straight-line interpolation was used only to evaluate calculated responses at those coordinates. These are multichannel-derived site datasets, not single-station measurements. However, the raw shot gathers, a spatial sequence of local dispersion curves, reverse-shot comparisons, and a co-registered lateral velocity section are unavailable to this reanalysis. Digitization and plotting resolution are additional uncertainty sources.

The two sites were selected because Senkaya *et al.*⁴⁰ published dispersion and refraction observations together with layered velocity interpretations, allowing the same coupled objective and optimizer comparison. Beşirli is a coastal filled site represented by five layers, including a shallow low- V_s interval; MAE is represented by two finite layers over a deeper competent half-space. The published reference profiles come from the three-dataset joint inversion (3JI) of surface waves, refraction, and electrical sounding in Ref.⁴⁰ These layered descriptions express the one-dimensional model parameterization, not observed interface geometry. The published regional borehole and seismic-profile context does not supply the spatially matched observations needed here to identify dip or build a two-dimensional section at either site. We therefore interpret these results as representative one-dimensional profiles and leave the actual flat, dipping, or more complex geometry unresolved.

Beşirli's coastal position makes fluid saturation a plausible influence on near-surface elastic properties, but the velocity response is not a universal decrease of both wave speeds. Increased water saturation commonly raises V_p , whereas V_s may decrease or change little depending on the sediment frame and effective stress. Accordingly, the inverted low- V_s interval cannot by itself establish seawater infiltration. Low-velocity layering can also complicate dispersion-mode interpretation.^{41,42} The site is used here to assess inversion stability under this interpretive ambiguity, not to diagnose salinity from seismic velocity alone.

The ESCA, SCA, and PSO used the same Beşirli observations, joint objective, population size, iteration limit, and parameter-by-parameter bounds in Table 4. These intervals do not fix the model to the solution of Senkaya *et al.*⁴⁰ all 14 published 3JI thickness, V_s , and V_p estimates fall strictly inside them. For example, the

third-layer V_s interval is 40–300 m/s around their 107 m/s estimate, and the first-layer V_p interval is 100–900 m/s around 473 m/s. Senkaya *et al.*⁴⁰ reported local-search solutions and an independent neighborhood search, but not a matching table of lower and upper bounds. Their model-space volume therefore cannot be compared quantitatively with ours. Our experiment isolates relative optimizer behavior on a common explicit domain; it does not test uniqueness or invariance to still wider bounds.

Table 4. Parameter search bounds for the Beşirli layered model

Layers	V_s (m/s)	h (m)	V_p (m/s)
1	50~400	0.5~5	100~900
2	130~760	1~7	200~1,500
3	40~300	0.8~5	300~2,200
4	100~600	0.6~4	500~3,000
Half-space	90~550		400~2,500

Note: The search bounds were established based on the information reported in Ref.⁴⁰ and applied consistently to enhanced sine cosine algorithm, sine cosine algorithm, and particle swarm optimization. All 14 parameter values reported for the 3JI model in Table 5 of Ref.⁴⁰ fall strictly within these bounds.

Figure 8 compares calculated and observed field responses. All methods obtain reasonable fits, and the differences between joint and separate residuals are modest. The parameter models in Figure 9 provide a clearer test because field noise allows multiple structures to explain the observations with similar residual levels.

The ESCA profiles in Figure 9 are broadly consistent with the published reference model in Ref.⁴⁰ but that model also informed the search bounds and is not an independent benchmark. Joint inversion gives a more coherent deep structure than the individual-data inversions under the adopted five-layer parameterization. The recovered low- V_s interval below about 6 m is compatible with weak material; its cause, including any contribution from saturation or seawater, is not established by these seismic data alone.

Figure 10 reports the 30-run mean best-so-far histories. The final mean objectives are 0.0343 for ESCA, 0.0393 for SCA, and 0.0404 for PSO. Tables 5–7 provide parameter means and standard deviations; ESCA has the lowest spread for most estimated parameters. These statistics support greater repeatability within the adopted bounds, not uniqueness of the field model.

The Beşirli results therefore support ESCA as a repeatable search strategy within the specified five-layer model and parameter bounds. Agreement with Ref.⁴⁰

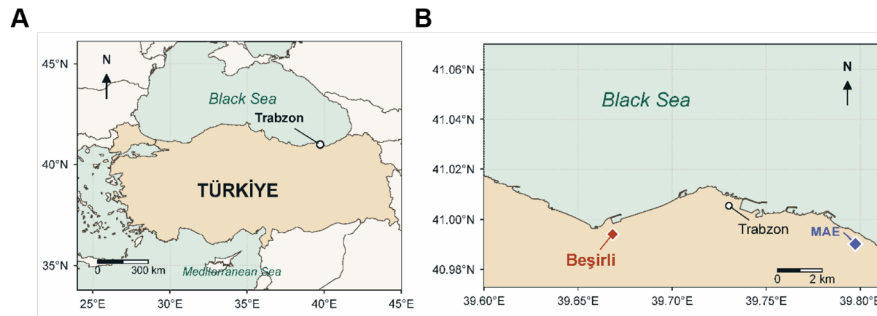


Figure 7. Map of the study area. (A) Location in Türkiye. (B) Beşirli and MAE field locations in Trabzon. Base data: Natural Earth (public domain); © OpenStreetMap contributors (ODbL). Locality markers provide geographic context, not survey coordinates; the map does not represent new field acquisition in this study.

