

Nonlinear dynamics of a discrete-time banking monopoly model with delayed risk effects

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ABSTRACT

Bank lending dynamics are strongly influenced by internal adjustment mechanisms and accumulated risk exposure, both of which can contribute to financial instability if they are not properly managed. In this study, we propose a discrete-time banking monopoly model with delayed risk effects, in which the current lending decision depends on both current profitability and the previous-period loan portfolio. The delayed component is introduced to capture the economic impact of accumulated credit risk, regulatory capital pressure, and expected loss provisioning associated with loan expansion. Using a gradient-based bounded-rationality framework, the model was formulated as a two-dimensional, nonlinear map. We derive the equilibrium points and establish explicit local stability conditions using the Jury criterion. Furthermore, bifurcation analysis shows that the positive equilibrium may lose stability through flip and Neimark–Sacker bifurcations, depending on the interaction between the adjustment speed and the delayed risk intensity. Numerical simulations, including bifurcation diagrams, Lyapunov exponents, isoperiodic diagrams, and basin-of-attraction plots, reveal rich nonlinear phenomena such as periodic oscillations, quasi-periodicity, and chaos. These results show that delayed risk exposure can be an endogenous cause of instability in the lending dynamics. It also outlines the role of prudential adjustment policies in ensuring stability in the banking sector.

Keywords: Banking dynamics; Delayed risk effects; Discrete-time systems; Bifurcation analysis; Chaos



1. Introduction

Bank lending dynamics are a significant factor in financial stability, as they increase economic fluctuations by expanding the credit supply in a procyclical manner. Other related factors, such as excessive lending and interbank connections, also contribute to the buildup of systemic risk and susceptibility.^{1–4} Therefore, sound regulatory and macroprudential policies are required to strike a balance

between credit growth and resilience, particularly as new challenges, namely, fintech growth and structural risks, tend to evolve.^{5,6}

Modern bank lending is influenced not only by current profitability but also by a client's past exposure to credit risk and to capital constraints. Specifically, increased credit risk and rising capital requirements are likely to restrict banks' lending capacity and affect risk-taking behavior.^{7–9} Regulatory pressures also influence these outcomes by altering

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portfolio and credit allocation decisions under constrained conditions.^{9,10} Understanding how numerous intertemporal factors affect lending behavior is crucial for both theoretical and policy designs. In this context, mathematical modeling provides an ideal framework for describing the diverse interactions and qualitative behavior of banking systems across various economic conditions.

Continuous- and discrete-time models have been used in several studies to analyze the dynamics of banking, including profit maximization, balance-sheet interactions, and regulatory constraints. For example, continuous nonlinear models and banking system models have been helpful in modeling complicated risk interactions, system stability, and regulatory-based controls.^{11–16} Discrete-time nonlinear models with bounded-rationality are particularly suitable for characterizing adaptive decision-making processes with limited information and adjustment frictions.^{17,18} Simple nonlinear financial models also exhibit rich dynamics, including cycles and chaotic behavior, arising from mechanisms driven by bifurcations and feedback effects.^{19–21} These properties naturally account for endogenous credit market fluctuations, in which internal dynamics and heterogeneity give rise to boom-and-bust cycles without external shocks.^{22,23}

Recently, particular attention has been given to the discrete-time dynamics of a banking monopoly or duopoly, examining how policy parameters affect the nonlinear behavior of banks in allocating loans. The seminal work was started by Fanti²⁴ who studied a banking duopoly incorporated with capital policy. The banks were studied under naïve and bounded-rational expectations, in which their nonlinear dynamics are strongly influenced by adjustments to the capital policy parameter. Brianzoni *et al.*²⁵ further developed this model to analyze capital policy in Italy, where banks' operating costs are assumed to be asymmetric.^{25,26} A similar approach was used to study the effect of non-performing loans on banking duopoly dynamics.²⁷ In the case of banking duopoly with bounded rational expectations, Ansori *et al.*²⁸ evaluated the role of a macroprudential policy in Indonesia, namely the loan-to-deposit ratio-based reserve requirement.²⁸ The results showed that, under a particular economic scenario, the model can produce discontinuous maps, resulting in the presence of border collision bifurcations. Meanwhile, in a monopoly banking system, the Fanti model was used to assess several banking policies, including reserve requirements, capital adequacy ratios, and benchmark interest rates.²⁹ Another similar approach, involving the adaptive dynamics of deposits and non-performing loans, was studied and calibrated using real banking data.³⁰

Despite these advances, many existing models assume that costs and risks depend only on current variables, neglecting the potential impact of delayed effects arising from past lending decisions. However, banks face delayed risk effects associated with accumulated loan exposure, including increased default risk, capital adequacy requirements, and loan-loss provisioning. These factors can constrain current lending by increasing costs and limiting credit supply, particularly when the loan portfolio is large.^{31–33} Consequently, intertemporal feedback mechanisms emerge, whereby higher risk and provisioning today reduce future lending capacity and amplify credit cycles.^{34,35}

The limited attention paid to such delayed interactions in discrete-time banking models stems from several challenges. From a nonlinear dynamics perspective, the inclusion of time delays introduces analytical and computational complexity, leading to phenomena such as bifurcations and

chaos, which are difficult to analyze.^{36,37} In addition, difficulties in parameter estimation and empirical validation, together with the widespread reliance on linear approximations and focus on immediate interactions, have further hindered the incorporation of delay effects.^{38,39}

This gap motivates the present study, which aims to investigate how delayed risk exposure influences bank lending stability and long-run behavior. To this end, we propose a discrete-time banking monopoly model in which the profit function incorporates a delayed risk term that depends on current and past loan volumes. The model was formulated within a bounded-rationality framework in which the bank modified its lending policy using a gradient-based rule. The result of this formulation was a two-dimensional nonlinear difference system describing the interaction between current decisions and prior exposures.