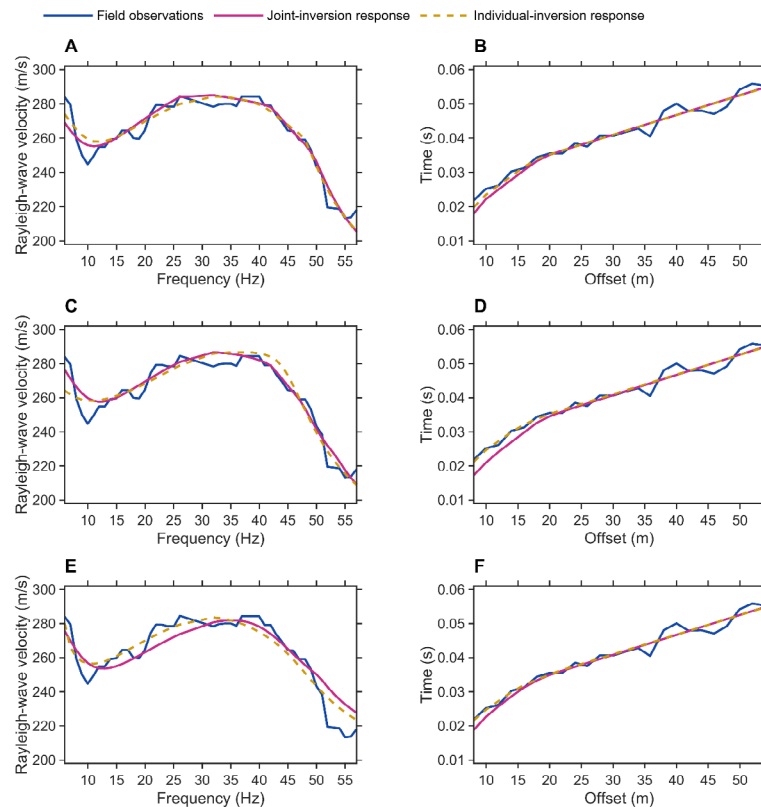


Figure 8. Beşirli response fits obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Blue solid curves denote the field observations, pink solid curves the joint-inversion responses, and gold dashed curves the responses obtained from the corresponding individual-data inversions. (A) Rayleigh-wave dispersion fit obtained with ESCA. (B) Refraction first-arrival time fit obtained with ESCA. (C) Rayleigh-wave dispersion fit obtained with SCA. (D) Refraction first-arrival time fit obtained with SCA. (E) Rayleigh-wave dispersion fit obtained with PSO. (F) Refraction first-arrival time fit obtained with PSO.

is a within-source consistency check, not independent validation or evidence that the recovered model is unique.

The second example uses the MAE measurements published by Senkaya *et al.*,⁴⁰ with the digitization procedure described above. The MAE site is represented by a three-layer parameterization, compared with five layers at Beşirli, and its model includes a deeper high-velocity

unit. All three optimizers were compared using the same observations, joint objective, population and iteration settings, and Table 8 bounds. All eight published MAE 3JI values lie strictly inside these bounds; for example, the half-space V_s range is 400–1,600 m/s around the reported 997 m/s, and its V_p range is 1,000–3,200 m/s around 2,107 m/s. These are comparisons on shared non-degenerate

Table 5. Beşirli model statistics obtained with enhanced sine cosine algorithm: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	235.7	0.58	2.3	0.01	447.3	11.51
2	439.3	6.49	4.0	0.05	716.2	17.09
3	154.6	3.54	2.7	0.13	1,264.6	67.46
4	318.3	2.14	2.1	0.01	1,732.2	42.66
Half-space	302.4	0.82			1,417.5	26.53

Table 6. Beşirli model statistics obtained with sine cosine algorithm: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	230.8	20.71	2.2	0.10	465.6	6.64
2	428.8	20.13	3.9	0.16	713.9	57.64
3	158.8	2.17	2.8	0.08	1,198.7	8.92
4	334.9	35.89	2.1	0.09	1,696.2	203.34
Half-space	312.6	17.45			1,447.1	177.18

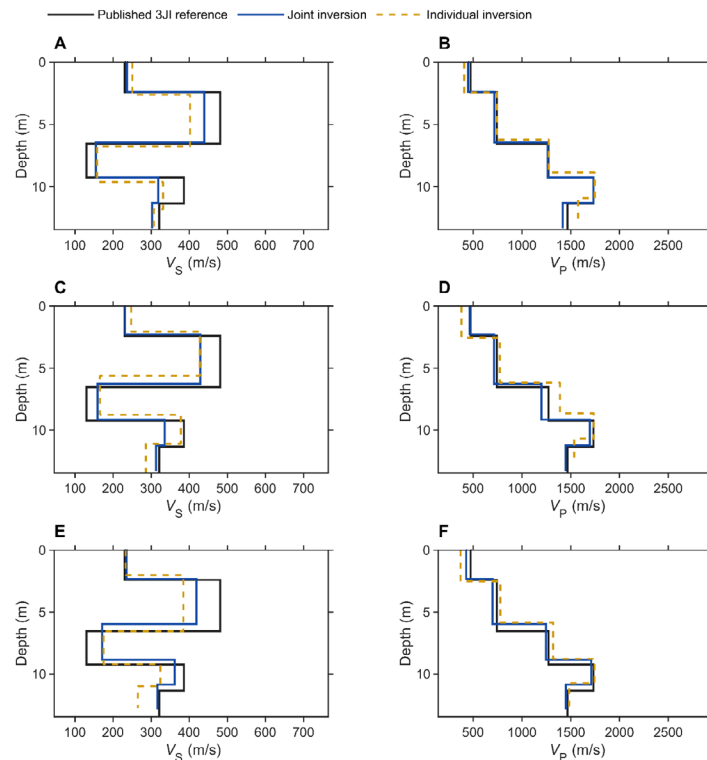


Figure 9. Beşirli shear-wave velocity (V_s) and P-wave velocity (V_p) profiles obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Black solid curves are the published three-dataset joint-inversion (3JI) reference profiles from Ref.⁴⁰; blue solid curves and gold dashed curves are the joint-inversion and individual-data inversion solutions generated in this study, respectively. (A) Shear-wave velocity (V_s) profile obtained with ESCA. (B) P-wave velocity (V_p) profile obtained with ESCA. (C) V_s profile obtained with SCA. (D) V_p profile obtained with SCA. (E) V_s profile obtained with PSO. (F) V_p profile obtained with PSO. The published 3JI reference profiles are not independently measured ground truth. All curves are one-dimensional parameterizations and do not establish lateral interface geometry.

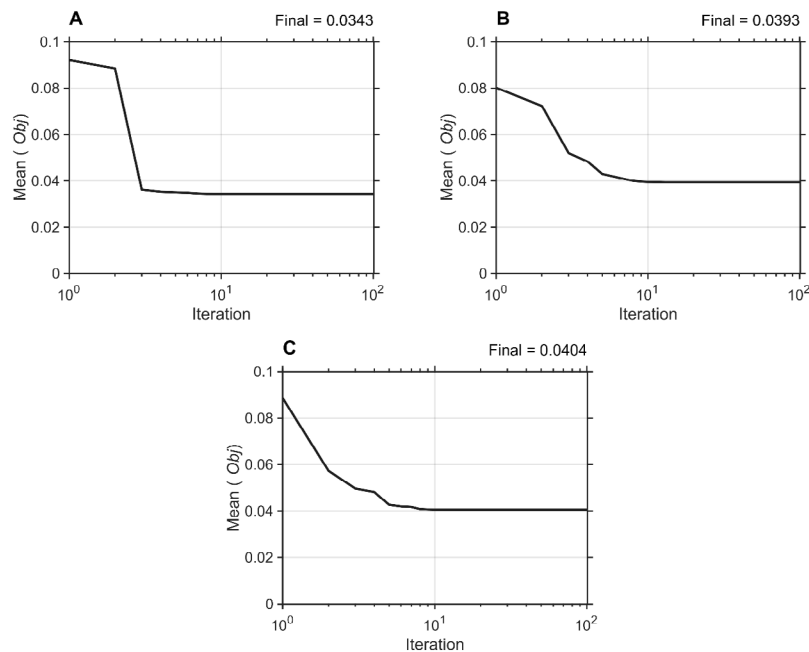


Figure 10. Mean best-so-far joint-inversion objective histories over 30 independent runs for the Beşirli dataset: (A) enhanced sine cosine algorithm, (B) sine cosine algorithm, and (C) particle swarm optimization. The plotted joint objective is dimensionless. The final mean objective is indicated above each panel.

Table 7. Beşirli model statistics obtained with particle swarm optimization: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	235.2	10.64	2.3	0.22	427.5	9.99
2	418.2	32.88	3.6	0.07	697.5	10.91
3	171.3	4.19	2.8	0.36	1,247.4	153.17
4	361.1	5.88	2.0	0.18	1,710.2	19.52
Half-space	316.1	4.64			1,450.1	101.26

intervals, not inversions from the published solution as a fixed model.