From a nonlinear dynamics perspective, delayed interactions are well known for generating complex behaviors in discrete-time systems. By introducing memory effects, time delays increase the system's effective dimensionality and may alter the stability of equilibria. Consequently, delayed feedback can induce local bifurcations, including flip and Neimark–Sacker bifurcations, leading to periodic, quasi-periodic, or chaotic dynamics. In banking systems, accumulated credit risk and regulatory pressures naturally create such delayed feedback mechanisms. Therefore, investigating their impact on lending dynamics is important not only from an economic perspective but also for understanding the nonlinear processes that may generate endogenous fluctuations and instability.

Accordingly, this study aims to evaluate the existence and local stability of equilibria using the Jury stability criterion and to perform a bifurcation analysis with respect to the delayed risk parameter. The results showed that the system could undergo flip and Neimark–Sacker bifurcations, resulting in periodic, quasiperiodic, and chaotic dynamics. The main contribution of this study is to outline the role of delayed risk effects as endogenous sources of instability in the banking system. Furthermore, combining the analytical outcomes with numerical simulations revealed that the interaction between adjustment speed and delayed risk intensity significantly changed the system's behavior. These results highlight the dynamic implications of risk accumulation and can inform the development of more effective lending practices and regulatory measures to ensure financial stability.

The remainder of this paper is organized as follows. Section 2 presents the formulation and derivation of the discrete-time banking monopoly model with delayed risk effects. Section 3 investigates the existence and local stability of equilibrium points using the Jury stability criterion. Section 4 analyzes the bifurcation structure of the model with respect to the delayed risk parameter and derives the conditions for flip and Neimark–Sacker bifurcations. Section 5 presents numerical simulations, including bifurcation diagrams, Lyapunov exponents, isoperiodic diagrams, and basins of attraction, to illustrate the theoretical findings of this study. Finally, Section 6 concludes the paper and discusses potential directions for future research.

2. Formulation of the mathematical model

2.1. Loan demand and profit function

Assume that a monopolistic bank provides loans at discrete times $t = 0, 1, 2, \dots$, and let $L_t \geq 0$ represent the outstanding loan portfolio at the time t . The lending market is presumed to have a linear inverse loan demand as follows:

$$r_{L,t} = a - bL_t \quad a > 0, \quad b > 0 \quad (1)$$

where $r_{L,t}$ is the loan interest rate. This specification implies that an increase in loan supply depresses lending rates.

The bank earns interest income $r_{L,t}L_t$ and incurs a linear operational funding cost cL_t where $c > 0$. In addition, to capture the delayed effect of credit risk exposure, regulatory capital pressure, and expected loss provisioning associated with loan expansion, we introduce a dynamic risk cost term that depends on both past and current loan volumes. The bank's profit is assumed to be calculated as follows:

$$\pi_t = r_{L,t}L_t - cL_t - \phi L_{t-1}L_t \quad \phi > 0 \quad (2)$$

The interaction term $\phi L_{t-1}L_t$ represents a reduced-form approximation of the idea that expanding new lending becomes more costly when the bank already has a large existing loan portfolio. Hence, the marginal cost of new lending increases with prior exposure.

The delayed risk term can be seen as a simplified model of various mechanisms found in banking operations. Firstly, a larger loan portfolio typically leads to higher expected credit losses and capital charges, which in turn diminish the profitability of future loans. Secondly, regulatory standards such as Basel III mandate that banks hold sufficient capital reserves for risk-weighted assets, suggesting that an aggressive increase in loans today might limit future lending options. Thirdly, standards for expected-loss provisioning incur additional costs as credit exposure accumulates. As a result, current lending choices are inherently linked to past portfolio decisions. The bilinear interaction term effectively encapsulates these cumulative impacts while preserving analytical simplicity.

Substituting Equation 1 into Equation 2, we obtain:

$$\begin{aligned} \pi_t &= (a - bL_t)L_t - cL_t - \phi L_{t-1}L_t \\ &= (a - c)L_t - bL_t^2 - \phi L_{t-1}L_t \end{aligned} \quad (3)$$

Differentiating Equation 3 with respect to the decision variable L_t gives the marginal profit:

$$\frac{\partial \pi_t}{\partial L_t} = a - c - 2bL_t - \phi L_{t-1} \quad (4)$$

2.2. Gradient-based loan adjustment and dynamics

Following the bounded-rationality approach commonly adopted in economic dynamics,^{24-26,28} we assume that the bank does not instantaneously maximize profit, but instead adjusts its lending gradually in response to observed marginal profitability. Specifically, loan volume evolves according to the gradient adjustment mechanism:

$$L_{t+1} = L_t + \alpha L_t \frac{\partial \pi_t}{\partial L_t} \quad \alpha > 0 \quad (5)$$

where α denotes the speed of adjustment. The multiplicative factor L_t implies proportional adjustment relative to the current scale of lending and ensures that the zero-loan state remains unchanged.

Substituting Equation 4 into Equation 5, we obtain:

$$\begin{aligned} L_{t+1} &= L_t + \alpha L_t (a - c - 2bL_t - \phi L_{t-1}) \\ &= L_t (1 + \alpha(a - c) - 2\alpha bL_t - \alpha\phi L_{t-1}) \end{aligned} \quad (6)$$

Equation 6 defines a second-order nonlinear difference equation. To facilitate the dynamical analysis, we rewrite it as a first-order system in $\mathbb{R}^2$ by introducing:

$$x_t = L_t, \quad y_t = L_{t-1}$$

Then Equation 6 can be written as:

$$\begin{cases} x_{t+1} = x_t (1 + \alpha(a - c) - 2\alpha b x_t - \alpha\phi y_t), \\ y_{t+1} = x_t \end{cases} \quad (7)$$

with the initial conditions $x_0 = L_0 \geq 0$ and $y_0 = L_{-1} \geq 0$.