The calculated responses in Figure 11 match the measured dispersion and refraction trends for all three methods. As in the Beşirli example, residual differences alone are small; the principal distinctions appear in the recovered velocity-depth models shown in Figure 12.

The ESCA gives V_s and V_p structures broadly consistent with the published MAE interpretation in Ref.⁴⁰ which is a source-informed reference rather than an independent ground truth. The joint solutions are also more coherent than their individual-inversion counterparts, particularly for the deeper unit. The first layer is compatible with clay

containing sand and/or gravel, consistent with the borehole description reported for the area.

Figure 13 and Tables 9–11 evaluate convergence and parameter repeatability. The dimensionless final 30-run mean joint objective is 0.0717 for ESCA, 0.0740 for SCA, and 0.0722 for PSO. The ESCA–PSO difference is only 0.0005 and is not evidence of a statistically significant advantage. Only aggregate convergence statistics are reported here; these means are not accompanied by confidence intervals or formal across-run significance tests. The parameter standard deviations in Tables 9–11 describe optimizer repeatability within the chosen bounds, not posterior uncertainty or uncertainty in the objective means.

Table 8. Parameter search bounds for the MAE layered model

Layers	V_s (m/s)	h (m)	V_p (m/s)
1	50~300	0.5~3	140~500
2	100~500	1~7	300~1,000
Half-space	400~1,600	∞	1,000~3,200

Note: The search bounds were established based on the information reported in Ref.⁴⁰ and applied consistently to ESCA, SCA, and PSO. All eight seismic velocity and layer-thickness estimates reported for the 3JI model in Table 2 of Ref.⁴⁰ fall strictly within these bounds. The half-space is assumed to have infinite thickness and therefore has no thickness bounds.

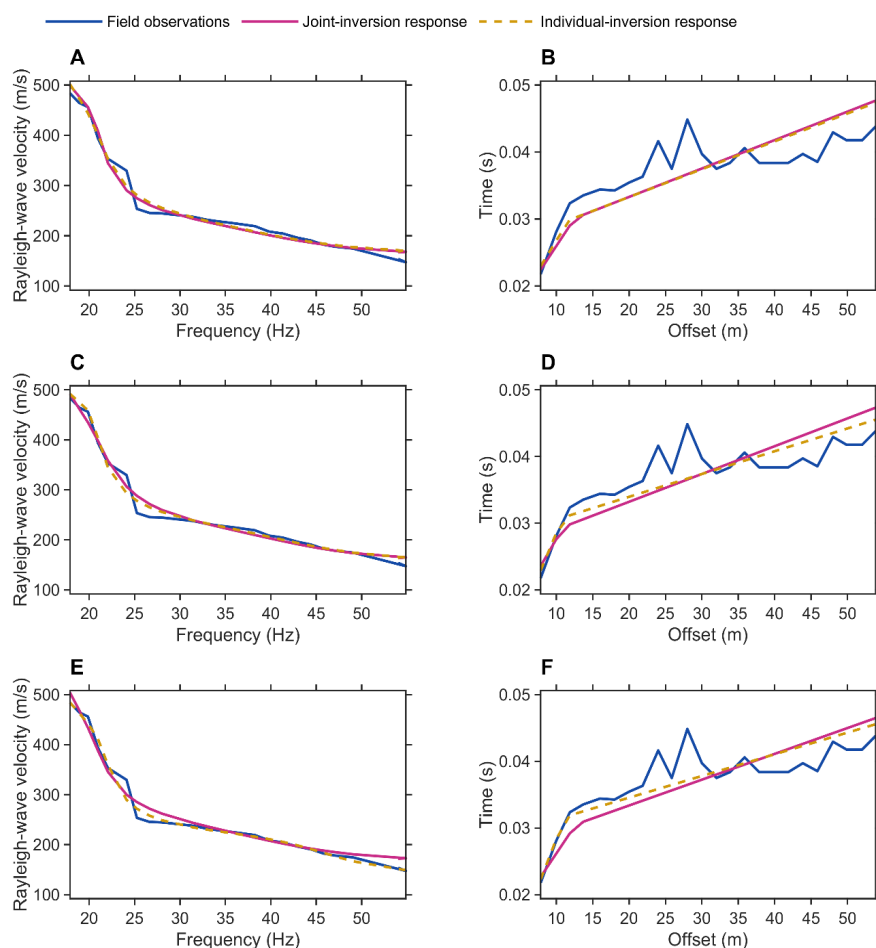


Figure 11. MAE response fits obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Blue solid curves denote the field observations, pink solid curves the joint-inversion responses, and gold dashed curves the responses obtained from the corresponding individual-data inversions. (A) Rayleigh-wave dispersion fit obtained with ESCA. (B) Refraction first-arrival time fit obtained with ESCA. (C) Rayleigh-wave dispersion fit obtained with SCA. (D) Refraction first-arrival time fit obtained with SCA. (E) Rayleigh-wave dispersion fit obtained with PSO. (F) Refraction first-arrival time fit obtained with PSO.