3. Local stability analysis of equilibria

A steady state (x^*, y^*) of Equation 7 satisfies $x_{t+1} = x_t = x^*$ and $y_{t+1} = y_t = y^*$. From the second equation, we obtain $y^* = x^*$. Substituting into the first equation yields:

$$x^* = x^* (1 + \alpha(a - c) - \alpha(2b + \phi)x^*)$$

Hence, the system admits two equilibria:

$$E_0 = (0, 0), \quad E_1 = \left(\frac{a - c}{2b + \phi}, \frac{a - c}{2b + \phi} \right)$$

The positive equilibrium E_1 exists whenever $a > c$, which means the maximum attainable lending rate exceeds the bank's marginal operating cost.

Next, we investigate the local asymptotic stability of the equilibria of the planar map in Equation 7. The Jacobian matrix of the system at a generic point (x, y) is given by:

$$J(x, y) = \begin{pmatrix} 1 + \alpha(a - c) - 4\alpha b x - \alpha\phi y & -\alpha\phi x \\ 1 & 0 \end{pmatrix} \quad (8)$$

For a 2×2 matrix with characteristic polynomial:

$$\lambda^2 + p\lambda + q = 0 \quad (9)$$

both eigenvalues lie inside the unit disk, that is, $|\lambda| < 1$, if and only if the Jury conditions hold:

$$1 + p + q > 0; \quad 1 - p + q > 0; \quad 1 - q > 0 \quad (10)$$

3.1. Stability of the boundary equilibrium

We first consider the boundary equilibrium $E_0 = (0, 0)$.

Theorem 1. *The equilibrium $E_0 = (0, 0)$ is locally asymptotically stable if and only if:*

$$-2 < \alpha(a - c) < 0. \quad (11)$$

In particular, if $a > c$, then E_0 is unstable.

Proof. Evaluating Equation 8 at $(0, 0)$ gives:

$$J(E_0) = \begin{pmatrix} 1 + \alpha(a - c) & 0 \\ 1 & 0 \end{pmatrix}$$

Hence, the eigenvalues are:

$$\lambda_1 = 1 + \alpha(a - c), \quad \lambda_2 = 0$$

Since $|\lambda_2| = 0 < 1$, the equilibrium E_0 is locally asymptotically stable if and only if $|\lambda_1| < 1$, namely:

$$|1 + \alpha(a - c)| < 1$$

This is equivalent to:

$$-2 < \alpha(a - c) < 0$$

If $a > c$, then $1 + \alpha(a - c) > 1$, and thus, E_0 is unstable. $\square$

3.2. Stability of the positive equilibrium

Assume that $a > c$, such that a positive equilibrium exists. For convenience, let:

$$d := a - c$$

Then the positive equilibrium is:

$$L^* = \frac{a - c}{2b + \phi} = \frac{d}{2b + \phi} \quad E_1 = (L^*, L^*)$$

Evaluating Equation 8 at $(x, y) = (L^*, L^*)$, we obtain:

$$J(E_1) = \begin{pmatrix} A & B \\ 1 & 0 \end{pmatrix}$$

where:

$$A = 1 - \frac{2\alpha bd}{2b + \phi}, \quad B = -\frac{\alpha\phi d}{2b + \phi}$$

Therefore, the characteristic polynomial is:

$$\lambda^2 - A\lambda - B = 0 \quad (12)$$

Equivalently, in Equation 9:

$$p = -A, \quad q = -B$$

Theorem 2. Assume that $a > c$, such that E_1 exists. Then the positive equilibrium E_1 is locally asymptotically stable if and only if:

$$\frac{\alpha\phi(a - c)}{2b + \phi} < 1 \quad (13)$$

and:

$$2 + \frac{\alpha(a - c)(\phi - 2b)}{2b + \phi} > 0 \quad (14)$$

Equivalently, the following cases are valid:

- If $\phi \geq 2b$, then E_1 is locally asymptotically stable whenever:

$$0 < \alpha(a - c) < \frac{2b + \phi}{\phi} \quad (15)$$

- If $0 < \phi < 2b$, then E_1 is locally asymptotically stable whenever:

$$0 < \alpha(a - c) < \min \left\{ \frac{2b + \phi}{\phi}, \frac{2(2b + \phi)}{2b - \phi} \right\} \quad (16)$$

Proof. Applying the Jury conditions of Equation 10 to Equation 12, we obtain:

$$1 - A - B > 0; \quad 1 + A - B > 0; \quad 1 + B > 0$$

We simplify these inequalities individually. First,

$$1 + B > 0 \Leftrightarrow \frac{\alpha\phi(a - c)}{2b + \phi} < 1$$

This gives Equation 13.

Next,

$$1 - A - B = \frac{\alpha d(2b + \phi)}{2b + \phi} = \alpha d > 0$$

This condition is automatically satisfied.

Finally,

$$1 + A - B > 0 \Leftrightarrow 2 + \frac{\alpha(a - c)(\phi - 2b)}{2b + \phi} > 0$$

This yields Equation 14.

Thus, E_1 is locally asymptotically stable if and only if Equations 13 and 14 hold.

To derive the explicit parameter ranges, we distinguish two cases.

If $\phi \geq 2b$, then $\phi - 2b \geq 0$, so Equation 14 holds automatically. Hence, only Equation 13 is required, which gives Equation 15.

If $0 < \phi < 2b$, then $\phi - 2b < 0$, so Equation 14 becomes:

$$\alpha d < \frac{2(2b + \phi)}{2b - \phi}$$

Combining this with Equation 13 yields Equation 16. This completes the proof. $\square$

The boundary of the stability region of E_1 is determined by the cases $1 + B = 0$ and $1 + A - B = 0$, which will be used in the bifurcation analysis in the next section.

4. Bifurcation analysis with respect to the delayed risk parameter

In this section, we study the local bifurcation behavior of the positive equilibrium with respect to the delayed risk or capital intensity parameter $\phi > 0$, while keeping the remaining parameters fixed. For convenience, define:

$$k := \alpha d = \alpha(a - c)$$

The positive equilibrium exists if and only if $a - c > 0$.

Recall that the system admits the equilibria $E_0 = (0, 0)$ and:

$$E_1(\phi) = \left(\frac{d}{2b + \phi}, \frac{d}{2b + \phi} \right)$$

Since the Jacobian matrix at E_0 is independent of ϕ , the stability of the boundary equilibrium does not depend on ϕ . Therefore, any ϕ -driven local bifurcation can only occur at the positive equilibrium $E_1(\phi)$.

Next, write:

$$A(\phi) = 1 - \frac{2bk}{2b + \phi}, \quad D(\phi) = \frac{k\phi}{2b + \phi}$$

4.1. Critical values of the delayed risk parameter

Local bifurcations occur when an eigenvalue of $J(E_1(\phi))$ lies on the unit circle.

A flip (period-doubling) bifurcation occurs when $\lambda = -1$, or equivalently, when:

$$1 + A(\phi) + D(\phi) = 0$$

Solving this equation gives:

$$\phi_{PD} = \frac{2b(k - 2)}{k + 2} \quad (17)$$

Hence, $\phi_{PD} > 0$ if and only if $k > 2$.

A Neimark-Sacker bifurcation may occur when the eigenvalues form a complex conjugate pair on the unit circle, and for a planar map, this requires:

$$D(\phi) = 1 \text{ and } -2 < A(\phi) < 2$$

Solving $D(\phi) = 1$ yields:

$$\phi_{NS} = \frac{2b}{k - 1}$$

which is positive if and only if $k > 1$.

At $\phi = \phi_{NS}$, we have:

$$A(\phi_{NS}) = 1 - \frac{2bk}{2bk(k - 1)} = 2 - k \quad (18)$$

Therefore, the condition $-2 < A(\phi_{NS}) < 2$ becomes:

$$-2 < 2 - k < 2$$

which is equivalent to:

$$1 < k < 4.$$

Thus, a genuine Neimark–Sacker bifurcation can occur only when $1 < k < 4$.

A fold bifurcation would require $\lambda = 1$, equivalently:

$$1 - A(\phi) + D(\phi) = 0$$

However, this cannot occur because:

$$1 - A(\phi) + D(\phi) = k > 0$$

4.2. Main bifurcation result

Theorem 3. Assume that $d = a - c > 0$, so that the positive equilibrium $E_1(\phi)$ exists for all $\phi > 0$, and set $k = \alpha(a - c) > 0$.

- (i) If $0 < k \leq 1$, then $E_1(\phi)$ is locally asymptotically stable for all $\phi > 0$. Hence, no ϕ -driven local bifurcation occurs.
- (ii) If $1 < k < 2$, then $E_1(\phi)$ is locally asymptotically stable for:

$$0 < \phi < \phi_{NS}$$

and loses stability at $\phi = \phi_{NS}$ through a Neimark–Sacker bifurcation.

- (iii) If $2 < k < 4$, then $\phi_{PD} < \phi_{NS}$. In this case, $E_1(\phi)$ is locally asymptotically stable for:

$$\phi_{PD} < \phi < \phi_{NS}$$

It undergoes a flip bifurcation at $\phi = \phi_{PD}$, and then loses stability again at $\phi = \phi_{NS}$ through a Neimark–Sacker bifurcation.

- (iv) If $k \geq 4$, then $E_1(\phi)$ is unstable for all $\phi > 0$.

Proof. First, consider the function $D(\phi)$. This function is strictly increasing in ϕ , with $D(\phi) \rightarrow 0$ as $\phi \rightarrow 0^+$ and $D(\phi) \rightarrow k$ as $\phi \rightarrow \infty$. Therefore, the condition $D(\phi) < 1$ holds for all $\phi > 0$ when $0 < k \leq 1$, while for $k > 1$, it holds precisely on:

$$0 < \phi < \phi_{NS}$$

Next, define:

$$F(\phi) := 1 + A(\phi) + D(\phi) = 2 + \frac{k(\phi - 2b)}{2b + \phi}$$

At $\phi = 0$, we have $F(0) = 2 - k$, and as $\phi \rightarrow \infty$, $F(\phi) \rightarrow 2 + k > 0$. Hence, if $0 < k \leq 2$, then $F(\phi) > 0$ for all $\phi > 0$; if $k > 2$, then $F(\phi) = 0$ has a unique positive solution ϕ_{PD} given by **Equation 17**, and:

$$F(\phi) < 0 \quad \text{for } 0 < \phi < \phi_{PD}$$

and:

$$F(\phi) > 0 \quad \text{for } \phi > \phi_{PD}$$

Combining these observations gives the following cases.

If $0 < k \leq 1$, both nontrivial Jury conditions hold for all $\phi > 0$. Therefore, $E_1(\phi)$ is stable for all $\phi > 0$.

If $1 < k < 2$, then $F(\phi) > 0$ for all $\phi > 0$, while $D(\phi) < 1$ holds exactly for $0 < \phi < \phi_{NS}$. Thus $E_1(\phi)$ is stable for $0 < \phi < \phi_{NS}$. At $\phi = \phi_{NS}$, we have $D(\phi_{NS}) = 1$ and, by **Equation 18**,

$$A(\phi_{NS}) = 2 - k \in (0, 1)$$

Therefore, the eigenvalues form a complex conjugate pair on the unit circle. Hence, the loss of stability occurs through a Neimark–Sacker bifurcation.