The inferred competent unit begins below approximately 6 m, within the depth interval reported by previous site investigations.^{40,43} Taken together, the two field examples show that ESCA can provide repeatable solutions for different layered settings when the model bounds are informed by existing site knowledge.

Figure 14 compares the representative V_s - and V_p -depth profiles of both sites on the same axes, making the layer depths and velocity contrasts directly comparable. The dashed profiles are the published 3JI interpretations of Senkaya *et al.*,⁴⁰ whereas the blue profiles use the ESCA joint-inversion parameter means in Tables 5 and 9. Unlike

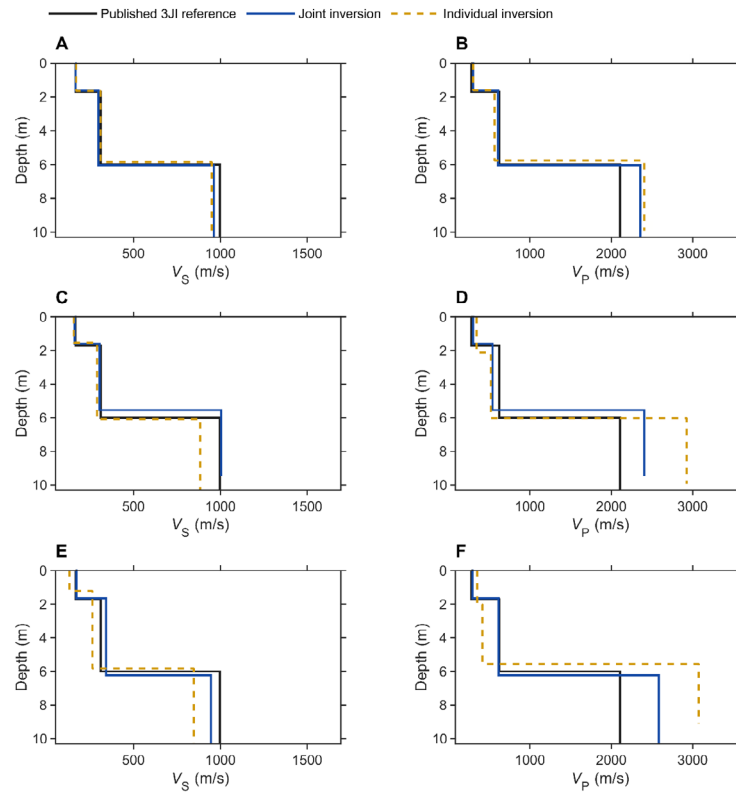


Figure 12. MAE shear-wave velocity (V_S) and P-wave velocity (V_P) profiles obtained with the enhanced sine cosine algorithm (ESCA), sine cosine algorithm (SCA), and particle swarm optimization (PSO). Black solid curves are the published three-dataset joint-inversion (3JI) reference profiles from Ref.⁴⁰; blue solid curves and gold dashed curves are the joint-inversion and individual-data inversion solutions generated in this study, respectively. (A) V_S profile obtained with ESCA. (B) V_P profile obtained with ESCA. (C) V_S profile obtained with SCA. (D) V_P profile obtained with SCA. (E) V_S profile obtained with PSO. (F) V_P profile obtained with PSO. The published 3JI reference profiles are not independently measured ground truth. All curves are one-dimensional parameterizations and do not establish lateral interface geometry.

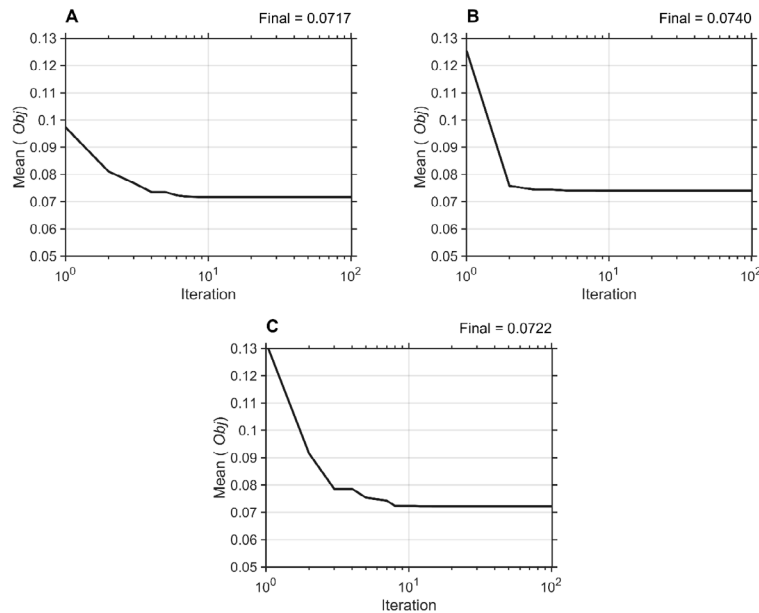


Figure 13. Mean best-so-far joint-inversion objective histories over 30 independent runs for the MAE dataset: (A) enhanced sine cosine algorithm, (B) sine cosine algorithm, and (C) particle swarm optimization. The plotted joint objective is dimensionless. The final mean objective is indicated above each panel.