If $2 < k < 4$, then $F(\phi) < 0$ for $0 < \phi < \phi_{PD}$ and

$F(\phi) > 0$ for $\phi > \phi_{PD}$, whereas $D(\phi) < 1$ holds for $0 < \phi < \phi_{NS}$. Since $\phi_{PD} < \phi_{NS}$ for $2 < k < 4$, the equilibrium is stable exactly on $\phi_{PD} < \phi < \phi_{NS}$.

At $\phi = \phi_{PD}$, the condition $1 + A(\phi) + D(\phi) = 0$ is equivalent to $\lambda = -1$, so a flip bifurcation occurs. At $\phi = \phi_{NS}$, we have:

$$A(\phi_{NS}) = 2 - k \in (-2, 0)$$

so the eigenvalues are a complex conjugate pair on the unit circle, and a Neimark–Sacker bifurcation occurs.

Finally, if $k \geq 4$, then at the candidate point $D(\phi) = 1$, we obtain:

$$A(\phi_{NS}) = 2 - k \leq -2$$

Therefore, the strict Neimark–Sacker condition fails. Moreover, when $k > 4$, the interval where $F(\phi) > 0$ lies strictly to the right of the interval where $D(\phi) < 1$, so the Jury conditions cannot be satisfied simultaneously. Hence, $E_1(\phi)$ is unstable for all $\phi > 0$. The case $k = 4$ is the degenerate boundary case where:

$$\phi_{PD} = \phi_{NS} = \frac{2b}{3}$$

Therefore, again, there is no open stability interval. This completes the proof. $\square$

Remark 1. The parameter ϕ has two distinct effects on this system. On the one hand, it lowers the equilibrium loan level:

$$L^*(\phi) = \frac{d}{2b + \phi}$$

Therefore, stronger delayed risk pressure reduces the steady-state loan volume. On the other hand, it also changes the local stability of the equilibrium through the effective adjustment parameter $k = \alpha(a - c)$. In particular, for $1 < k < 4$, increasing ϕ may destabilize the equilibrium via a Neimark–Sacker bifurcation, indicating the emergence of oscillatory or quasi-periodic loan dynamics.

5. Numerical simulations

In this section, we present the analytical results from numerical simulations of the discrete-time system given by **Equation 7**. Unless otherwise stated, the common economic parameters are taken as:

$$a = 8, \quad c = 4, \quad b = 1$$

with initial condition:

$$x_0 = 0.4, \quad y_0 = 0.3$$

The parameter values used in the numerical simulations were selected to illustrate the qualitative dynamics predicted by the analytical results rather than to calibrate the model for a specific banking institution. Because the model is formulated in a reduced-form, dimensionless setting, the parameters may be interpreted as normalized economic quantities. In particular, $a = 8$ represents the maximum attainable lending rate or gross return from loan demand, whereas $c = 4$ denotes the marginal operational and funding cost. Thus, $a > c$ ensures a positive net-lending margin and guarantees the existence of a positive equilibrium. The value $b = 1$

normalizes the sensitivity of the lending rate to loan expansion, such that a larger loan supply reduces the lending rate through the inverse demand function. With these choices, $a - c = 4$, which allows the effective adjustment parameter $k = \alpha(a - c)$ to enter different stability and bifurcation regimes for moderate values of α . Therefore, the selected baseline values provide a mathematically transparent setting for observing stable equilibrium, periodic oscillations, quasi-periodicity, and chaos while preserving the economic interpretation of positive profitability and downward-sloping loan demand.

For the one-parameter bifurcation diagrams, we used $N_{trans} = 1,500$ transient iterations and retained the next $N_{keep} = 250$ iterates. The largest Lyapunov exponent was computed after $N_{lyap,trans} = 1,000$ transient steps using $N_{lyap} = 3,000$ iterations. For the two-parameter isoperiodic diagram in the (α, ϕ) -plane, we use $\alpha \in [0.2, 0.8]$, $\phi \in [0.001, 6.0]$, and classify the asymptotic dynamics up to a period 20. For the basin plots, the initial conditions were taken from the square $[0, 2.5] \times [0, 2.5]$ using a 220×220 grid.

5.1. One-parameter bifurcation diagrams and Lyapunov exponents

We first present two representative one-parameter bifurcation scenarios with respect to the delayed risk parameter ϕ .

Figure 1A corresponds to the parameter choice $\alpha = 0.6$, so that $k = \alpha(a - c) = 2.4$ which lies in the interval $2 < k < 4$. This is the regime predicted by Theorem 3 to admit both flip and Neimark–Sacker bifurcations. The numerical bifurcation diagram confirmed this behavior. For small ϕ , the positive equilibrium is unstable, and the orbit approaches a period-two cycle. As ϕ increases and passes the red dashed line ϕ_{PD} , the system transitions through the flip boundary. Subsequently, an intermediate range of ϕ was observed to have stable equilibrium branches. As ϕ tends towards the blue dashed line ϕ_{NS} , the equilibrium becomes unstable, and oscillatory dynamics appear. The bifurcation diagram shows invariant-curve oscillations above the equilibrium point, followed by more complex nonlinear dynamics, including chaotic behavior.

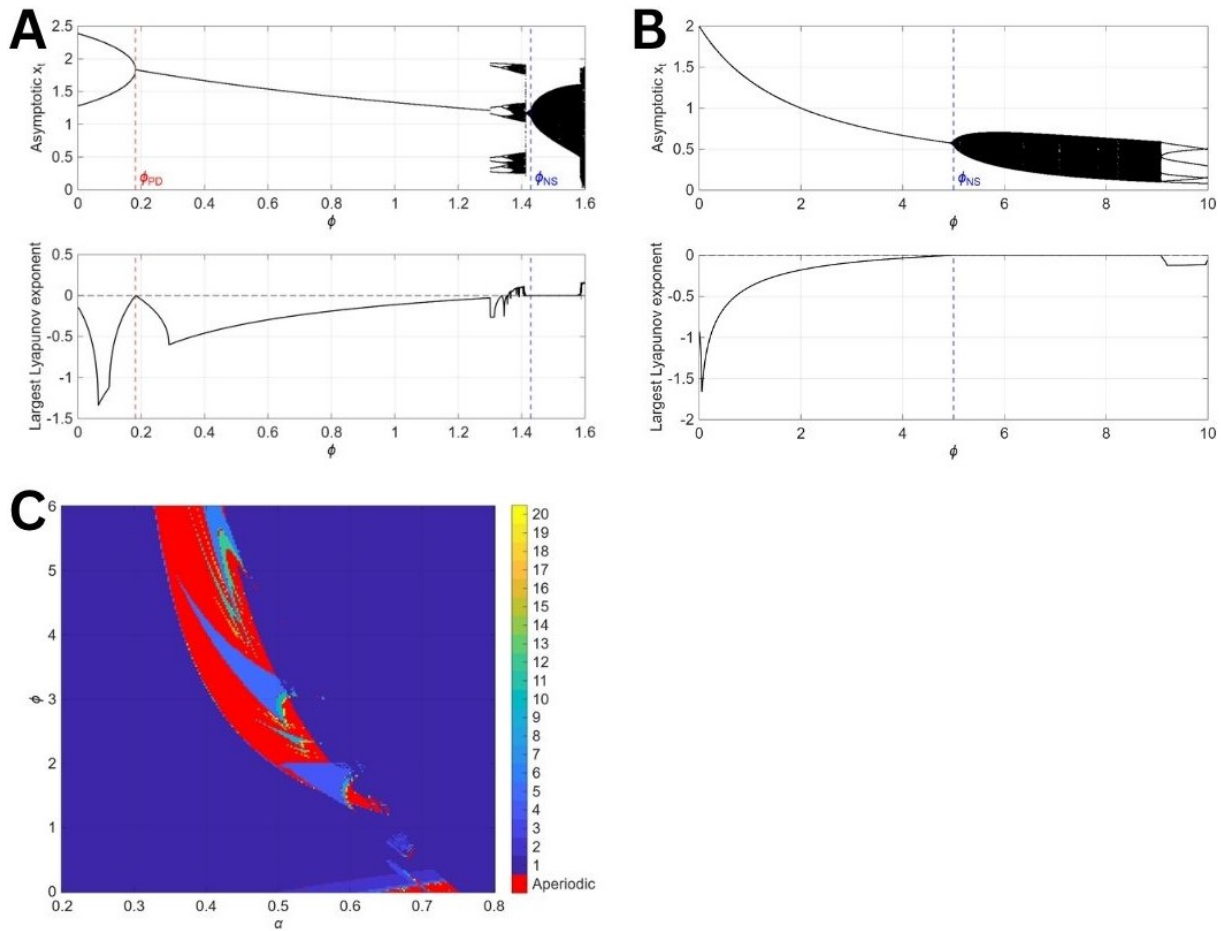


Figure 1. Numerical bifurcation structure of the model with respect to the delayed risk parameter ϕ and the adjustment speed α . Panels (A) and (B) show the bifurcation diagram of x_t with its corresponding largest Lyapunov exponent. Panel (A) uses $\alpha = 0.6$, showing both flip and Neimark–Sacker bifurcations. Panel (B) uses $\alpha = 0.35$, showing only a Neimark–Sacker bifurcation. Panel (C) presents a two-parameter isoperiodic diagram in the (α, ϕ) -plane. The red region corresponds to aperiodic dynamics, while the other colors represent periodic motions of different periods.

This interpretation is supported by the corresponding Lyapunov exponent in **Figure 1A**. The stable periodic and equilibrium regimes have negative values that tend towards zero near the bifurcation thresholds. A positive value was

observed in a portion of the right-hand region, which reflected chaotic dynamics. Therefore, the numerical results support the theoretical prediction that in $2 < k < 4$, the

system can undergo period doubling behavior, followed by Neimark–Sacker destabilization.

Figure 1B shows the case $\alpha = 0.35$, where $k = \alpha(a - c) = 1.4$, so that $1 < k < 2$. In this regime, the analysis predicted no flip bifurcation, and the equilibrium was lost only via a Neimark–Sacker bifurcation, as reflected in the bifurcation diagram. The equilibrium branch was stable at $\phi < \phi_{NS}$, and the critical point is denoted by the dashed blue line, after which a closed oscillatory band appears. The associated Lyapunov exponent was near zero post-transition, which is a characteristic of quasi-periodic dynamics on an invariant closed curve. Moreover, no period-doubling limit was observed.

Figure 1C summarizes the global structure of dynamics depicted by the two-parameter isoperiodic diagram. Based on this perspective, the diagram shows that the broad areas of periodic behavior were divided by aperiodic areas, represented in red. The dynamics at relatively small α are predominantly low-period, and as α increases, the aperiodic region grows, potentially corresponding to chaotic or quasi-periodic dynamics. This implies that the destabilizing impact of delayed risk increases with a faster adjustment speed, thereby motivating the development of complex dynamics. The tongue-like forms of various colors also showed that periodic windows of different orders were interwoven with more irregular regions, a characteristic feature of nonlinear discrete systems.

The isoperiodic structure offers valuable insights into the stability of lending policies. Regions characterized by low-period dynamics span substantial portions of the parameter space, suggesting that moderate adjustment speeds typically yield predictable lending patterns. Nevertheless, as the intensity of delayed risk rises, the boundaries between different dynamic regimes become more complex. As a result, even minor changes in parameters can lead to significant shifts in long-term outcomes. This sensitivity underscores the need to precisely evaluate the combined effects of credit risk, provisioning costs, and regulatory constraints when formulating lending strategies.