Table 9. MAE model statistics obtained with enhanced sine cosine algorithm: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	164.6	0.99	1.6	0.02	299.7	9.57
2	296.7	21.27	4.3	0.17	607.3	25.32
Half-space	962.1	38.88			2,356.0	72.80

Table 10. MAE model statistics obtained with sine cosine algorithm: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	159.9	19.18	1.6	0.27	300.6	2.81
2	302.5	7.27	3.9	0.32	539.9	78.87
Half-space	1004.3	141.51			2,403.4	157.26

Table 11. MAE model statistics obtained with particle swarm optimization: parameter means and standard deviations (SDs)

Layers	V_s (m/s)		h (m)		V_p (m/s)	
	Mean	SD	Mean	SD	Mean	SD
1	170.3	12.20	1.6	0.02	292.3	6.33
2	341.3	48.98	4.5	0.29	613.9	24.10
Half-space	945.6	53.91			2,583.0	79.40

the algorithm-by-algorithm displays in [Figures 9 and 12](#), this comparison isolates the agreement and differences between our mean profiles and the published reference interpretation. It is a graphical synthesis of existing numerical results, not an additional inversion experiment.

At Beşirli, both profiles contain a low- V_s third layer at approximately 6–9 m depth ([Figure 14A](#)), but its ESCA mean velocity is 154.6 m/s compared with the published 107 m/s. Thus, the weak-layer feature is retained without reproducing its velocity contrast exactly. The V_p profiles are closer through the third and fourth layers ([Figure 14B](#)). At MAE, the competent half-space begins at 5.9 m in the ESCA mean model and 6.0 m in the published model. Its V_s values are 962.1 and 997 m/s, respectively ([Figure 14C](#)), whereas the V_p values differ more clearly, at 2,356.0 and 2,107 m/s ([Figure 14D](#)). These comparisons show which layered features are consistent and where the parameter estimates differ. Because the reference interpretation informed our bounds and uses additional electrical-sounding data, the agreement is not independent geological validation. Neither profile provides the lateral coordinate

needed to determine interface dip or reconstruct a two-dimensional section.

6. Discussion

The ablation results suggest that the ESCA changes affect complementary parts of the search. Tent mapping improves initial coverage, reverse learning evaluates opposite regions before the first update, and the best-solution-centered exponential schedule increasingly concentrates the population around the retained optimum. This combination can explain the lower final objectives and reduced run-to-run variation observed in many tests. It can also reduce diversity too quickly when the current best model lies in a suboptimal basin, so the method is not expected to dominate every problem or parameter setting.

The improved search has a measurable computational cost. Reverse learning doubles the initialization evaluations from N to $2N$, so ESCA uses 30,600 objective evaluations compared with 30,300 for SCA and PSO at $N = 300$ and $T = 100$. This 0.99% difference means the available comparisons are not exactly evaluation-budget matched.