Analyzing the situation economically, the shift from a stable state to periodic and chaotic behavior suggests that instability might originate internally from the bank's adjustment processes. Without external macroeconomic disturbances, strong reactions to built-up risks can lead to ongoing variations in lending activities. These variations mirror the credit cycles observed in real financial systems, characterized by phases of rapid growth followed by periods of contraction. Consequently, the findings endorse the idea that internal feedback loops alone can cause significant volatility in credit markets.

Considering financial stability, the presence of extensive aperiodic zones suggests that being overly reactive to accumulated risks may unintentionally heighten instability rather than mitigate it. Although risk management strategies aim to protect banks from adverse shocks, overly aggressive adjustments can intensify internal volatility and lead to erratic lending behavior. Thus, achieving an appropriate balance between risk management and lending adaptability is crucial to sustaining stable credit markets. The isoperiodic diagram demonstrates that this equilibrium depends not only on the extent of delayed risk impacts but also on how swiftly banks adapt to shifts in profitability conditions.

5.2. Basins of attraction

We now examine how the long-run behavior depends on the initial conditions. For this purpose, we compute the basins

of attraction on the square $[0, 2.5] \times [0, 2.5]$ with $\alpha = 0.6$. The attractor in each case is overlaid on the corresponding basin plots.

In **Figure 2A**, we show the basin of attraction of the positive equilibrium for $\phi = 0.6$. The blue region occupies almost the entire computational domain, indicating that E_1^* is robust with respect to perturbations in the initial loan variables. Only a relatively small outer region corresponds to the other outcomes. This suggests that, at this parameter value, the system is strongly attracted to the steady state for a wide range of economically meaningful initial conditions. Next, in **Figure 2B**, we display the basin of attraction of an 8-period orbit for $\phi = 1.31$. In contrast to the equilibrium case, the basin is no longer simply connected in the non-equilibrium case. Instead, the purple region forms several narrow, curved bands distributed across the state space, indicating a greater reliance on the initial conditions. The periodic orbit is represented by eight yellow points. Additionally, the complex basin geometry proved that when the system reached a higher-period regime, long-run dynamics were significantly more sensitive to the initial configuration.

In **Figure 2C**, we present the basin of attraction of a chaotic attractor with $\phi = 1.4$. The red basin is torn and mixed within areas that give rise to bounded movements. The black overlaid set shows a chaotic attractor with an irregular geometry rather than finitely many isolated points. This confirms that the model produced chaotic dynamics around and above the Neimark–Sacker transition. Economically, the results imply that loan dynamics can exhibit chaotic behavior despite the absence of external random shocks.

Finally, **Figure 2D** shows the basin of attraction of a quasi-periodic orbit at $\phi = 1.5$. The orange basin occupies a major part of the domain, and the overlaid black closed curve is compatible with the movement of an invariant closed loop. The quasi-periodic basin was less fragmented than the chaotic case but still had nontrivial basin boundaries. This suggests that when equilibrium stability is lost, the system can either develop irregular chaos or regular oscillations with incommensurate frequencies, depending on the parameter values and the initial state.

Basin structures offer insights that cannot be derived from local stability analysis alone. While bifurcation theory pinpoints parameter values where stability shifts occur, basin analysis uncovers the practical probability of observing a specific attractor. This distinction is crucial in economic contexts, as banking systems are constantly influenced by market fluctuations, policy shifts, and managerial choices. A large basin of attraction suggests that the associated long-term behavior is relatively stable, whereas fragmented basins indicate increased sensitivity to initial conditions and external influences.

The intricate basin geometries across various parameter settings imply that predicting lending dynamics can be challenging, even when the economic environment seems stable. Notably, the presence of multiple bounded outcomes suggests that similar banking systems might follow different long-term paths despite having the same parameter values. These findings underscore the need to consider global dynamical properties in addition to local stability conditions. In practice, understanding the basin structure can aid in identifying operational regions where lending policies are less susceptible to unexpected shifts among stable, periodic, and chaotic regimes.

In contrast to earlier models of banking monopoly and duopoly,^{24–26,28,29} the current framework incorporates a lagged interaction between existing and previous loan

portfolios. This alteration leads to more complex dynamic behavior, as the effective marginal profitability is influenced by past exposure rather than by present lending choices alone. The findings suggest that the accumulation of risk over time can serve as an internal destabilizing factor. As a

result, macroprudential policies that focus solely on static equilibrium targets may be inadequate if the pace of adjustment is too rapid. Therefore, policymakers should take into account both the extent of risk exposure and the rate at which banks adapt to shifts in profitability conditions.

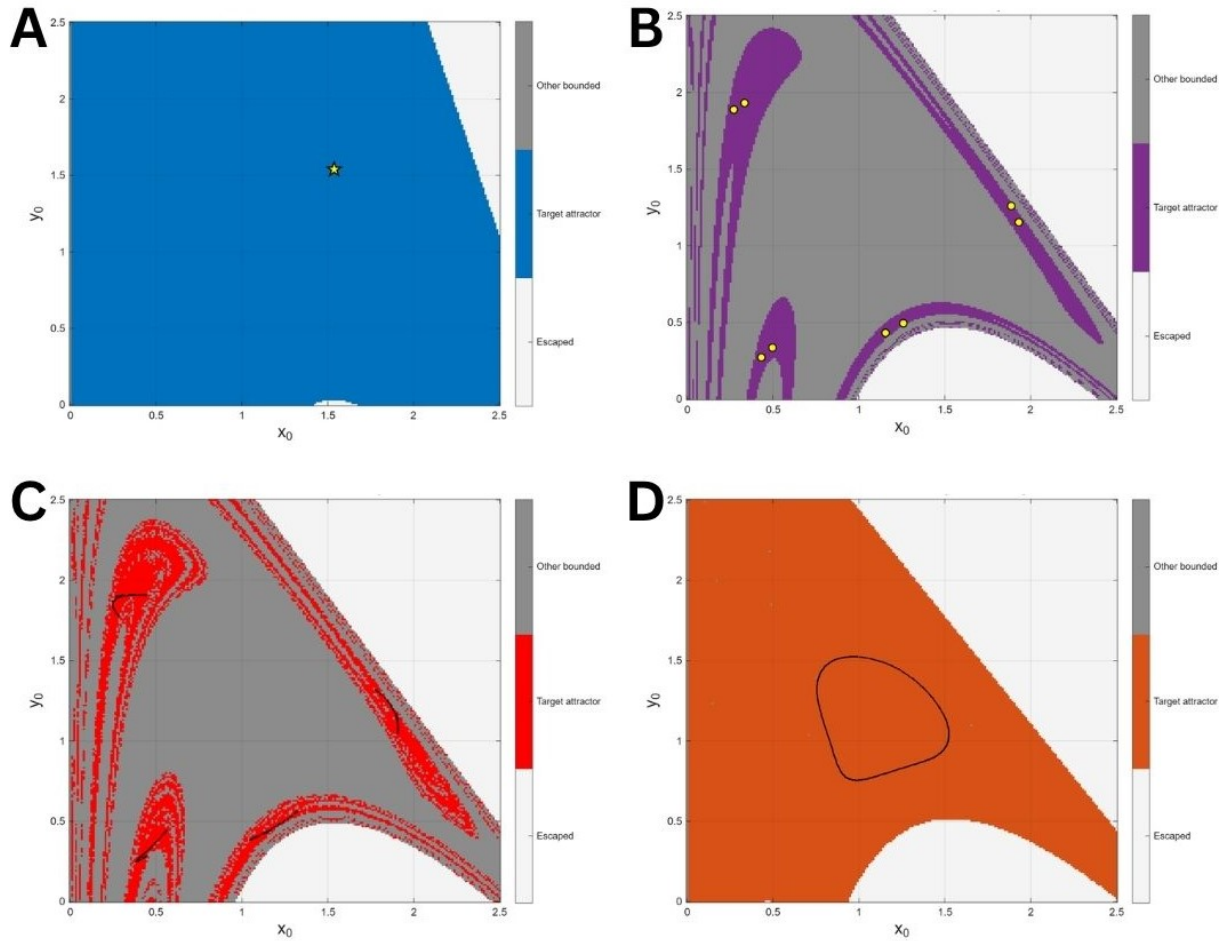


Figure 2. Basins of attraction for several asymptotic regimes of the model with $\alpha = 0.6$. The colored region indicates the basin of the target attractor, while the gray region corresponds to other bounded motions, and the light region represents escaped trajectories. (A) Basin of attraction of E_1^* when $\phi = 0.60$, (B) basin of attraction of 8-period when $\phi = 1.31$, (C) basin of attraction of chaotic when $\phi = 1.40$, (D) basin of attraction of quasi-periodic when $\phi = 1.50$.

5.3. Limitations

Although the model exhibited rich dynamic behavior, it faced several limitations. The current framework focuses exclusively on monopolistic banks and only considers loan dynamics. However, this has not yet been fully integrated to include other balance sheet items such as deposits, liquidity reserves, or capital accumulation. The delayed risk effect was modeled as a reduced-form interaction term. This was analytically convenient but stylized compared to the complexity of actual banking systems. The distinct simplifications did not diminish the significance of the study; instead, they facilitated the dynamic isolation of the role of delayed risk exposure and graphically showed how internal adjustment mechanisms tended to produce sustained fluctuations and instability, even without external random shocks.

6. Conclusion

In conclusion, this study created a discrete-time banking monopoly model with delayed risk effects, in which current

loan expansion is determined by contemporaneous profitability and the burden of past lending exposure. This formulation plays a significant role in describing how cumulative credit risk and regulatory pressure could be recycled into future lending behavior within a nonlinear framework. The analysis showed that the model has a boundary equilibrium and a positive equilibrium, and that stability was extremely sensitive to the interaction between the adjustment speed and the delayed risk parameter. The theoretical results provide clear criteria for local stability and demonstrate the potential occurrence of flip and Neimark-Sacker bifurcations. Numerical simulations supported these results by showing transitions from stable equilibrium behavior to periodic, quasi-periodic, and chaotic dynamics, as well as nontrivial basin structures.

Methodologically, this research illustrates the integration of delayed feedback mechanisms into bounded-rational banking models without losing analytical simplicity. By employing local stability analysis, bifurcation theory, and numerical investigation, the study offers a comprehensive

framework for understanding the impact of risk accumulation on lending dynamics.

Several directions can be considered for future research. For example, future studies should include additional state variables, such as deposits, equity, or regulatory capital, so that the model can fully capture the interactions between the balance sheet and state variables. Additionally, the strategic interaction of multiple banks in an oligopolistic environment, where lagged risk impacts competition and stability, should be examined in future studies. Other adjustment rules, time-varying regulatory strengths, and empirical calibrations using banking data should also be considered to assess the robustness of current bifurcation patterns under more realistic specifications. From a dynamical systems perspective, future research should investigate the model's global properties, including the characterization of global domains of attraction, the existence of global bifurcations, and the emergence of complex basin structures. Such analyses would provide a deeper understanding of the nonlinear mechanisms governing transitions between stable, periodic, quasi-periodic, and chaotic lending regimes and would complement the local stability and bifurcation results established in the present study.

In practical terms, the results showed that delayed risk exposure was regarded as both a fixed element of costs and a dynamic element that changed the stability of the lending process. Banks that responded too aggressively to the accumulated risk by modifying their loan portfolios inadvertently produced cyclical or non-cyclical credit flows. Therefore, the results can inform the formulation of lending policies or other macroprudential regulations that consider dynamic feedback effects rather than equilibrium targets. The present study offers a mathematical foundation for understanding how delayed risk processes affect the long-run conduct of bank lending, including the significance of prudential regulation of adjustment speed and risk intensity on financial stability.

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Author contributions

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