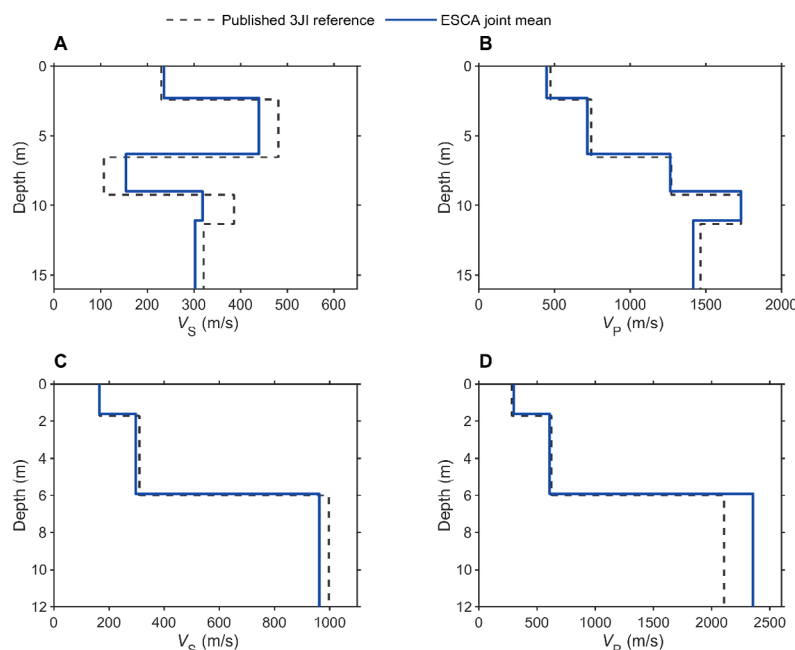


Figure 14. Comparison of representative one-dimensional velocity-depth profiles for Beşirli and MAE. (A) Beşirli shear-wave velocity (V_s) profiles. (B) Beşirli P-wave velocity (V_p) profiles. (C) MAE V_s profiles. (D) MAE V_p profiles. Dashed curves are the published three-dataset joint-inversion (3JI) reference profiles of Senkaya *et al.*⁴⁰ (Tables 2 and 5); blue solid curves are the enhanced sine cosine algorithm joint-inversion means in Tables 5 and 9 of this study. Curves are redrawn from the reported numerical values; half-space profiles are extended to the plotting limit. Vertical segments indicate constant velocity within each modeled layer, whereas horizontal segments mark velocity changes at layer boundaries. These one-dimensional profiles do not establish lateral interface geometry.

In the theoretical noise-free model, the 30-run mean $\pm$ standard-deviation runtime was 145.32 ± 3.46 s for ESCA, 137.89 ± 5.64 s for SCA, and 210.33 ± 4.65 s for PSO. ESCA was therefore 5.39% slower than SCA and 30.91% faster than PSO in this implementation, but runtime reflects both the forward calculations and optimizer overhead. The runtime comparison is restricted to this noise-free test and its implementation; no runtime comparison is reported for the noisy or field cases. Unequal evaluation counts and implementation-dependent overhead preclude interpreting these times as an exactly budget-matched, hardware-independent benchmark.

Li *et al.*²¹ addressed refraction–dispersion inversion with an improved sparrow-search method that also uses Tent-map initialization and opposition-based learning. Its perturbation and population update differ from the sine cosine update used here. Thus, reverse learning is an established principle; the tested contribution is its incorporation into this coupled seismic formulation. Published objectives and runtimes cannot be ranked directly across different forward solvers, preprocessing, bounds, and evaluation budgets.

The role of model-space width must be separated from the controlled algorithm comparison. Ai *et al.*³¹ widened the search domain in a multimode glacier inversion while

holding the optimizer settings fixed and observed a marked deterioration in the modified Barnacles Mating Optimizer result. This is direct evidence that bound choice changes inversion difficulty; it is not evidence that every optimizer deteriorates equally or that relative rankings remain fixed. A larger domain generally admits more competing layered models at a fixed evaluation budget. In the present Beşirli and MAE tests, ESCA, SCA, and PSO search the identical intervals in Tables 4 and 8, so differences among them cannot be attributed to unequal parameter bounds. The ranking and run-to-run spread are conditional on these domains. Because the field observations were digitized from Ref.⁴⁰ and the bounds were informed by its interpretation, agreement with its models is a within-source consistency check, not independent geological validation or a uniqueness claim.

Dispersion-mode identification remains a limitation. Mode kissing and low-velocity layering can complicate interpretation.^{41,42} Improved dispersion imaging, picking, and multimode inversion address this issue,^{44–48} while transdimensional parameterizations address layer-number uncertainty.^{49,50} Neither refinement removes lateral-geometry errors when the forward model remains one-dimensional. Our fundamental-mode results are limited accordingly.

The geometric interpretation of the field profiles depends on the dimensionality of the forward model. A one-dimensional, horizontally layered approximation remains a standard starting point for conventional dispersion-curve inversion in near-surface site characterization.^{1,37,38,51,52} It is also the explicit setting of constrained dispersion-refraction joint inversion.¹⁸ Under this approximation, each layer has uniform elastic properties and thickness along the survey, and the inversion estimates their depth variation. The resulting profile represents the finite survey footprint rather than an infinitesimal point. This established practice motivates our model choice, but does not establish that either field site has horizontal geological interfaces.

Previous studies have advanced methods for seismic-wave modeling and multimodal analysis in layered media and laterally heterogeneous media. Chen⁵³ developed a normal-mode solver for multilayered half-spaces. Wang *et al.*⁵⁴ derived frequency-Bessel dispersion imaging for flat, layered isotropic media, while Wu *et al.*⁵⁵ explicitly inverted local one-dimensional shear-velocity profiles from multimodal observations. Li and Chen⁵⁶ additionally tested lateral heterogeneity and showed how spatially restricted windows can distinguish neighboring structures. The dimensions of wave propagation, receiver-array geometry, and the estimated velocity model must therefore be distinguished: three-dimensional propagation through a layered medium, or a two-dimensional frequency-velocity image, does not itself imply inversion of a laterally variable velocity section.

For weak, smooth lateral changes, a local horizontal approximation may give a useful effective profile if changes within the aperture are small. Its validity depends on wavelength, aperture, contrasts, and propagation direction; we cannot assign a universal acceptable dip angle from these experiments. A dipping refractor can produce different apparent traveltimes slopes in opposite shooting directions so that horizontal ray tracing may trade dip against V_p or interface depth. Surface-wave phase estimates can also depend on which part of a dipping or laterally varying structure is illuminated. Because the two datasets sample different volumes and paths, a common one-dimensional fit can redistribute those geometry errors into thickness and velocity. Joint inversion adds complementary constraints but does not remove this modeling bias.^{18,51,52}

Strong dips, abrupt thickness changes, faults, basin edges, or localized velocity anomalies can cause scattering, reflected surface waves, mode conversion, and nonuniform refraction paths. A single layered dispersion relation and horizontal traveltimes model then need not represent the

measured response. A low objective or small run-to-run spread would describe a repeatable fit within the restricted model class, not recovery of the true two-dimensional structure. With multifold records and spatially distributed dispersion measurements, laterally constrained local inversions can resolve smooth variation.⁵⁷ For stronger heterogeneity or topography, two-dimensional wave-equation methods explicitly account for the relevant propagation effects.⁵⁸ Their data requirements and forward physics differ from the present curve-based inversion.

The available Beşirli and MAE curve pairs cannot establish which of these geometric cases applies. Figures 9, 12, and 14 therefore show representative one-dimensional interpretations rather than mapped geological cross-sections. We do not infer flat interfaces from those profiles or claim a tested capability to recover dip. Spatially registered multiple spreads or overlapping windows, reciprocal refraction measurements, raw waveforms, and topographic information would be needed to evaluate lateral structure and select a corresponding forward model. Holding the current model class and bounds common supports the reported optimizer comparison, while the geological interpretation remains conditional on that class.

7. Conclusion

This study examined a reverse-learning ESCA for coupled inversion of Rayleigh-wave phase velocities and seismic-refraction first arrivals in a one-dimensional, horizontally layered medium. Component-wise benchmarks, controlled synthetic tests, and reanalyses of the published Beşirli and MAE datasets yielded the following key findings and limitations:

(i) Joint constraints on the layered velocity model

Using dispersion and refraction observations within a common objective improved recovery of layer thickness, V_s , and V_p relative to the corresponding individual-data inversions. The improvement was clearest for ESCA, showing that the benefit of complementary data also depends on effective exploration of the shared model space.

(ii) Effect of the ESCA modifications

Tent-map initialization, reverse learning, and the best-solution-centered exponential update produced lower or joint-lowest benchmark objectives and reduced variability in repeated inversions. ESCA also reached the smallest mean final objective in the noise-free, noisy, and Beşirli comparisons; its MAE advantage over PSO was small (0.0717 versus 0.0722) and was not tested for statistical significance. Reverse learning added 300 initialization evaluations. On the noise-free synthetic model, ESCA

required 145.32 ± 3.46 s per inversion, versus 137.89 ± 5.64 s for SCA and 210.33 ± 4.65 s for PSO. These results do not establish universal or exactly budget-matched superiority.

(iii) Evidence from the two field reanalyses

For Beşirli and MAE, ESCA yielded velocity profiles broadly consistent with the interpretations in Ref.⁴⁰ and generally showed smaller run-to-run standard deviations than SCA and PSO. Because the observations were digitized and the bounds were informed by the same publication, these cases test consistency and repeatability within the adopted model space rather than independent field validity. The field profiles represent one-dimensional interpretations; the available curves do not resolve interface dip or lateral geometry.

(iv) Scope and remaining limitations

Applicability to dipping interfaces, strong lateral heterogeneity, and complex two-dimensional velocity structures has not been demonstrated. Horizontal layering, a fixed layer number, source-informed bounds, fundamental-mode selection, digitization uncertainty, and unequal objective-evaluation counts limit the geometric and performance interpretation. The reported runtimes apply only to the noise-free theoretical model, and optimizer repeatability statistics do not quantify full model uncertainty.

Within those limits, reverse-learning-assisted ESCA is a practical candidate for estimating common-layer V_s , V_p , and thickness parameters. The reported comparisons support this use within the adopted model class and bounds, without establishing general two-dimensional applicability or universal performance superiority.

Acknowledgment

We acknowledge Senkaya *et al.*⁴⁰ for publishing the Beşirli and MAE field datasets from Türkiye, from which the observations used in this study were digitized.

Funding

None.

Conflict of interest

The authors declare no potential conflict of interest.

Author contributions

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Availability of data

The code for the benchmark functions used in this study was obtained from <http://www.sfu.ca/~ssurjano>. Additional data are available from the corresponding author upon reasonable request.

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